Stochastic Single-Allocation Hub Location Routing Problem for the Design of Intra-City Express Systems

Yuehui Wu[1][a], Hui Fang[1], Ali Gul Qureshi[2][b] and Tadashi Yamada[2][c]
[1]College of Transportation Engineering, Dalian Maritime University, Dalian, 116026, Liaoning, China
[2]Department of Urban Management, Graduate School of Engineering, Kyoto University, Kyoto, 615-8246, Japan

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Abstract: The paper concentrates on designing an intra-city express system in a practical environment. In the target networks, flows of parcels are exchanged between branch offices via a less-than-truckload hub-and-spoke network in a stochastic environment. Hub and vehicle capacities are considered, and the flows between all pairs of branch offices are assumed to be stochastic variables. The problem is modelled as a multi-stage recourse model, named capacitated single-allocation hub location routing problem with stochastic demands (CSAHLRPSD). A sample average approximation (SAA) framework is proposed, in which two variants of adaptive large neighbourhood search algorithms are used to solve the SAA problem and to calculate the recourse cost. The SAA framework is tested on benchmark instances, proving that it can efficiently deal with the CSAHLRPSD. Also, the results indicate that employing the CSAHLRPSD can cut the operation cost in comparison with the deterministic model in the practical and stochastic environment.

1 INTRODUCTION

Express service network design is significant in urban logistics management as it can help reduce operation costs and improve service levels. With the development of e-commerce, intra-city express has become an increasingly essential segment in urban logistics systems. As a result, various cargo companies are offering “delivery within the same day in the city service”, “next day delivery service”, or “delivery within 24 hours service”, e.g., SF Express, Yamato Transport, Japan Post, and so on. For these companies, how to satisfy the intra-city express requests in a practical environment via a cost-efficient way arises as an important issue. Moreover, this issue is also significant for the urban management department, as the delivery of intra-city expresses has caused various social problems, e.g., traffic jams, air pollution, and so on (Zhao et al., 2019).

In this study, we focus on the design of an intra-city express system in a practical environment. Parcels are transported from the origin branch offices to the destination branch offices, resulting a many-to-many distribution system. As the parcel and mail flows are usually less-than-truckload (LTL), it is very costly to link them directly, both from the economic and social points of view (Gelareh & Nickel, 2011; Sun, 2013). Instead, one method is to use the network shown in Figure 1 to realize the flow exchange.

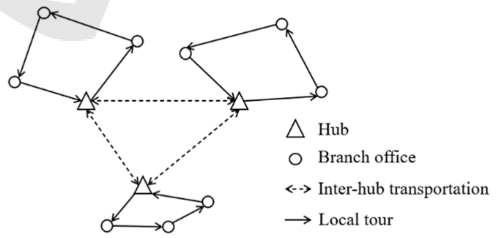

Figure 1: Hub-and-spoke network for intra-city express systems.

This network is a variant of hub-and-spoke networks specially designed for LTL transportation. The hubs and branch offices are connected by local tours instead of direct links, which is generally very

[a] https://orcid.org/0000-0002-1755-3360
[b] https://orcid.org/0000-0002-2832-2015
[c] https://orcid.org/0009-0005-9937-951X

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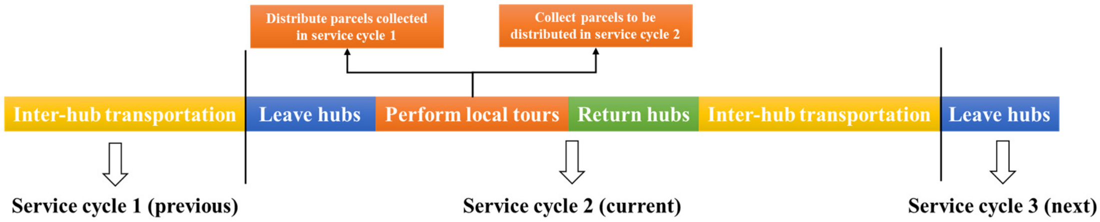

Figure 2: Hub-and-spoke network for intra-city express systems.

