A Three-Valued Semantics for Negotiated Situation of Multi-Agent System Based on BATNA and WATNA

Yang Song[a] and Ken Satoh[b]
National Institute of Informatics, Japan

Keywords: Multi-Agent System, Negotiation, BATNA and WATNA, Gödel Logic, Modal Logic.

Abstract: Negotiation plays a crucial role in the dispute resolution systems. In the negotiation, the agents usually need to compromise with each other because their preferences are different. To provide the best or acceptable suggestion in the negotiation, BATNA (Best Alternative To a Negotiated Agreement) and WATNA (Worst Alternative To a Negotiated Agreement) can be a method to express the preference of each agent. In this paper, our aim is to formalize the negotiated situations of multi-agent system in a logical method based on BATNA and WATNA. We consider each given suggestion as a possible world in modal logic, and provide a 3-valued valuation based on Gödel logic to judge whether a suggestion is over the BATNA, below the WATNA, or between the BATNA and WATNA of each agent, which is to show whether the suggestion is acceptable, rejectable or undecided to the agent. Moreover, by using the modal operator we can check whether there exists a best or acceptable suggestion for all agents in a negotiated situation.

1 INTRODUCTION

Today, Online Dispute Resolution (ODR) became more important because it could reduce the costs to resolve the dispute and due to demand for non face-to-face negotiation. In such systems, negotiation plays a crucial role since the ultimate goal of the system is to provide a satisfactory solution among disputing agents.

Negotiation is also an important topic in multi-agent systems (Kraus, 1997), and therefore many studies are aimed to formalize the negotiated situation. Dunne (Dunne et al., 2005; Dunne, 2005) considered the negotiation as resource allocation between agents, and therefore provided the definition of resource allocation setting and the model of resource allocation in which several agents exchange resources. Ragone (Ragone et al., 2006) gave a logic-based framework to automate the one-shot bilateral negotiation considering the demand and preference of agents. Endriss (Endriss and Pacuit, 2006) developed a dynamic modal logic that can be used to model scenarios where agents negotiate over the allocation. Yang (Yang et al., 2018) forced on the system of personalized product supply chain and provided a multi-agent negotiation mechanism based on personalized index.

In this research, we want to provide logical semantics to express the negotiated situations of multi-agent systems in ODR. Here, we consider the negotiation as a selection of several suggestions among the agents. Normally, a lot of suggestions by deputing agents and neutral third party are suggested in the negotiation. One agent may be glad to accept a suggestion while the other agent rejects it because they have different preference, therefore sometimes agents need to compromise with each other. In this paper, we consider that not each suggestion is either accepted or rejected, but some of them are not decided at first. As a famous example of negotiation, the case **Two sisters arguing over an orange** (Follett, 2011) can be considered as obtaining four suggestions as follows:

- The elder sister has the whole orange while the younger sister has none (suggested by elder sister).
- The younger sister has the whole orange while the elder sister has none (suggested by younger sister).
- The orange is cut into half and each sister has half of the orange (they compromise with each other).
- The elder sister has the peel (for cooking) and the younger has the juice (for breakfast), as the best

[a] https://orcid.org/0009-0006-1020-130X
[b] https://orcid.org/0000-0002-9309-4602

Song, Y. and Satoh, K.
A Three-Valued Semantics for Negotiated Situation of Multi-Agent System Based on BATNA and WATNA.
DOI: 10.5220/0012394200003636
Paper published under CC license (CC BY-NC-ND 4.0)
In *Proceedings of the 16th International Conference on Agents and Artificial Intelligence (ICAART 2024) - Volume 1*, pages 333-340
ISBN: 978-989-758-680-4; ISSN: 2184-433X
Proceedings Copyright © 2024 by SCITEPRESS – Science and Technology Publications, Lda.

solution.

In this example, we can see that each sister wants to have the whole orange at first, which is their desire. If the fourth suggestion is not mentioned, they can accept the third suggestion as the result of compromise, otherwise they prefer the forth suggestion to solve this negotiation. Using BATNA and WATNA, we can explain this situation. The BATNA and WATNA are important concept in negotiation and the meanings are as follows (Notini, 2005):

- BATNA: Best Alternative To a Negotiated Agreement.
- WATNA: Worst Alternative To a Negotiated Agreement.

In this example, the BATNA of the elder sister is to have the peel for cooking, therefore she is glad to accept the first and forth suggestion. Also, the BATNA of the younger sister is to have the juice for breakfast, therefore she is glad to accept the second and forth suggestion. In this example, both of their WATNAs are to have half of the orange (peel or juice), therefore they can accept the third suggestion as a compromise, while rejecting the first or second one because it is below one sister's WATNA.

