Heuristic Methods for the Antenna-Constrained Beam Layout Optimization on Multibeam Broadcasting Mission

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Abstract: In this paper, we tackle a payload design problem for a broadcasting mission where a telecommunication satellite must provide television services to distinct regions defined as polygons. To cover these polygons, several telecommunication beams are emitted by the satellite, with the risk that they mutually degrade their performance while also being hard to accommodate mechanically on the spacecraft. The problem is to determine a set of non-conflicting beams that cover all the regions and optimize a performance metric related to the sizes of the beams used. The first method is a matheuristic exploiting iterative solv of an ILP model. The second method, called the merge-and-split heuristic, is inspired by Iterated Local Search and reuses a fast graph coloring algorithm to analyze conflicts among selected beams. These two methods are evaluated on realistic instances, the largest one involving more than one hundred regions to cover.

1 INTRODUCTION

Telecommunication satellites are major assets used by operators to provide coverage in remote areas at a lower cost than terrestrial networks. As operators require an increasing quantity and quality of the services they provide, satellites must optimize their onboard resources. Therefore, satellite manufacturers need the support of optimization techniques to help them find payload design solutions that reach the mission requirements while complying with the full set of operational and design constraints. One of these solutions consists in using multiple beams to increase the capacity without expanding the frequency spectrum, as the same frequency band can be used by several beams. Moreover, using multiple beams requires using dedicated antenna technologies such as Single-Feed-Per-Beam (SFPB), which is cost-effective and can reach high-performance radio frequencies. However, SFPB introduces beam layout constraints. Each beam is created by a so-called feed horn (or feed, see Fig. 1), whose size and shape can be computed beforehand given the target region to be served, and each beam is allocated to a certain reflector (parabolic antenna available onboard). As shown in Fig. 2, one issue is that it can turn out that the positions required for two feeds on the same reflector are not compatible with each other for geometrical reasons if the target regions are too close.

To optimize the design of telecommunication satellites while considering beam layout constraints, several approaches have been studied. (Kyrgiazos et al., 2013) explores non-uniform beam sizes and bandwidth allocation among beams to maximize inter-beam distance. (Camino, 2017) incorporates antenna constraints into a non-uniform beam layout, leveraging graph coloring techniques for beam-to-reflector allocation. A patent by (Hammill and Dishaw, 2004) presents a method for generating non-uniform beams based on population density within specific polygons. (Contardo and Hertz, 2021) proposes an exact algorithm to cover polygons with a set of disks, but do not address the beam-to-reflector allocation problem.

In this paper, we introduce two heuristic meth-

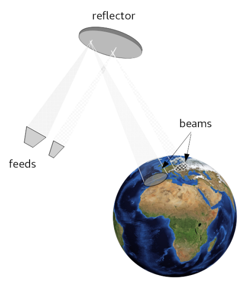

Figure 1: Beams of different sizes and their associated feeds.

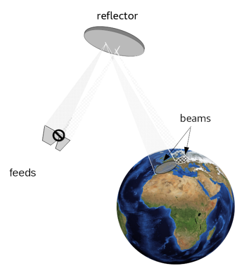

Figure 2: A conflict between two feeds.

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ods for beam layout design in SFPB antennas, aiming to fulfill the requirements of telecommunication missions. Our primary objective is to provide service to as many requested regions as possible while ensuring a satisfactory quality of service. In this decomposed optimization scheme where the beam layout is the first step, the quality of service cannot be accurately evaluated at this stage where important design steps subsequent to the beam layout optimization are missing (frequency allocation, definition of all payload routes, TV channels assignment, amplifiers sizing, power distribution, etc.). However, we can still create beams having a small size to maximize the power spectral densities and enhance signal quality for ground receivers. In the end, our goal is to select small beams that cover as many regions as possible, while preventing feed conflict assignments among beams allocated to the same reflector.

The rest of the paper is organized as follows: Section 2 presents the formal problem definition using Integer Linear Programming (ILP). Section 3 introduces a matheuristic approach that iteratively enlarges a set of candidate beams. Section 4 describes a heuristic algorithm called merge-and-split. Section 5 provides experimental results on various instances. Finally, Section 6 concludes the paper and outlines potential future developments.