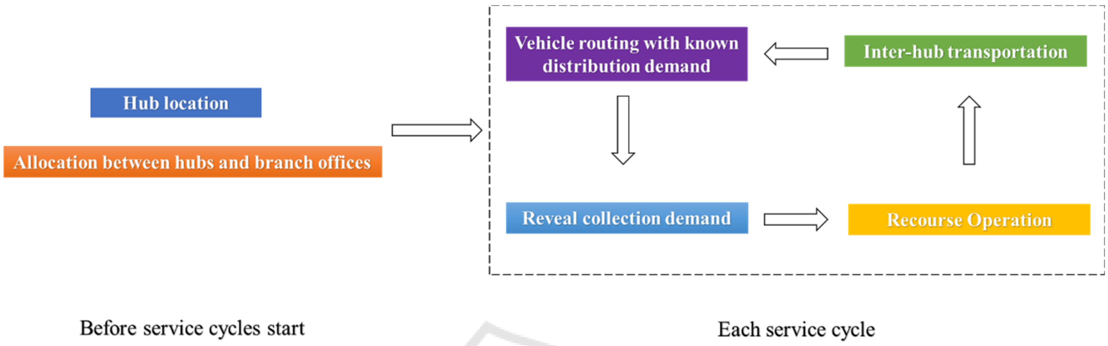

Figure 3: Stochastic decision process for intra-city express systems.

expensive (Kartal et al., 2017; Alumur et al., 2020).

The parcels are picked up at the origin branch offices, sorted in the first hub, possibly transported to the second hub, and delivered to the destination branch offices. Moreover, the collection and distribution processes are conducted at the same time along local tours.

Branch offices usually do not own enough sorting resources (e.g., labour, machines, spaces, and so on). Therefore, parcels are collected in a mixed status and have to be sorted based on their destinations in hubs for further delivery. Consequently, the flow cannot be exchanged directly, even along the same local tour. More specifically, in the current service cycle (for example, this morning), each vehicle leaves its corresponding hub and traverses a subset of branch offices, while distributing the parcels collected in the previous service cycle (for example, yesterday morning) and collecting the parcels to be distributed in the next service cycle (for example, tomorrow morning), i.e., the service system is a warmed-up transportation system. Furthermore, inter-hub transportation is conducted after the vehicles return to the hubs (for example, at night). The whole procedure is illustrated in Figure 2. Similar settings have been applied in various studies related to the intra-city express system design (such as in Sun, 2013; Karimi, 2018; Wu et al., 2023).

Based on the above descriptions, one can find that the main decisions of the planning problem for the referred system include hub location, allocation between branch offices and hubs, and vehicle routing, which should be resolved jointly. Moreover, the following three practical conditions are considered:

(i) Capacity. Capacitated hubs and vehicles should be employed due to the limitation of land resources and the limitation of the use of large-volume trucks in urban areas.

(ii) Single-allocation. In practical applications, each branch office is usually served by precisely one hub, as branch offices generally do not have enough sort capacities.

(iii) Stochastic demand. The express company might not know the parcel flows beforehand. For instance, business activities might result in the uncertainty of parcel flows, i.e., the intra-city express demands are stochastic rather than deterministic. One natural process to deal with the uncertainty is that the hub location and allocation between hubs and branch offices are decided before any random variable is revealed (before service cycles start) since changing these decisions for a warmed-up system is expensive. In each service cycle, the vehicle routing is determined with known distribution demand (since these parcels have been collected in the previous service cycle) and unknown collection demand. Finally, recourse operations and inter-hub transportation are conducted to finish the distribution procedure. This process is shown in Figure 3.

With these considerations, we propose the planning problem for the intra-city express system, named capacitated single-allocation hub location routing problem with stochastic demand (CSAHLRPSD), belonging to the field of the hub location routing problem (HLRP). This problem has been applied to the design of various many-to-many systems, such as postal service systems (Bostel et al., 2015), communication systems (Catanzaro et al., 2015), ship cargo systems (Fontes & Goncalves, 2021), and so on. Please find more details of the HLRP in Section 2.