There are several studies that employ BATNA and WATNA to the ODR systems. Lodder (Lodder and Zelznikow, 2005) considered calculating BATNAs as the first step of their three-step model of the negotiation support systems. Andrade (Andrade et al., 2010) developed an architecture supported by a JADE platform based on BATNA and WATNA, while considering that they are useful to take into account when making or accepting a proposal. In this paper, we consider a logical method based on BATNA and WATNA, that provide a 3-valued valuation as follows:

- The value 1 means that the suggestion is over the BATNA, which will be accepted by the agent.
- The value 0.5 means that the suggestion is between the BATNA and the WATNA, which may be accepted or rejected by the agent.
- The value 0 means that the suggestion is below the WATNA, which must be rejected by the agent.

In this paper, our aim in this paper is not how to reach the solution, but to define what the negotiated point is under the assumption that every information is provided about each agent's desire, for example. We then try to consider how to reach such a negotiated point. We give a suggestion model like modal logic and show that we can use the modal operators to compare the suggestions. Moreover, we provide some formulas which can express the best suggestion and best acceptable suggestion of one agent and of a group. Also, we can express some features of negotiation by our logic.

The structure of the rest of the paper is as follows. In Section 2, we introduce the previous studies, Gödel propositional and modal logic as the technical background. In Section 3, we propose the syntax and semantics of our logic. We regard the symbols of agents as the atomic propositions informally, and then provide a 3-valued suggestion model. Moreover, we provide an explanation of the concrete example "two sisters arguing over an orange" by our semantics In Section 4, we consider other normal and dynamic operators to extend our semantics and give the axioms. Finally, in Section 5, we conclude and give some directions for future works.

2 PREVIOUS STUDY

2.1 Gödel 3-Valued Logic

In classical logic, the valuation is 2-valued, i.e., each formula is either true or false. t is natural for humans, however, that sometimes two values seem to be not enough, e.g., when considering paradoxes like the liar sentence.

Gödel provided a many-valued propositional logic with finite or infinite values between 0 and 1, where 0 and 1 are considered as false and true, respectively. Gödel 3-valued logic is the simplest Gödel logic whose valuation is $\{1, 0.5, 0\}$. The language is built over a countable set of propositional variables with binary connectives $\wedge, \vee, \rightarrow$ and constant $\bot$. $\top$ is defined as $\bot \rightarrow \bot$ and the negation $\neg\varphi$ is defined as $\varphi \rightarrow \bot$. In Gödel 3-valued logic, we have the following truth tables for the operators $\neg, \wedge, \vee$ and $\rightarrow$: (Robles, 2014)

A	$\neg A$
1	0
0.5	0
0	1

$A \wedge B$	1	0.5	0
1	1	0.5	0
0.5	0.5	0.5	0
0	0	0	0

$A \vee B$	1	0.5	0
1	1	1	1
0.5	1	0.5	0.5
0	1	0.5	0

$A \rightarrow B$	1	0.5	0
1	1	0.5	0
0.5	1	1	0
0	1	1	1

Gödel 3-valued logic is axiomatized by the following axioms and rules (Robles, 2014):

- A1. $A \rightarrow A$
- A2. $(A \wedge B) \rightarrow A$ and $(A \wedge B) \rightarrow B$
- A3. $A \rightarrow (A \vee B)$ and $B \rightarrow (A \vee B)$
- A4. $[A \wedge (B \vee C)] \rightarrow [(A \wedge B) \vee (A \wedge C)]$
- A5. $[(A \rightarrow B) \wedge A] \rightarrow B$
- A6. $A \rightarrow (B \rightarrow A)$

- A7. $(A \to \neg B) \to (B \to \neg A)$
- A8. $\neg(A \wedge B) \to (\neg A \vee \neg B)$
- A9. $\neg A \to (A \to B)$
- A10. $(A \vee \neg B) \vee (A \to B)$
- (Adj) From A and B, infer $A \wedge B$
- (MP) From A and $A \to B$, infer B
- (Trans) From $A \to B$ and $B \to C$, infer $A \to C$
- (CI$\wedge$) From $A \to B$ and $A \to C$, infer $A \to B \wedge C$
- (E$\vee$) From $A \to C$ and $B \to C$, infer $A \vee B \to C$

2.2 Gödel Modal Logic

In modal logic, we use the modal operators $\Box$ and $\Diamond$ to express the necessity and possibility. $\Box\varphi$ stands for "φ is necessary" while $\Diamond\varphi$ stands for "φ is possible". A Kripke modal $M = (S, R, V)$ is usually used for the semantics of modal logic, where S is a set of possible worlds (states), R is an arbitrary function, and V is the valuation function.