2 PROBLEM DEFINITION

2.1 Telecommunication Mission

The mission is defined by a set of regions on the Earth's surface. The geometrical definition of a region is a polygon defined by the set of successive vertices placed on its boundary, and each region has its own demand expressed in terms of number of television channels. The segmentation of the market in geographical areas can be made for linguistic reasons or because of the economical context. In the following, the set of polygons to cover is referred to as P.

2.2 Beams

General Beams. We can define a beam as a disk that covers a specific area on the Earth's surface served by the satellite communication system. More formally, a beam is a pair $b = (c_b, r_b)$ where c_b is the center of the beam (defined by a longitude and a latitude) and r_b is the radius of the disk associated with the beam, expressed in degree angle. For signal quality reasons, the radius of all the beams considered must not be greater than a maximum value referred to as $MaxRadius$.

Smallest Covering Beams. In the following, we consider specific kinds of beams that are directly defined by the set of polygons they cover. Using the Welzl algorithm (Welzl, 2005), we can find the smallest beam $b = (c_b, r_b)$ covering a subset $P' \subseteq P$ in $O(n)$ time. Here, n is the number of polygons in P'. The Smallest Covering Beam for a set $P' \subseteq P$ is denoted as $SCB(P')$, and the covered polygons are denoted as P_b.

One combinatorial challenge arises: if a beam can be defined from any set $P' \subseteq P$, then there are $2^{|P|}$ candidate beams.

Conflicts Between Beams. As mentioned in the introduction, the SFPB antenna is characterized by a limited set of *reflectors*, usually three or four. Each reflector is associated with a cluster of feeds, as shown in Figure 2. Each feed is the hardware equipment associated with a single beam, and the reflector focuses the signal and reflects the beam on the Earth's surface.

Two beams whose disks are close on the Earth's surface and that are allocated to the same reflector can create a conflict on their associated feeds. To prevent conflicts, a minimum separation distance is imposed on the Earth's surface. Formally, for two beams $b_1 = (c_{b_1}, r_{b_1})$ and $b_2 = (c_{b_2}, r_{b_2})$ assigned to the same reflector, the $\text{CONFLICT}(.,.)$ predicate is defined as follows:

$$\text{CONFLICT}(b_1, b_2) : dist(c_{b_1}, c_{b_2}) < \kappa(r_{b_1} + r_{b_2}) \quad (1)$$

Here, $dist(c, c')$ is the Euclidean distance between points c and c' in the longitude-latitude plane, and $\kappa > 1$ is a fixed parameter determined by the satellite manufacturer. In this context, $\kappa = \sqrt{3}$ is considered, reflecting recent observations in related satellite manufacturing activities.

From an operations research perspective, the beam separation constraints can be represented using a graph coloring problem. Indeed, let B_s denote the set of beams selected. We can build a graph $G(B_s)$ called the feed conflict graph, containing one node per beam in B_s and one edge between two nodes associated with beams b_1, b_2 such that $\text{CONFLICT}(b_1, b_2)$ takes value true. Then, a set of beams is mechanically implementable if the chromatic number of $G(B_s)$, referred to as $\gamma(G(B_s))$, is less than or equal to the number of reflectors, so that the beams can be distributed among the different reflectors.

2.3 Input Data

For a particular beam layout problem, we consider the following input data:

- $P = \{p_1, p_2, \dots, p_{N_P}\}$: set of polygons to cover;
- $R = \{1, 2, \dots, N_R\}$: set of reflector indices;
- $B = \{b_1, b_2, \dots, b_{N_B}\}$: set of candidate beams; the radius of all these candidate beams is assumed to be consistent with parameter *MaxRadius*;
- $I \subset B \times B$: pairs of beams which cannot be assigned to the same reflector (incompatible beams).

Moreover, for every $p \in P$, we denote as $B_p \subseteq B$ the set of beams in B that cover polygon p, that is the set of beams $b \in B$ such that $p \in P_b$.