The main contributions lay in three points: i) A multi-stage recourse model is introduced to formulate the CSAHLRPSD, which models the HLRP with stochastic demand for the first time. ii) A sample average approximation (SAA) framework, which is embedded with two variants of the adaptive large neighbourhood search (ALNS) algorithm, is introduced as the solution approach. iii) Numerical experiments are performed to prove the proposed framework's efficiency.

The remainder of the paper is structured as follows: Section 2 reviews the HLRP and compares our study with the existing ones. Section 3 defines the CSAHLRPSD via a multi-stage recourse model. Section 4 provides the solution methodology, whose efficiency is tested in Section 5. Finally, Section 6 concludes the study.

2 LITERATURE REVIEW

This section mainly reviews the works on the single-allocation hub location routing problem (SAHLRP), which is closely concerning to the CSAHLRPSD. Nagy and Salhi (1998) first proposed the SAHLRP with route length constraints to limit working hours. They proposed an integer linear programming formulation for this problem and utilised a *locate first–route second* heuristic algorithm to solve it on a single instance with 249 clients.

So far, most studies related to the SAHLRP have concentrated on postal service networks, where the collection and distribution processes usually coincide. Bostel et al. (2015) focused on an SAHLRP where the length of each vehicle route is constrained by a maximum number of visited clients. A memetic algorithm (MA) was introduced to solve instances with up to 100 clients. Kartal et al. (2017) investigated the operational characteristics of a leading cargo company in Turkey. Three variants of formulations were introduced, and a multi-start simulated annealing algorithm and an ACO algorithm were introduced to solve the problem. Numerical results indicated that the proposed algorithms could find high-quality solutions for instances with up to 200 nodes in reasonable computational time. Karimi (2018) studied a capacitated SAHLRP with simultaneous pickup and delivery for a warmed-up postal system. The study introduced a polynomial-sized mixed integer programming formulation and several valid inequalities. Moreover, a tabu-search-based heuristic was proposed to solve the problem. The results from computational tests showed that the proposed valid inequalities and algorithm worked well for their model.

The pickup and delivery process can be distinct for logistical or scheduling reasons, e.g., the case for general freight forwarders. Sun (2015) investigated a problem similar to the one in Sun (2013), in which pickup and deliveries were assumed to be distinct. An endosymbiotic evolutionary algorithm was developed, simultaneously solving hub location and vehicle routing problems. The algorithm's performance was tested on 20 instances with 100 and 200 customers. Experimental results showed that the proposed algorithm could be used for supply-chain network planning. More recently, Yang et al. (2019) investigated the capacitated SAHLRP with distinct collection and delivery processes. Moreover, they proposed a new MILP model and developed a memetic algorithm (MA) to solve larger-sized problems. Numerical experiments showed that the MA could find high-quality solutions in acceptable computational time.

Most studies have employed heuristic algorithms (Danach et al., 2019; Ratli et al., 2020; Pandiri & Singh, 2021), and there are only a few attempts to solve the problem exactly. de Camargo et al. (2013) introduced a new SAHLRP model with simultaneous collections and distributions. They assumed that a fixed cost was imposed upon the hubs and vehicles. Moreover, they decomposed the problem into two subproblems: a transportation problem and a feasibility problem. Then the problem was optimally solved by a tailored Benders decomposition algorithm. The results were compared to the CPLEX solver, proving that this method was able to find optimal solutions for instances with 100 clients. Later, Rodriguez-Martin et al. (2014) investigated a variant of SAHLRP in which a cyclical path connected the uncapacitated hubs. In the problem, each cluster of clients and assigned hub was connected by precisely one local route cycle. Furthermore, the number of visited clients of each local route cycle is limited as a length constraint. The problem was solved by a branch-and-cut algorithm. Wu et al. (2023) provided a branch-and-price-and-cut algorithm to solve the capacitated SAHLRP, which were tested on benchmark instances. Numerical results proved that the branch-and-price-and-cut algorithm could efficiently deal with the capacitated SAHLRP.