In Gödel modal logic, the semantics is similar with normal modal logic, where the frame may be a fuzzy or a crisp Kripke frame and the valuation is a Gödel many-valued valuation. Here, since our semantics is similar to the crisp model, we introduce the semantics of crisp Gödel modal logic as follows (Rodriguez and Vidal, 2021):

Definition 1 (Semantics)**.** A crisp Gödel Kripke model is a tuple (S, R, V), where S is a set of possible worlds, $R : S \times S \to \{0, 1\}$ is an arbitrary function, and $V : \mathsf{Prop} \times S \to [0, 1]$ (closed interval) is a Gödel valuation function where Prop is a non-empty set of propositions. We can extend the valuation V to interpretations I by the following conditions where $p \in \mathsf{Prop}$:

$$I(\bot, s) = 0$$
$$I(p, s) = V(p, s)$$
$$I(\neg\varphi, s) = \begin{cases} 1 & I(\varphi, s) = 0 \\ 0 & I(\varphi, s) > 0 \end{cases}$$
$$I(\varphi \wedge \psi, s) = min(I(\varphi, s), I(\psi, s)),$$
$$I(\varphi \vee \psi, s) = max(I(\varphi, s), I(\psi, s)),$$
$$I(\varphi \to \psi, s) = \begin{cases} 1 & I(\varphi, s) \leq I(\psi, s) \\ I(\psi, s) & Otherwise \end{cases}$$
$$I(\Box\varphi, s) = inf((I(\varphi, t) : sRt \text{ and } t \in S), 1),$$
$$I(\Diamond\varphi, s) = sup((I(\varphi, t) : sRt \text{ and } t \in S), 0),$$

The axioms for modal operators are shown as follows: (Rodriguez and Vidal, 2021)

- $K_\Box : \Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$
- $K_\Diamond : \Diamond(\varphi \vee \psi) \to (\Diamond\varphi \vee \Diamond\psi)$
- $T_\Box : \Box\varphi \to \varphi$
- $T_\Diamond : \varphi \to \Diamond\varphi$
- $4_\Box : \Box\varphi \to \Box\Box\varphi$
- $4_\Diamond : \Diamond\Diamond\varphi \to \Diamond\varphi$
- $B_1 : \varphi \to \Box\Diamond\varphi$
- $B_2 : \Diamond\Box\varphi \to \varphi$
- $5_1 : \Diamond\varphi \to \Box\Diamond\varphi$
- $5_2 : \Diamond\Box\varphi \to \Box\varphi$
- $D : \Diamond\top$

3 LANGUAGE AND SEMANTICS

3.1 Language

Usually, the symbol of agent is sometimes used as a subscript to combine the modal operator to the formula, however, the propositions are usually independent from agents. In this paper, we combine the agents and the atomic propositions, i.e., we use the symbol of agents as the propositions and then provide the language of our logic.

Definition 2 (Language)**.** The language L is written as follows in BNF:

$$L \ni \varphi ::= i|\bot|\varphi \wedge \varphi|\varphi \vee \varphi|\varphi \to \varphi|\Box\varphi|\Diamond\varphi$$

where $i \in Ag$ and Ag is a non-empty set.

We consider the meaning of the elements in syntax as follows where $i, j \in Ag$:

- i means that agent i accepts the suggestion.
- Ag is a set of propositions and each element $i \in Ag$ means that "agent i accepts the suggestion".
- $\bot$ can be considered as an agent that rejects all suggestions.
- $i \wedge j$ means that agent i and j accept the suggestion.
- $i \vee j$ means that either agent i or j accepts the suggestion.
- $i \to j$ means that if agent i accepts the suggestion, then agent j will also accept the suggestion.
- $\Box i$ means that agent i accepts all given suggestions.
- $\Diamond i$ means that agent i accepts some given suggestion.

Remark 1. In our research, the idea that using the symbol of agents as the propositions came from Ågotnes (Ågotnes et al., 2011). However, here we provide different readings.

Here, we define the negation as $\neg\varphi \equiv \varphi \to \bot$ and $\top \equiv \bot \to \bot$ as normal logic. Therefore, we can read the formula $\neg i$ and $\top$ as follows:

- $\neg i$ means that agent i rejects the suggestion.
- $\top$ can be considered as an agent that always accepts all suggestions.

Remark 2. It may seem strange that we don't define suggestions in the language. Actually, we consider the suggestions as possible worlds, therefore "agent i accepts the suggestion" is the same as "proposition i is valid in the possible world" in modal logic.

3.2 Semantics

In the negotiation, each agent will accept it, reject it or hesitate for a given suggestion, therefore we can provide a 3-valued valuation as follows

- The value 1 means that the suggestion is over the BATNA, which will be accepted by the agent.
- The value 0.5 means that the suggestion is between the BATNA and the WATNA, which may be accepted or rejected by the agent.
- The value 0 means that the suggestion is below the WATNA, which must be rejected by the agent.

Then, we provide our suggestion model based on Gödel 3-valued as follows:

Definition 3 (Suggestion model)**.** A suggestion model M is a pair (S,V), where S is a non-empty finite set of possible worlds, and $V : Ag \times S \to \{1, 0.5, 0\}$ is a 3-valued valuation.