2.4 ILP Definition

Defining a solution of the beam layout problem means (1) selecting a subset of the candidate beams to cover polygons, and (2) associating a reflector with each selected beam so that the reflector-to-beam allocation is feasible. Such a problem can be formalized as an Integer Linear Program using the following variables:

- $z_p \in \{0,1\}, p \in P$: binary variable taking value 1 if and only if polygon p is covered by a selected beam;
- $x_{b,r} \in \{0,1\}, b \in B, r \in R$: binary variable taking value 1 if and only if beam b is allocated to reflector r.

The ILP model proposed is given in Equations 2 to 5.

$$\text{maximize} \quad M \cdot \sum_{p \in P} z_p - \frac{1}{|B|} \sum_{b \in B, r \in R} r_b^2 x_{b,r} \tag{2}$$

$$\text{subject to} \quad \forall p \in P, \sum_{(b,r) \in B_p \times R} x_{b,r} \geq z_p \tag{3}$$

$$\forall p \in P, \forall b \in B_p, \sum_{r \in R} x_{b,r} \leq z_p \tag{4}$$

$$\forall r \in R, \forall (b_1, b_2) \in I, x_{b_1,r} + x_{b_2,r} \leq 1 \tag{5}$$

The objective function given in Equation 2 tries to both maximize the number of polygons covered (term $\sum_{p \in P} z_p$) and minimize the size of the beams used (term $-\frac{1}{|B|} \sum_{b \in B, r \in R} r_b^2 x_{b,r}$). To express that the main goal is to maximize the coverage of the polygons, a large constant $M = MaxRadius^2$ is used to weight the first term, which leads to a lexicographic objective function. Constraint 3 ensures that if a polygon is covered, then at least one of its covering beams is selected. Constraint 4 ensures that if a polygon p is not covered, then none of the beams covering p is selected. As z_p is a binary variable, it also states that a beam is associated with at most one reflector. Last, Constraint 5 expresses that two conflicting beams cannot use the same reflector.

2.5 Complexity

While we plan to prove the NP-hardness of the beam layout problem considered, we can refer to closely related problems from the literature that are already known to be NP-hard. One notable similarity can be observed with various NP-complete covering and packing problems (Fowler et al., 1981). For example, the 3-colorable unit disk covering problem involves finding a set of unit disks and assigning a distinct color to each disk such that the union of the disks covers all given points, and overlapping disks have different colors. This problem is proven NP-hard in (Biedl et al., 2021). In another direction, the beam layout problem addressed in (Camino, 2017) considers a set of points to be covered by beams, with each point having a traffic demand. The objective is to find a beam layout, including the beam-to-reflector assignment, that maximizes the total traffic. This problem is proven NP-hard through a polynomial reduction from the Circle-Covering problem. It is important to note that our problem differs in that we aim to maximize the number of polygons covered by beams of small sizes, rather than maximizing the total traffic associated with a set of covered points.

3 MATHEURISTIC METHOD

As mentioned before, enumerating all candidate beams is exponential in the number of polygons and is not practicable. For example, for an instance involving only 20 polygons, there can be up to 1048576 possible subsets of polygons and as many potential beams. This is why we propose a matheuristic method that solves the ILP several times, on a restricted pool of candidate beams B that evolves during the iterations. Iterations are performed until a solution covering all the polygons is found or until a maximum CPU time is reached. Also, one of the modelling issues in the ILP presented in Section 2.4 is the fractional part of the criteria in Equation 2 ($\frac{1}{|B|} \sum_{b \in B, r \in R} r_b^2 x_{b,r}$), referred as MSRS for Mean Squared Radius Sum in this paper. The latter is fractional and difficult to implement, we will then use the following criterion in this first method:

$$\text{maximize} \quad M \cdot \sum_{p \in P} z_p - \sum_{b \in B, r \in R} r_b^2 x_{b,r}$$