All the above works have focused on the deterministic HLRPs, and there is only one work on the stochastics HLRP. Mohammadi et al. (2013) investigated a multi-objective chance-constrained model of a stochastic green HLRP. In their problem, stochastic travel time and service time were considered. A multi-objective invasive weed optimisation was introduced to solve the problem, which was then compared with other multi-objective algorithms on randomly generated instances.

As reviewed above, stochastic HLRP-related literature is extremely limited. Our study is the first one to investigate the HLRP with stochastic demands. Moreover, our work is the first attempt to model the stochastic HLRP via the recourse model.

3 MODEL FORMULATION

The CSAHLRPSD is defined on a complete graph $G = (V, A)$, in which V and A are vertex set and edge set, respectively. Vertex set V consists of potential hub set H and client (branch office) set C, while edge set A consists of edges between all vertices. For each pair of clients $i \in C$ and $j \in C$, d_{ij} represents the flow to be transported from i to j through local tours and hubs, which is assumed to be a random variable with known and independent distribution. Without loss of the generality, we assume that all realizations of d_{ij} are greater than 0 and they do not exceed the vehicle capacity. Moreover, the collection demand and distribution demand of client $i \in C$ is denoted as $O_i = \sum_{j \in C} d_{ij}$ and $D_i = \sum_{j \in C} d_{ji}$, respectively.

Each potential hub has a capacity Q_k and a fixed cost F_k. As in Ernst and Krishnamoorthy (1999), Hu et al. (2021) and Ghaffarinasab (2022), it is assumed that only receiving flows from clients consumes hub capacity since parcels are generally sorted in the origin hubs and then transported to destination hubs without further sorting operations. Local tours are operated by an unlimited fleet of identical vehicles, and each vehicle is associated with a capacity q and a fixed cost f. Furthermore, inter-hub transportation is assumed to be realised by an unlimited fleet of identical trucks, and there is no capacity limitation and fixed cost of the trucks.

Each edge $(i, j) \in A$ is addressed with a nonnegative travel distance c_{ij}, satisfying the triangle inequality. Local tour cost is dependent on the sum of travel distances of the travelled edges, while inter-hub transportation cost is calculated based on travel distances and transferred flows (Karimi, 2018; Yang et al., 2019). In addition, the unit inter-hub transportation cost (¥/km.t) and unit local tour cost (¥/km) are denoted as α and β, respectively.

The CSAHLRPSD belongs to the field of stochastic programming, which is generally formulated by chance-constrained models and recourse models. Based on the descriptions in Section 1, we model the CSAHLRPSD via a multi-stage recourse model as follows:

i) In the first stage, the hub locations and the allocation between clients and hubs (long-term decisions) are determined before the random variables $(d_{ij} | i, j \in C)$ are realised.

ii) Then, in the second stage, the flows to be delivered to each client $i \in C$ $(d_{ji} | j \in C)$ are revealed first (since these parcels have been collected in the previous service cycle, as shown in Section 1), forming the distribution demands $(D_i | i \in C)$. After the distribution demands are known, the vehicles are routed to link the hubs and clients (short-term decisions) before knowing the collection demands $(O_i | i \in C)$.

iii) In the third stage, the collection demands are revealed, and a predetermined recourse policy is applied when a failure occurs. The classical recourse policy is employed, in which the vehicles return to the hub, drop off the collected parcels, and continue their planned route at the point of failure. Furthermore, if the total collection demand assigned to a hub exceeds its capacity due to uncertainty, a penalty cost must be paid, representing the overwork cost. The unit overwork cost is expressed as ω. Note that the inter-hub transportation costs are also calculated in this stage.

In other words, after the hub location and the allocation between hubs and clients are determined, a VRPSDSP is solved for each installed hub and the clients assigned to it. Although these VRPSDSPs need to be solved multiple times for all the service cycles, we only model them once for simplicity, and the fixed costs are distributed into each service cycle to make long-term and short-term costs comparable.