Semantically speaking, we read the model as follows:

- the set of possible worlds S is the set of given suggestions in the system;
- the 3-valued valuation expresses whether each suggestion is over BATNA, between BATNA and WATNA or below WATNA of each agent as we showed before.

Definition 4 (Interpretation)**.** Given a suggestion model $M = (S,V)$, we can extend the valuation V to interpretations I by the following conditions:

$$
\begin{aligned}
&I(\bot,s)=0\\
&I(i,s)=V(i,s)\\
&I(\neg\varphi,s)=\begin{cases}1 & I(\varphi,s)=0\\ 0 & I(\varphi,s)\geq 0.5\end{cases}\\
&I(\varphi\wedge\psi,s)=min(I(\varphi,s),I(\psi,s)),\\
&I(\varphi\vee\psi,s)=max(I(\varphi,s),I(\psi,s)),\\
&I(\varphi\to\psi,s)=\begin{cases}1 & I(\varphi,s)\leq I(\psi,s)\\ I(\psi,s) & Otherwise\end{cases}\\
&I(\Box\varphi,s)=min(I(\varphi,t):t\in S),\\
&I(\Diamond\varphi,s)=max(I(\varphi,t):t\in S),
\end{aligned}
$$

Remark 3. In this research, we define the suggestion model considering the Kripke model, however, we don't define the relation R because we consider that all suggestions are shown to all agents, therefore they can judge every suggestion is over BATNA, between BATNA and WATNA, or below WATNA, and moreover they can compare them.

As we use a Gödel 3-valued valuation, there are some important differences from classical (modal) logic:

- Double negation cannot be removed. Actually, $\neg\neg i$ here means that agent i **may** accept the suggestion (suggestion s is over i's WATNA) because from semantics we can see $I(\neg\neg i,s)=1$ iff $I(\neg i,s)=0$ iff $I(i,s)\geq 0.5$.
- $\varphi\wedge\psi$ is not the same as $\neg(\neg\varphi\vee\neg\psi)$. One counterexample is that, assume $I(\varphi,s)=I(\psi,s)=0.5$, then we have $I(\varphi\wedge\psi,s)=0.5$ while $I(\neg(\neg\varphi\vee\neg\psi),s)=1$. Also, $\varphi\vee\psi$ and $\neg(\neg\varphi\wedge\neg\psi)$ are different and the counterexample is similar.
- $\Box\varphi$ is not the same as $\neg\Diamond\neg\varphi$. one counterexample is that, assume $S=\{s\}$ therefore $I(\Box\varphi,s)=I(\Diamond\varphi,s)=I(\varphi,s)$. Let $I(\varphi,s)=0.5$, then we have $I(\Box\varphi,s)=0.5$ while $I(\neg\Diamond\neg\varphi,s)=1$. Also, $\Diamond\varphi$ and $\neg\Box\neg\varphi$ are different and the counterexample is similar.

Remark 4. Actually, there exist other 3-valued logics, e.g., strong Kleene 3-valued logic, that can also be used as a 3-valued valuation. In our research, the main reason we choose Gödel three-valued logic in our semantics is that we consider that the formula $i\to i$ should always hold (it is not a tautology in strong Kleene logic).

From the interpretation, we can define the satisfaction relation $\models$ as model logic as follows:

$$M,s\models\varphi \quad \text{iff} \quad I(\varphi,s)=1$$

Also, we can give a weaker relation as:

$$M,s\models_w\varphi \quad \text{iff} \quad I(\varphi,s)\neq 0$$

As we remarked, we can see that $M,s\models_w\varphi$ iff $M,s\models\neg\neg\varphi$.

By this semantics, we can formalize some statements as follows:

- agent i will accept suggestion s (suggestion s in M is over i's BATNA): $M,s\models i$.
- agent i may accept suggestion s (suggestion s in M is over i's WATNA): $M,s\models_w i$.
- agent i may accept and may reject suggestion s (suggestion s in M is between i's BATNA and WATNA): $M,s\not\models i$ and $M,s\not\models\neg i$.
- group G will accept suggestion s (suggestion s is over the group G's BATNA): $M,s\models\bigwedge_{i\in G} i$.

Moreover, we can define $M \models \varphi$ and $\models \varphi$ as follows:

$M \models \varphi$ iff for all $s \in S$ that $M,s \models \varphi$

$\models \varphi$ iff for all M that $M \models \varphi$.

By using such definition we can express the feature of the all given suggestions in model M as follows:

- agent i will accept each given suggestion (all suggestions in M are over i's BATNA): $M \models i$.
- agent i may accept each given suggestion (all suggestions in M are over i's WATNA): $M \models_w i$.
- agent i will accept at least one suggestion s (there exists some suggestion that over i's BATNA): $M \models \Diamond i$.
- the features for a group of agents are similar as above.