with $M = \sum_{b \in B} r_b^2 + 1$

3.1 Detailed Description

The matheuristic is provided in Algorithm 1. Initially, the set of candidate beams B only contains all the smallest beams covering a single polygon in P and all the smallest beams covering two polygons in P. Based on these candidate beams, it is possible to compute the set I containing the pairs of incompatible beams and to solve the ILP model given before for input data P, R, B, I. In Algorithm 1, this is achieved through a call to a function solveILP, that returns the set of beams B_s selected in the solution of the ILP presented in Section 2.4. More formally, if $\bar{y}$ denotes the value of a variable y after optimization, then $B_s \leftarrow \text{solveILP}(P,R,B,I)$ is equivalent to $B_s \leftarrow \{b \in B \,|\, \sum_{r\in R} \overline{x_{b,r}} = 1\}$. Altogether, the beams in B_s cover a set of polygons $P_s = \cup_{b\in B_s} P_b$. The algorithm iteratively improves the current solution by generating a set of new beams to expand the set B while there are uncovered polygons and available computation time. It updates beam incompatibilities and solves the enlarged ILP model, potentially refining the beam selection strategy. The algorithm returns the last beam selection found and, if there's a maximum CPU time for each solveILP call, it can also return the best solution across iterations based on the objective function provided in Equation 2. Notably, in Algorithm 1, cpuTime() denotes the current computation time, and *TimeLim* represents the global CPU time limit.

```
Input: P: set of polygons to cover; R: set of
       reflector indices
B ← initBeams(P);
I ← initIncomp(B);
B_s ← solveILP(P,R,B,I) ;
P_s ← ∪_{b∈B_s} P_b;
while P_s ≠ P and cpuTime() ≤ TimeLim do
    B_g ← generateNewBeams(B_s, P);
    I_g ← generateNewIncomp(B_g, B);
    (B,I) ← (B ∪ B_g, I ∪ I_g);
    B_s ← solveILP(P,R,B,I) ;
    P_s ← ∪_{b∈B_s} P_b
end
return B_s
```

Algorithm 1: Pseudocode of the matheuristic.

3.2 Beam Generation Heuristic

In the proposed matheuristic method, the main challenge lies in devising an effective strategy for generating beams that can be added to the current pool. Several approaches have been tested, but in this paragraph we only discuss the one that yielded the best results thus far. The method involves enlarging the pool of beams with new beams that cover subsets of polygons with increasing cardinality during each iteration. To achieve this, we introduce a parameter called *NbPolyLimit*. Initially, we set *NbPolyLimit* to 2, enabling the generation of beams that individually cover one or two polygons. In each iteration, solving the Integer Linear Programming (ILP) problem produces a set of selected beams denoted as B_s. If B_s does not cover all the polygons, we increment *NbPolyLimit* by one unit and generate beams b by merging two beams from the solution. The resulting beams b cover at most *NbPolyLimit* polygons and are added to the pool. Formally, we compute

$$B_g = \{b = \text{SCB}(P_{b_1} \cup P_{b_2}) \,|\, (b_1, b_2 \in B_s) \wedge (r_b \leq MaxRadius) \wedge (|P_b| \leq NbPolyLimit)\} \tag{6}$$

4 MERGE-AND-SPLIT

The ILP manipulated in the previous sections has limitations on instances containing more than 100 polygons. To overcome this limitation, we define a specific heuristic that is independent of the ILP formalization, called the merge-and-split heuristic.

4.1 Global Description

Globally, the merge-and-split heuristic is inspired from Iterated Local Search (Lourenço et al., 2003), which alternates between optimization phases where local moves are performed to try and improve the current solution (*beam merging* moves in our case), and a perturbation phase where the features of the current solution are randomly updated (*split* operations in our case). The merging phase consists in replacing two beams b_1 and b_2 by the smallest beam covering all the polygons covered by b_1 and b_2. The selection of the beams involves randomness to diversify the exploration of the search space. Beam merging operations aim at providing a new conflict graph that is easier to color than the current graph and are performed until reaching a colorable graph or until a maximum number of merging operations is reached. At that point starts the splitting process of some beams, where splitting a beam b covering a set of polygons P_b means replacing b with the set of individual beams $\{SCB(\{p\}) \,|\, p \in P_b\}$ covering each polygon served by b. Each time a feasible solution has been found, it is evaluated according to the MSRS criteria. We fix a total time limit, and we return the best solution found at the end of the process. The merge-and-split heuristic proposed is described in Figure 3.

It starts from a set of beams B containing the smallest covering beam associated with each polygon in P, i.e. $B = \{SCB(\{p\}) \mid p \in P\}$. By definition, this set of beams covers all the polygons in P. In the following, we detail the three main components of the algorithm, that is the coloring, merging, and split procedures.