For each edge $(i, j) \in A$, x_{ij} is a binary variable equal to 1 if there is a vehicle travelling directly from vertex i to vertex j. $z_{ik} (i \in C, k \in H)$ is a binary variable equal to 1 if client i is allocated to hub k. For each vertex $i \in V$, let v_i be the delivery load on the vehicle just after having served vertex i. b_k is a binary variable equal to 1 if potential hub $k \in H$ is open. Moreover, y_{ijkl} denotes the fraction flow from client $i \in C$ to client $j \in C$ passing hub $k \in H$ and hub $l \in H$. Finally, e_k denotes the overwork load of hub $k \in H$.

The CSAHLRPSD is modelled as (1)-(21), in which $Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi})$ and $Q_2(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi})$ are the optimal value of the second stage problem and the third stage problem. Random vector $\boldsymbol{\xi}$ contains the flow d_{ij} to be transported from client $i \in C$ to $j \in C$.

$$Stage\ 1\ min \sum_{k \in H} F_k b_k + E[Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi})] \tag{1}$$

$$s.t. \sum_{k \in H} z_{ik} = 1\ \forall i \in C \tag{2}$$

$$z_{ik} \le b_k\ \forall i \in C, k \in H \tag{3}$$

$$z_{ik} \in \{0, 1\}\ \forall i \in C, k \in H \tag{4}$$

$$b_k \in \{0, 1\}\ \forall k \in H \tag{5}$$

Objective function (1) minimises the operation cost, consisting of the hub fixed cost and expected recourse cost. Constraint (2) guarantees the single-allocation between clients and hubs. Only open hubs can serve clients, which is ensured by Constraint (3). Constraints (4) and (5) are variable domains.

$$\begin{aligned} Stage\ 2\ Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi}) = \min \sum_{k \in H} \sum_{j \in V} f x_{kj} \\ + \sum_{i \in V} \sum_{j \in V} \beta c_{ij} x_{ij} + E[Q_2(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\xi})] \end{aligned} \tag{6}$$

$$s.t. \sum_{j \ne i \in V} x_{ij} = 1\ \forall i \in C \tag{7}$$

$$\sum_{j \in V} x_{ji} = \sum_{j \in V} x_{ij}\ \ \forall i \in V \tag{8}$$

$$x_{ik} \le z_{ik}\ \forall i \in C, k \in H \tag{9}$$

$$x_{ki} \le z_{ik}\ \forall i \in C, k \in H \tag{10}$$

$$x_{ij} + z_{ik} + \sum_{l \ne k \in H} z_{jl} \le 2\ \forall i \in C, j \ne i \in C, k \in H \tag{11}$$

$$v_i - D_j + q(1 - x_{ij}) \ge v_j\ \forall i \in V, j \ne i \in C \tag{12}$$

$$v_i \le q\ \forall i \in V \tag{13}$$

$$x_{ij} \in \{0,1\}\ \forall i \in V, j \in V \tag{14}$$

$$v_i \ge 0\ \ \forall i \in V \tag{15}$$

Objective function (6) minimises the vehicle fixed cost, local tour cost, and expected recourse cost. Each client should be visited by exactly one vehicle, which is guaranteed by Constraint (7). Constraint (8) balances the vehicle flow at each vertex. Constraints (9)-(11) link the allocation variables with routing variables. Constraint (12) describes the delivery load on vehicles. Vehicle capacity constraints are imposed via Constraint (13). Decision variables are defined by Constraints (14)-(15).

$$\begin{aligned} Stage\ 3\ Q_2(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\xi}) = min\ R(\boldsymbol{x}, \boldsymbol{\xi}) + \sum_{k \in H} \omega e_k \\ + \sum_{i \in C} \sum_{j \in C} \sum_{k \in H} \sum_{l \in H} \alpha d_{ij} c_{kl} y_{ijkl} \end{aligned} \tag{16}$$

$$s.t. \sum_{l \in H} y_{ijkl} = z_{ik}\ \forall i \in C, j \in C, k \in H \tag{17}$$