3.3 Best Suggestion and Best Acceptable Suggestion

One of the advantage of our logic is that, using our semantics, we can define the best suggestion and best acceptable suggestion by formulas as follows:

Definition 5 (Best suggestion)**.** In a suggestion model, if $M,s \models \Diamond i \to i$, then suggestion s is (one of) the best suggestion for agent i in model M. And for a group $G \subseteq Ag$, if $M,s \models \bigwedge_{i \in G}(\Diamond i \to i)$, then suggestion s is (one of) the best suggestion for group G in model M.

We explain why we define the above semantically. $M,s \models \Diamond i \to i$ means that $I(\Diamond i \to i, s) = 1$, which holds if and only if $I(\Diamond i,s) \leq I(i,s)$. $I(\Diamond i,s)$ stands for the biggest value of agent i among the set of suggestions S, therefore $I(\Diamond i,s) \leq I(i,s)$ means that there doesn't exist a better suggestion than s, thus s is considered as (one of) the best suggestion. The consideration of the best suggestion for group G is similar.

However, sometimes even the best suggestion will also be rejected by the agent since it may be below the WATNA of this agent. This case occurs when $I(\Diamond i,s) = 0$, in other words, all of the suggestions are below the WATNA therefore every suggestion is the best suggestion and would be rejected. To avoid this case, we can define the best acceptable suggestion as follows:

Definition 6 (Best acceptable suggestion)**.** In a suggestion model, if $M,s \models \Diamond i \to i \wedge \neg\neg i$, then suggestion s is the best acceptable suggestion for agent i in model M. And for a group $G \subseteq Ag$, if $M,s \models \bigwedge_{i \in G}(\Diamond i \to i \wedge \neg\neg i)$, then suggestion s is the acceptable best suggestion for group G i in model M.

Here, we add the condition $\neg\neg i$ to express the acceptable. The reason is that we read $M,s \models \neg\neg i$ as "agent i may accept the suggestion s" in our semantics.

There are some properties for the best suggestion and best acceptable suggestion.

- A best acceptable suggestion is also a best suggestion for an agent or a group. It is easy to see from the two definitions.
- In every model, there always exists at least one best suggestion for each agent, while there may not exist one for a group. The reason is that by the semantics there always exist $s \in S$ that for one agent i: $I(i,s) = I(\Diamond i,s)$, while a group of agents may not have the same best suggestion.
- In every model, there may exist more than one best acceptable suggestion and may not exist one for an agent or a group. The counter case is that we showed above if $I(\Diamond i,s) = 0$.
- If $I(i,s) = 1$ $(M,s \models i)$, then suggestion s must be one of the best (acceptable) suggestions for agent i.
- If $I(i,s) = 0$ $(M,s \models \neg i)$, then suggestion s cannot be the best acceptable suggestion for agent i (but may be the best suggestion).

Similar to the definition of best suggestion, we can also define the worst suggestion as follows:

Definition 7 (Worst suggestion)**.** In a suggestion model, if $M,s \models i \to \Box i$, then suggestion s is (one of) the worst suggestion for agent i in model M. And for a group $G \subseteq Ag$, if $M,s \models \bigwedge_{i \in G}(i \to \Box i)$, then suggestion s is (one of) the worst suggestion for group G i in model M.

The reason for this definition is similar since $I(\Box i,s)$ stands for the smallest value of agent i among the set of suggestions S.

We can give some meaningful formula by syntax to express the property of the negotiation as follows:

- $\not\models i \vee \neg i$: a suggestion is not either accepted or rejected, because it may be not decided(0.5).
- $\neg(i \wedge \neg i)$: a suggestion can not be accepted and rejected at the same time.
- $(\Diamond(i \wedge \neg j) \wedge \Diamond(\neg i \wedge j) \wedge \Diamond((\Diamond i \to i) \wedge (\Diamond j \to j))) \to \Diamond(i \wedge j)$: we can read it as "if there exists a suggestion that over i's BATNA while below j's WATNA, and there exists a suggestion that below i's WATNA while over j's BATNA, and there exists a best suggestion for group i and j, then there exists a suggestion over both i and j's BATNA".
- Also, we can prove that these formulas hold in all suggestion models.

3.4 Axiomatization

As a special case of Gödel modal logic, the axiomatization and proof theory can be easily inferred by the proof of general form (Caicedo and Rodríguez, 2015). Therefore, we don't show the soundness and completeness in this paper. Instead, we provide the new readings of some axioms of $S5$ Gödel crisp model as follows, which are to show that our interpretation is a proper reading for Gödel modal logic.