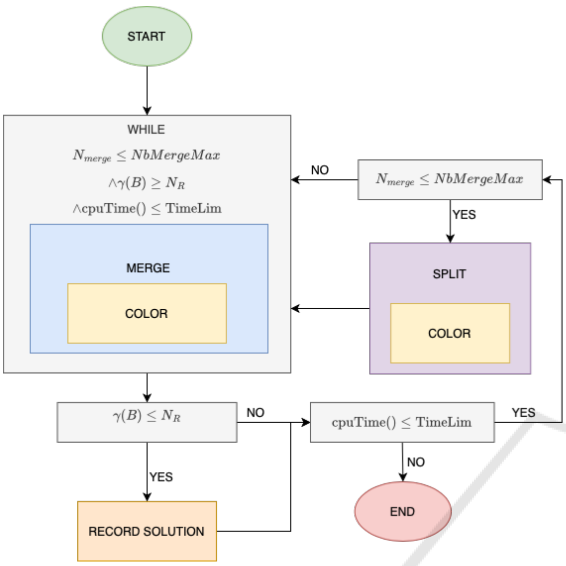


Figure 3: Merge-and-split algorithm.

4.2 Coloring Method

Each time the current set of beams B is updated, a function needs to color the corresponding feed conflict graph. This function is called many times during the algorithm and needs to be fast, even if determining whether a graph can be colored using a restricted number of colors is NP-complete. Therefore, we decided to reuse DSATUR (Brélaz,1979). DSATUR is a greedy algorithm that colors nodes with the highest degree first. Each node is assigned the lowest feasible color, considering its neighboring nodes' colors. If DSATUR manages to color the graph using no more than N_R colors, a consistent beam-to-reflector allocation exists. We denote the number of colors used by DSATUR as $\hat{\gamma}(G(B))$, which serves as an upper bound on the actual chromatic number $\gamma(G(B))$.

4.3 Merging Mechanisms

The pseudocode of the merging process is sketched in Algorithm 2. As expressed in the condition of the while loop, merging operations are performed while the chromatic number of current feed conflict graph $G(B)$ exceeds N_R and there is some computation time left and the number of merge operations does not exceed a limit referred to as $NbMergeMax$ [1].

At each step, the merging loop selects a candidate pair of beams (b_1, b_2) in $Cand$, according to a merging method randomly chosen (more details later on this point). The merged beam is accepted if and only if it is feasible according to parameter $MaxRadius$, but also if the estimated chromatic number of $G(B')$ with $B' \leftarrow (B \setminus \{b_1, b_2\}) \cup \{b_3\}$ is lower than or equal to the estimated chromatic number of the feed conflict graph $G(B)$ (i.e., $\hat{\gamma}(G(B')) \leq \hat{\gamma}(G(B))$). Indeed, merging two beams can also create new edges in the feed conflict graph. As illustrated in Figure 4, beam 10 created by merging beams 3 and 4 has a conflict with beam 9 that was not in conflict with the two beams merged. Finally, the set of candidate beams $Cand$ is updated to avoid selecting several times the same pair of beams, and the number of merging operations is incremented.

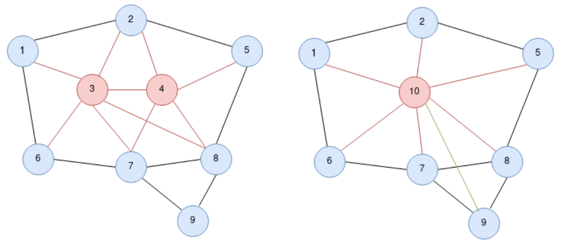
Input: B: set of beams in the current solution
$NbMerge \leftarrow 0$
$Cand \leftarrow \{(b_1, b_2) \mid b_1, b_2 \in B, b_1 \prec b_2\}$
while $\hat{\gamma}(G(B)) > N_R$ *and*
  $Cand \neq \emptyset$ *and*
  $cpuTime() \leq TimeLim$ *and*
  $NbMerge \leq NbMergeMax$ **do**
  $m \leftarrow$ select merging method$(ProbMergMethod)$
  $(b_1, b_2) \leftarrow$ select a pair in $Cand$ given m
  $Cand \leftarrow Cand \setminus \{(b_1, b_2)\}$
  $b_3 \leftarrow SCB(P_{b_1} \cup P_{b_2})$
  if $r_{b_3} \leq MaxRadius$ **then**
    $B' \leftarrow B \cup \{b_3\} \setminus \{b_1, b_2\}$
    if $\hat{\gamma}(G(B')) \leq \hat{\gamma}(G(B))$ **then**
      $B \leftarrow B'$
      $Cand \leftarrow Cand \cup \{(b_3, b) \mid b \in B' \setminus \{b_3\}\}$
      $NbMerge \leftarrow NbMerge + 1$
    end
  end
end