$$\sum_{k \in H} y_{ijkl} = z_{jl}\ \forall i \in C, j \in C, l \in H \tag{18}$$

$$e_k \ge \sum_{i \in C} \sum_{j \in C} \sum_{l \in H} d_{ij} y_{ijkl} - Q_k\ \ \forall k \in H \tag{19}$$

$$0 \le y_{ijkl} \le 1\ \forall i \in C, j \in C, k \in H, l \in H \tag{20}$$

$$e_k \ge 0\ \forall k \in H \tag{21}$$

Objective function (16) optimises the realised recourse cost ($R(\boldsymbol{x}, \boldsymbol{\xi})$) and overwork cost. Also, the inter-hub transportation cost is calculated via the third term of it. Constraints (17)-(18) correlate the flow variables and allocation variables. Note that Constraints (17)-(18), along with Constraints (9)-(11), connect the allocation variables $\boldsymbol{z}$, flow variables $\boldsymbol{y}$, and routing variables $\boldsymbol{x}$, ensuring the proper network flow assignment. Overwork cost for each hub $k \in H$ is calculated via Constraint (19). Constraints (20)-(21) are variable domains. Since there is no simple way to formulate the computation of $R(\boldsymbol{x}, \boldsymbol{\xi})$ via decision variables and linear relationships (Laporte et al., 2002), we do not provide a specific formulation here. However, one can find a way to calculate its expectation in Laporte et al. (2002) and Hernandez et al. (2019).

4 SOLUTION METHODOLOGY

4.1 Sample Average Approximation

The key to solving model (1)-(21) is calculating $E[Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi})]$, which is very difficult even under a discrete distribution. Thus, we present an SAA-based approach to approximate $E[Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi})]$. The SAA approach is presented by Kleywegt et al. (2002), whose principle is that sampling problems can approximate the numerical expectation. A random sample with size N is generated first. Then the CSAHLRPSD can be approximated as below:

$$SAA\ Problem{:}\ min \sum_{m \in H} F_m b_m + \frac{1}{N} \sum_{n=1}^{N} Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi}_n) \tag{22}$$

$$s.t.\ (2)\text{-}(5) \tag{23}$$

The obtained solution is evaluated on a larger sample with size N' ($N' \gg N$) by obtaining the approximate SAA gap and the variance of the gap estimator. If they are small enough, the solution is accepted as the CSAHLRPSD's solution. Otherwise, the sample sizes should be increased. This process is shown in Algorithm 1. In the algorithm, $z_{\xi_n}(\boldsymbol{b}, \boldsymbol{z})$ and $z_{N'}(\boldsymbol{b}, \boldsymbol{z})$ denote the objective function values of the solution $(\boldsymbol{b}, \boldsymbol{z})$ on scenario ξ_n and sample N', respectively. We define "sufficiently small" as: $\epsilon_{N,N'}(\boldsymbol{b}, \boldsymbol{z})^{\wedge}/z_{N'}(\boldsymbol{b}, \boldsymbol{z})^{\wedge} \leq 3\%$ and $\sigma_{\epsilon_{N,N'}}(\boldsymbol{b}, \boldsymbol{z})^{\wedge}/z_{N'}(\boldsymbol{b}, \boldsymbol{z})^{\wedge} \leq 5\%$.