- The $K_\Box$ axiom $\Box(i \to j) \to (\Box i \to \Box j)$ can be considered as " if agent i accepts a suggestion then j will also accept it for all suggestion, then if i accepts all suggestions then j will also accept all suggestions".
- The $K_\Diamond$ axiom $\Diamond(i \vee j) \to (\Diamond i \vee \Diamond j)$ can be considered as " if there exists a suggestion either agent i or j accepts, then there exists a suggestion that i will accept or there exists a suggestion that j will accept".
- The $T_\Box$ axiom $\Box i \to i$ can be considered as "if agent i accepts all given suggestions, then i will accept this suggestion".
- The $T_\Diamond$ axiom $i \to \Diamond i$ can be considered as "if agent i accepts this suggestions, then i will accept some given suggestion".

3.5 Example

Here, we use a concrete example, **two sisters arguing over an orange** that we have shown in Section 1, to explain our semantics. We mention the four given suggestion again as the possible worlds in the model as follows:

- s_1: The elder sister has the whole orange while the younger sister has none.
- s_2: The younger sister has the whole orange while the elder sister has none.
- s_3: Each sister has half of the orange.
- s_4: The elder sister has the peel while the younger sister has the juice.

We consider that there are three steps in this negotiation.

- Firstly, both sisters want to have the whole orange and suggestions s_1 and s_2 are suggested. We write the model M_1 in Figure 1:

  Here, e stands for the agent "elder sister" and y stands for the agent "younger sister". For suggestion s_1 is over e's BATNA and below y's WATNA, we have $V(e,s_1) = 1$ and $V(y,s_1) = 0$. Also, since suggestion s_2 is over y's BATNA and below e's WATNA, we have $V(e,s_2) = 0$ and $V(y,s_2) = 1$. We can see that $M_1 \not\models \Diamond(e \wedge y)$ and $M_1 \not\models_w \Diamond(e \wedge y)$, which means that there doesn't exist a suggestion that is over both sisters' BATNAs or WATNAs, therefore they cannot get a solution in the first step. Also, in model M_1, $M_1 \not\models \Diamond((\Diamond e \to e) \wedge (\Diamond y \to y))$, which means that there is no best suggestion for both sisters at the first step.

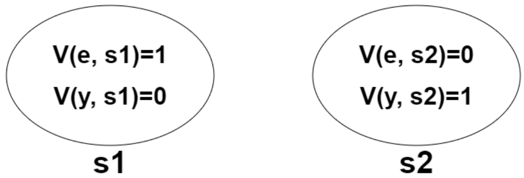

Figure 1: First model: M_1.

- Secondly, sisters notice that they need a compromise thus suggestion s_3 is provided. The second model is shown in Figure 2.

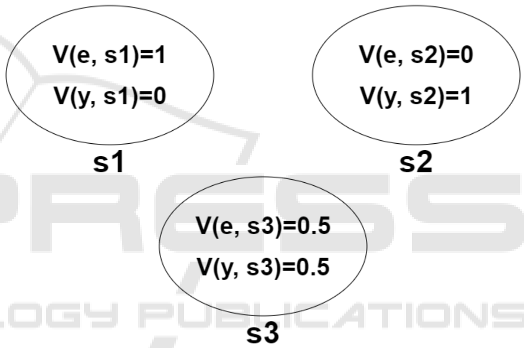

Figure 2: Second model: M_2.

  Since suggestion s_3 is between both sisters' BATNAs and WATNAs, we have $V(e,s_3) = V(y,s_3) = 0.5$. In model M_2, we have $M_2 \not\models \Diamond(e \wedge y)$ but $M_2 \models_w \Diamond(e \wedge y)$, which mean that there doesn't exist a suggestion that over sisters' BATNAs but exists one over their WATNAs in the second step. Also, $M_2 \not\models \Diamond((\Diamond e \to e) \wedge (\Diamond y \to y))$, which means that there is no best suggestion for two sisters.

- Finally, sisters notice the BATNA of each other therefore s_4 is suggested. The third model is shown in Figure 3.

  Since suggestion s_4 is over(just) both sisters' BATNA, we have $V(e,s_4) = V(y,s_4) = 1$. In model M_3, we have $M_3 \models_w \Diamond(e \wedge y)$, which means that there exists a suggestion over both sisters' BATNA in the final step. Also, $M_3 \models_w \Diamond((\Diamond e \to e) \wedge (\Diamond y \to y))$, which means that there exists a best suggestion for both sisters.

SCITEPRESS SCIENCE AND TECHNOLOGY PUBLICATIONS

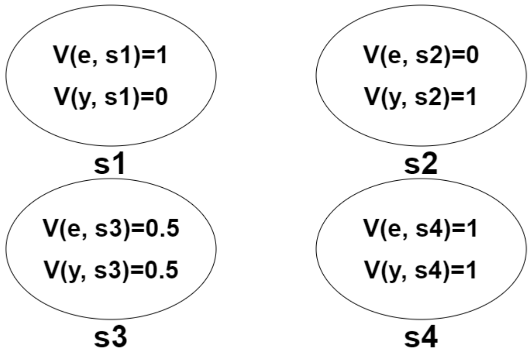

Figure 3: Third model: M_3.