Algorithm 2: Merging loop.

In the following, we define three heuristics to select the two beams b_1, b_2 to merge at each step.

Merging Method 1 (M1). The first merging heuristic favors the creation of small beams. To diversify the search process, we select a pair (b_1, b_2) leading to a merged beam b_3 that is among the $\lceil |B|^2 \times RatioSelectedBeams \rceil$ smallest ones. Here,

[1] As we only merge beams two by two, the highest number of merging operations is always $|P| - 1$. We consider a smaller value for $NbMergeMax$ in order to favor the exploration of solutions containing a larger number of beams



(a) Before merging. (b) After merging.

Figure 4: Impact of beam merging on the feed conflict graph.

RatioSelectedBeams $\in]0,1]$ is a parameter that allows us to control the number of candidate pairs considered at each step, that is the degree of diversification.

Merging Method 2 (M2). The second merging heuristic favors the selection of two beams b_1, b_2 that have the highest number of common neighbors in graph $G(B)$. As illustrated in Figure 4b, the underlying idea is to make the coloring of these common neighbors easier. In Figure 4, beams 3 and 4 have four common neighbors, the merging of this pair of beams deletes 10 edges and creates 7 ones, reducing the total number of edges in the graph. Similarly to **M1**, in order to diversify the search process, merging method M2 randomly selects a pair of beams among the $\lceil |B|^2 \times \mathit{RatioSelectedBeams} \rceil$ highest quality ones.

Merging Method 3 (M3). This third method first selects a beam b for which the index of the color assigned by DSATUR is strictly greater than the number of reflectors available. This beam is then merged with a beam b' that is selected following either a rule inspired from method M1, or a rule inspired from method M2, or a rule that merges b with its closest neighbor. The method for selecting the second beam is chosen randomly among these three methods.

4.4 Splitting Mechanisms

The splitting procedure selects $\lceil \beta \times |B| \rceil$ beams in B, with a selection according to a probability distribution that is proportional to the beam size. The procedure can be called in two different cases:

- DSATUR manages to color the current set of beams using no more than N_R colors, then $\beta = \beta_m$ and should be low enough to try and improve the current solution and not restart all over again, but not too low to favor the exploration of other parts of the search space
- *NbMergeMax* has been reached, then, $\beta = \beta_M$ and should be higher in order to try and remove inconsistencies in the current solution.

4.5 Parameters

In the end, the merge-and-split heuristic uses several parameters. TimeLim is the maximum duration of the process; *NbMergeMax* is the maximum number of merging operations performed before splitting some beams; $\mathit{ProbMergMethod} = [p_{M1}, p_{M2}, p_{M3}]$ is the selection probability for each merging method; *RatioSelectedBeams* is the proportion of beam pairs among which the beam merging method selects a good alternative; β_m is the ratio of beams to split when the current solution is colorable using no more than N_R colors; β_M is the ratio of beams to split when the current solution is not feasible.

5 EXPERIMENTS

Experimental Setup. The matheuristic and merge-and-split methods have been implemented in Python. The ILP problems have been solved using CPLEX 12.10. The runs were made on a server with 96 cores of an Intel(R) Xeon(R) Gold 5318Y CPU @2.10GHz processor and 62GB of RAM. CPLEX exploits all 96 cores, while the merge-and-split heuristic runs on a unique core. This difference must be considered in the analysis of the experimental results, but for the heuristic our goal is to build a fast method anyway. We consider a telecommunication satellite having $N_R = 4$ reflectors. The methods were tested on six instances, corresponding to sets of polygons P mapped on France, Italy, West Europe, and Central Europe. The last two instances correspond to the Central Europe instance split into two parts that are called Central Europe Part1 and Central Europe Part2. If we covered each of the polygons of the four instances using one beam per polygon, we would obtain a feed conflict graph $G_0 = (B_0, E_0)$ whose features are given in Table 1.