Input: the number of SAA replications M and the sample sizes, N and N' ($N' \gg N$)
Step 1:
For $m = 1,2, \ldots, M$, **do**:
Generate a sample with size N by realising $\xi_1^m, \xi_2^m, \ldots, \xi_N^m$;
Solve the SAA to get the solution $(\boldsymbol{b}, \boldsymbol{z})^m$ and the objective value z_N^m;
Obtain the statistical lower-bound $z_N = \frac{1}{m}\sum_{m'=1}^{m} z_N^{m'}$;
Obtain the variance of the statistical lower-bound $\sigma^2(z_N) = \frac{1}{m(m-1)}\sum_{m'=1}^{m}(z_N^{m'} - z_N)^2$;
Generate a sample with size N' and get the upper-bound $z_{N'}(\boldsymbol{b}, \boldsymbol{z})^m$and a estimate of variance of upper-bound $\sigma_{N'}^2(\boldsymbol{b}, \boldsymbol{z})^m = \frac{1}{N'(N'-1)}\sum_{n=1}^{N'}(z_{\xi_n}(\boldsymbol{b}, \boldsymbol{z})^m - z_{N'}(\boldsymbol{b}, \boldsymbol{z})^m)^2$;
Select the solution $(\boldsymbol{b}, \boldsymbol{z})^{\wedge}$ with best $z_{N'}(\boldsymbol{b}, \boldsymbol{z})^{\wedge}$ then ontain the SAA gap $\epsilon_{N,N'}(\boldsymbol{b}, \boldsymbol{z})^{\wedge} = z_{N'}(\boldsymbol{b}, \boldsymbol{z})^{\wedge} - z_N$;
Calculate the variance of the SAA gap $\sigma_{\epsilon_{N,N'}}^2(\boldsymbol{b}, \boldsymbol{z})^{\wedge} = \sigma^2(z_N) + \sigma_{N'}^2(\boldsymbol{b}, \boldsymbol{z})^{\wedge}$;
If $\epsilon_{N,N'}(\boldsymbol{b}, \boldsymbol{z})^{\wedge}$ and $\sigma_{\epsilon_{N,N'}}^2(\boldsymbol{b}, \boldsymbol{z})^{\wedge}$ sufficiently small:
Go to **Step 3**;
End
End
Step 2:
If $\epsilon_{N,N'}(\boldsymbol{b}, \boldsymbol{z})^{\wedge}$ and $\sigma_{\epsilon_{N,N'}}^2(\boldsymbol{b}, \boldsymbol{z})^{\wedge}$ not sufficiently small:
Increase the sample size N and/or N' and go to **Step 1**
End
Step 3: Output: $(\boldsymbol{b}, \boldsymbol{z})^{\wedge}$
Stop

Algorithm 1: SAA algorithm.

The SAA problem is a special variant of the HLP. More complex, calculating $Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi})$ is NP-hard even when $b_k (k \in H)$ and $z_{ik} (i \in C, k \in H)$ are fixed. As a result, two ALNS algorithms are introduced as the solution approach for solving the SAA problem and getting $Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi})$, respectively. These two algorithms are designed according to the one used by Wu et al. (2022), which has been proven to solve the HLRP efficiently. For notation simplicity, we name them ALNS-SAA and ALNS-RECOURSE, respectively.

4.2 Adaptive Large Neighbourhood Search

The ALNS algorithm has been successful in solving various routing problems, e.g., vehicle routing problem, pickup and delivery problem, location routing problem, and so on. We follow the procedure in Ropke and Pisinger (2006) to present the ALNS-SAA and ALNS-RECOURSE: In each iteration, a destroy method removes several clients from the current solution, and then a repair method inserts them into the destroyed solution to obtain a new solution. Each method is associated with a weight and is randomly selected based on their weights. The weights are adjusted adaptively based on their performance. The new solution is accepted is it is