4 NEW OPERATORS

In Section 3.2, we formalize the statement "agent i may accept or reject suggestion s" as $M,s \not\models i$ and $M,s \not\models \neg i$ by semantics. However, we cannot express this statement by syntax using the operators $\neg, \wedge, \vee, \to$. Therefore, we want to add a new operator to express such suggestions.

Since the negation $\neg$ is not symmetrical, a natural idea is to define a new negation $\sim$ as a dual of $\sim$.

$$I(\sim\varphi, s) = \begin{cases} 1 & I(\varphi, s) \leq 0.5 \\ 0 & I(\varphi, s) = 1 \end{cases}$$

Remark 5. Actually, Baaz (Baaz, 1996) gave this definition of negation operator already in the last of the paper in Gödel infinite-valued logic. However, the main he studied was the operator $\triangle$ which is defined as follows:

$$I(\triangle\varphi, s) = \begin{cases} 1 & I(\varphi, s) = 1 \\ 0 & Otherwise \end{cases}$$

It is easy to see that $\triangle\varphi \equiv \neg\sim\varphi$ and $\sim\varphi \equiv \neg\triangle\varphi$. The reason we don't use operator $\triangle$ is, from the semantics we can see that $M,s \models \varphi$ iff $M,s \models \triangle\varphi$, thus we cannot distinguish the readings of two formulas in syntax since both i and $\triangle i$ should be read as "agent i will accept the suggestion". However, since we can use $\triangle$ and $\neg$ to express the operator $\sim$, later we will provide the axioms of $\sim$ from the axioms of $\triangle$ in that paper.

By this definition, we read the $\sim i$ as "agent i may reject the suggestion (the suggestion is below the BATNA of agent i)". And therefore, we can express the statement "agent i may accept or reject suggestion" by formula $\neg\neg i \wedge \sim i$.

Also, we have some axioms of $\sim$ which comes from the axioms of $\triangle$ (Preining, 2010; Bílková et al., 2022):

- $\neg\sim\varphi \vee \neg\neg\sim\varphi$
- $\neg\sim(\varphi \vee \psi) \to (\neg\sim\varphi \vee \neg\sim\psi)$
- $\neg\sim(\varphi \to \psi) \to (\neg\sim\varphi \to \neg\sim\psi)$
- $\neg\sim\varphi \to \varphi$
- $\neg\sim\varphi \to \neg\sim\neg\sim\varphi$
- From φ infer $\neg\sim\varphi$

Also, since we consider the Gödel 3-valued logic, there exists other rule for example:

$$\text{From } \sim(\varphi \to \psi) \text{ and } \sim(\psi \to \chi), \text{ infer } \varphi$$

5 CONCLUSION AND FUTURE WORK

In this paper, we provided a 3-valued logical semantics to express the negotiated situation based. We divide the suggestions to three –must be accepted, must be rejected, may be accepted while may be rejected– by comparing it with BATNA and WATNA of each agent. We provided our semantics by using the suggestion model like the Kripke model, where we regard the possible worlds as suggestions and propositional variables as names of agents. Then, we formalized some statements of negotiation in our semantics and syntax, e.g., the best suggestion, and gave an example to show how we define a negotiated situation by our semantics. Later, we provide some new operators to express the statement that cannot be shown by normal operator, and finally, we showed that our semantics can express some negotiated states and situations by considering a concrete example.

Unfortunately, in this research, our semantics is based on Gödel modal logic and the axioms of the operators have been already studied. However, in this paper, we provide an informal reading of modal logic therefore we show that negotiated states can be shown by the basic frame which can be considered as an $S5$ Kripke frame. Therefore, we gave a new perspective to the study of negotiation and the multi-agent system of modal logic.

Other future works remained as follows:

- In this paper, we use the 3-valued valuation based on BATNA and WATNA. One of the future directions is to consider the valuation as the evaluation of suggestions from 0 to 1 and to give the bound of BATNA and WATNA. For example, if we define the bound of WATNA as 0.3 and that of BATNA is 0.8, suggestions with value 0.5 are considered to be between BATNA and WATNA, while those with value 0.9 are considered to be over BATNA. In this paper, we compare the bound of BATNA/WATNA and the suggestion as the pre-work and

SCITEPRESS SCIENCE AND TECHNOLOGY PUBLICATIONS

then give the 3-valued valuation. Since Gödel logic can have finite or infinite values, it is able to use a many-valued valuation (from 0 to 1) in the suggestion model.