Table 1: Features of the instances.

Instance	Notation	$\lvert P \rvert$	$\lvert E_0 \rvert$	$\gamma(G_0)$
France	F	22	144	10
Italy	I	30	128	8
West Europe	W	92	467	9
Central Europe	C	103	893	12
C Part 1	C1	52	294	10
C Part 2	C2	51	402	11

To evaluate the efficiency of each algorithm, we analyze the number of polygons covered by the solution found and the number of beams selected in this solution. We also analyze the Mean Squared Radius Sum given by

$$\mathrm{MSRS} = \frac{\sum_{b \in B_s} r_b^2}{|B_s|}$$

where B_s stands for the set of beams selected in the final solution. For the merge-and-split heuristic, we use 0.2 for β_m; 0.8 for β_M; $[0.2, 0.7, 0.1]$ for *ProbMergMethod* and finally 0.2 for *RatioSelectedBeams*.

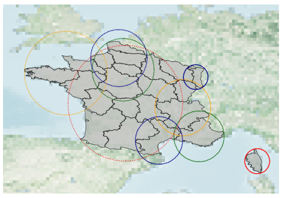


Figure 5: Matheuristic result on France.

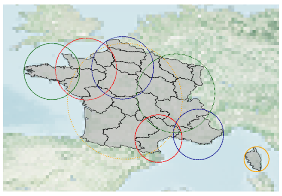


Figure 6: Merge-and-split heuristic on France.

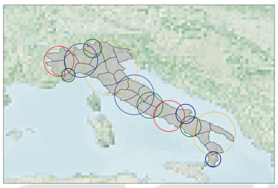


Figure 7: Matheuristic result on Italy.

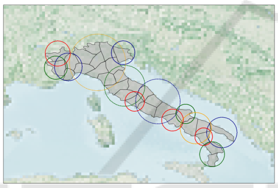


Figure 8: Merge-and-split heuristic on Italy.

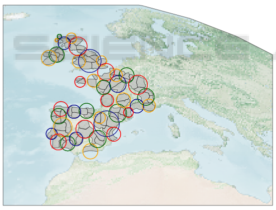


Figure 9: Matheuristic result on West Europe.

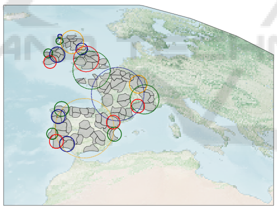


Figure 10: Merge-and-split heuristic on West Europe.

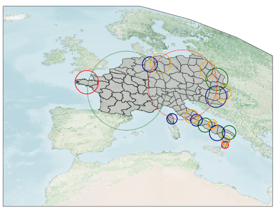


Figure 11: Matheuristic result on Central Europe.

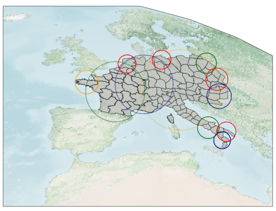
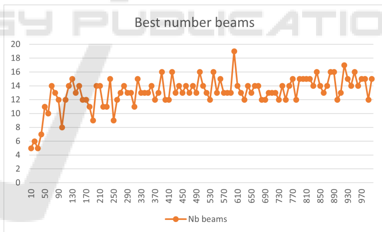


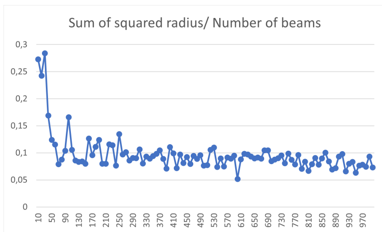
Figure 12: Merge-and-split heuristic on Central Europe.