- In our suggestion model $M = (S,V)$, we do not use the accessibility relation as Kripke model because R doesn't work since we consider only one suggestion can be selected and each suggestion can be compared with each other. If we can select more than two suggestions and combine them as a solution, we are able to use the relations to express whether two suggestions can be selected together or not. For example, we can use $s_1 \cup s_2$ shows the combine suggestion that includes s_1 and s_2, and we can define the satisfied relation as:
  - $M, s_1 \cup s_2 \models \bot$ iff $s_1 R s_2$
  - $M, s_1 \cup s_2 \models \varphi$ iff ($M, s_1 \models \varphi$ or $M, s_2 \models \varphi$) and not $s_1 R s_2$.
- In this paper, we employed our 3-valued semantics to express the negotiated situations in the ODR system. Actually, using this semantics we can show other situations, e.g., the strategy in game theory. If an agent benefits much from a strategy, then the agent would be glad to execute it; If an agent loses much from a strategy, then the agent would not execute it; and if an agent benefits or loses little from a strategy, the agent may hesitate whether to execute it or not. We can see the consideration is quite similar with our semantics based on BATNA and WATNA. In this case, since not all strategies can be noticed by every agent, we may need to add different relations for each agent as epistemic logic.

ACKNOWLEDGEMENTS

We would like to thank Takahiro Sawasaki for the helpful comments. We are also grateful to Wachara Fungwacharakorn and Kanae Tsushima for the useful discussions. Finally, but not the least, we would also like to thank the referees for their careful reading and helpful comments. This research is supported by AIP challenge.

REFERENCES

Ågotnes, T., van der Hoek, W., and Wooldridge, M. (2011). On the logic of preference and judgment aggregation. *Autonomous Agents and Multi-Agent Systems*, 22:4–30.

Andrade, F., Novais, P., Carneiro, D., Zeleznikow, J., and Neves, J. (2010). Using BATNAs and WATNAs in online dispute resolution. In *New Frontiers in Artificial Intelligence: JSAI-isAI 2009 Workshops, LENLS, JURISIN, KCSD, LLLL, Tokyo, Japan, November 19-20, 2009, Revised Selected Papers 1*, pages 5–18. Springer.

Baaz, M. (1996). Infinite-valued Gödel logics with 0-1-projections and relativizations. In *Gödel'96: Logical foundations of mathematics, computer science and physics—Kurt Gödel's legacy, Brno, Czech Republic, August 1996, proceedings*, volume 6, pages 23–34. Association for Symbolic Logic.

Bílková, M., Frittella, S., and Kozhemiachenko, D. (2022). Paraconsistent Gödel modal logic. In *International Joint Conference on Automated Reasoning*, pages 429–448. Springer.

Caicedo, X. and Rodríguez, R. O. (2015). Bi-modal Gödel logic over [0, 1]-valued kripke frames. *Journal of Logic and Computation*, 25(1):37–55.

Dunne, P. E. (2005). Extremal behaviour in multiagent contract negotiation. *Journal of Artificial Intelligence Research*, 23:41–78.

Dunne, P. E., Wooldridge, M., and Laurence, M. (2005). The complexity of contract negotiation. *Artificial Intelligence*, 164(1-2):23–46.

Endriss, U. and Pacuit, E. (2006). Modal logics of negotiation and preference. In *European Workshop on Logics in Artificial Intelligence*, pages 138–150. Springer.

Follett, M. P. (2011). Constructive conflict. *Sociology of Organizations: Structures and Relationships*, 417.

Kraus, S. (1997). Negotiation and cooperation in multi-agent environments. *Artificial intelligence*, 94(1-2):79–97.

Lodder, A. R. and Zelznikow, J. (2005). Developing an online dispute resolution environment: Dialogue tools and negotiation support systems in a three-step model. *Harv. Negot. L. Rev.*, 10:287.

Notini, J. (2005). Effective alternatives analysis in mediation:"BATNA/WATNA" analysis demystified. *URL: https://www. mediate. com/articles/notini1. cfm.[01/2022]*.

Preining, N. (2010). Gödel logics–a survey. In *International Conference on Logic for Programming Artificial Intelligence and Reasoning*, pages 30–51. Springer.

Ragone, A., Di Noia, T., Di Sciascio, E., and Donini, F. M. (2006). A logic-based framework to compute pareto agreements in one-shot bilateral negotiation. *FRONTIERS IN ARTIFICIAL INTELLIGENCE AND APPLICATIONS*, 141:230.

Robles, G. (2014). A simple Henkin-style completeness proof for Gödel 3-valued logic G3. *Logic and Logical Philosophy*, 23(4):371–390.

Rodriguez, R. O. and Vidal, A. (2021). Axiomatization of crisp Gödel modal logic. *Studia Logica*, 109(2):367–395.

Yang, C., Xu, T., Yang, R., and Li, Y. (2018). Multi-agent single-objective negotiation mechanism of personalized product supply chain based on personalized index. *Advances in Mechanical Engineering*, 10(10):1687814018795785.