Results of the Matheuristic. The solutions found by the matheuristic method are illustrated in Figures 5, 7, 9, 11, depicting the best sets of beams found, each beam being colored according to the index of the reflector to which it is assigned. From an operational point of view, the solutions appear to be of good quality for the telecommunication satellite designers. The detailed results of the matheuristic are given in Table 2. In this table, we indicate the time limits of the matheuristic method and the time limits specified for the call to the ILP solver at each iteration (column *ILP time limit*). This parameter is set manually following preliminary experiments. It is also worth noting that the matheuristic method manages to find good solutions on all instances, but can have a long computational time on the instance containing more than 100 polygons, where the time limit is increased to 1000 seconds.

Results of the Merge-and-Split Heuristic. Examples of solutions found by the merge-and-split heuristic are given in Figures 6, 8, 10, 12. The detailed results are given in Table 2. Even if the solutions found cover all polygons using a number of beams that is rather low given the number of polygons to be covered, the merge-and-split heuristic finds several feasible solutions during the process. Figure 13 displays the evolution of the best solution's quality for the *West Europe* instance. Various *TimeLimit* values ranging from 10s to 1000s (in increments of 10s) are considered. The number of selected beams (Figure 13a) and the mean squared radius sum (Figure 13b) represent the best values obtained for each run. The results indicate that the improvement of both criteria becomes less significant as the total time limit is increased.



(a) Number of beams.



(b) Squared radius sum.

Figure 13: Impact of the time limit on the performance of the merge-and-split heuristic (time limit on the x-axis).

Comparison Between the Two Methods. Given the time limit to consider for each instance, the matheuristic method outperforms the merge-and-split

Table 2: Results of the matheuristic and the merge-and-split heuristic.

Instance name	Time limit	Matheuristic results					Merge-and-split results				
		ILP time limit	Nb it.	Best solution			*NbMergeMax*	Nb sol.	Best solution		
				$\|P_s\|$	$\|B_s\|$	*MSRS*			$\|P_s\|$	$\|B_s\|$	*MSRS*
F	100	10	7	22/22	7	0.062	5	8	22/22	7	0.069
I	10	5	3	30/30	12	0.017	5	88	30/30	15	0.027
W	200	50	3	92/92	37	0.022	20	14	92/92	21	0.49
C	1000	300	7	103/103	18	0.069	50	5	103/103	12	0.12
C1	50	10	4	52/52	19	0.037	5	15	52/52	18	0.38
C2	50	10	2	51/51	6	0.13	5	1	51/51	8	0.10

heuristic in terms of number of beams selected and mean squared radius sum, except on instance C2. However, the merge-and-split heuristic finds several solutions covering all polygons, while the matheuristic takes a few iterations to cover all polygons. The heuristic method considers MSRS directly, while the matheuristic focuses on SRS, as the formulation of MSRS is not directly linear. We can show that the matheuristic improves the SRS criterion from iteration 2 while the MSRS is improved from iteration 2 to 3 on the Central Europe instance but then increases until the end of the process.

6 CONCLUSION

In this paper, we considered a problem associated with the design of a telecommunication satellite used for television broadcasting on regions, for which a set of beams of different sizes must be defined to cover a set of polygons, considering antenna mechanical constraints represented as a graph coloring problem. To face the combinatorial issues and find solutions that are industrially feasible, we proposed two different methods. The first one is a matheuristic method that is built upon an ILP formulation and uses an evolving pool of candidate beams until finding a feasible solution. This matheuristic produces good-quality solutions, but can require a long computational time. The second method, called the merge-and-split heuristic, iteratively constructs the beam layout by updating a set of beams step-by-step through local merging operations. This second method is faster and robust to large scale instances. Several perspectives can be listed for this work. For the matheuristic approach, we could design other methods to fill the pool with relevant beams, and some unused beams could be deleted to alleviate the ILP model. Moreover, to handle the mean squared radius sum in the matheuristic instead of the squared radius sum, we could use Linear-fractional Programming. We could also try other criteria, such as minimize the maximum radius or raise the radius to higher exponents. For the merge-and-split heuristic, we could parallelize the process to benefit from all cores and other merging methods should be looked for, in particular to better identify the reasons for the non-colorability at each merging step. A last perspective is to define a hybrid method where the matheuristic and the merge-and-split heuristic could share solutions or sets of relevant beams.

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