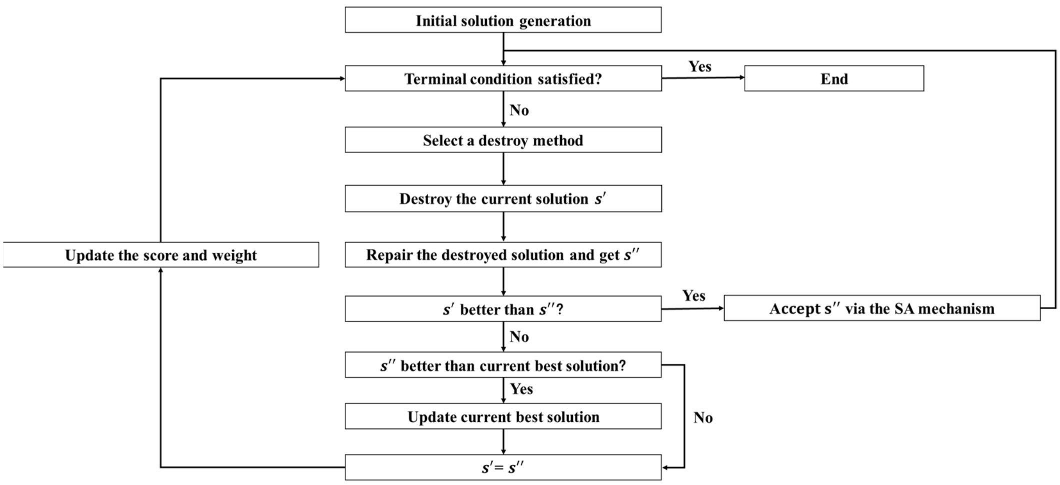

Figure 4: ALNS algorithm

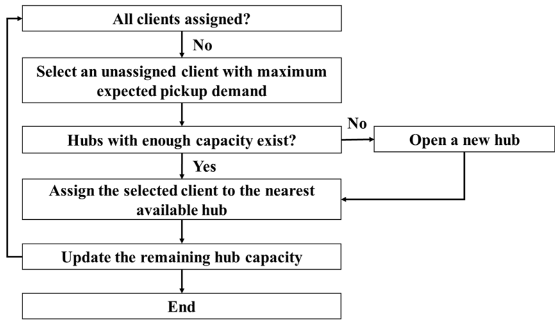

Figure 5: Greedy algorithm.

better than the current one. Otherwise, a simulated annealing mechanism is applied to determine whether the new solution is accepted. Although the ALNS-SAA and ALNS-RECOURSE have the same procedures, their destroy/repair methods and initial solution generation methods are different, which will be presented in Section 4.2.1 and Section 4.2.2. Please refer to Wu et al. (2022) for the common parts (e.g., weight adjustment, destroy/repair method selection, and simulated annealing mechanism).

4.2.1 ALNS-SAA

a. Initial Solution Generation

We use the following greedy algorithm (Figure 5) for the initial solution generation. The clients are allocated to the nearest open hubs one-by-one. If such hubs do not exist, a new hub is installed. The process continues until all clients are assigned.

b. Destroy Method

Random Hub Removal: This method randomly selects one open hub and closes it. All linked clients are deleted from the current solution and added into the client pool.

Worst Usage Hub Removal: This method closes the open hub with the least utilisation ratio. All clients allocated to it are deleted and added into the client pool.

Random Hub Opening: This method randomly selects one close hub and opens it. Then, several clients are randomly selected, deleted from the current solution, and then put in the client pool.

Random Allocation Change: This method aims to optimise the allocation between clients and hubs. The randomly-selected clients are deleted from the current solution and inserted into the client pool.

Worst Allocation Removal: This method delates some clients far from the hubs they are allocated to. The distance is randomised and normalised to avoid constantly selecting the same clients.

c. Repair Method

Greedy Insertion: The clients are inserted into the solution randomly, one after the other, into the position with minimum insertion cost.

4.2.2 ALNS-RECOURSE

a. Initial Solution Generation

The following nearest-neighbour algorithm (Algorithm 2) is used to generate initial solutions for calculating $Q_1(\boldsymbol{b}, \boldsymbol{z}, \boldsymbol{\xi})$.

For each open hub k:
    While unlinked clients allocated to hub k exist:
        Initialise vertex $v = k$
        Initialize pickup capacity $p = q$
        Initialize delivery capacity $d = q$
        While available unrouted clients exist:
            Select unrouted client i nearest to v
            $v = i$
            $p = p - E[O_i], d = \min(p - O_i, d - D_i)$
        End
    End
End

Algorithm 2: Nearest-neighbour algorithm.

b. Destroy Method

Random Removal: This method chooses several clients randomly and adds them into the client pool.

Worst Cost Removal: This method deletes some clients far from the vertexs visited just before and ahead of them.

Shaw Removal: This method aims to remove clients similar to each other.

Random Route Removal: This method deletes a randomly-selected route and adds its visited clients into the client pool.

c. Repair Method

The same **Greedy Insertion** is used. However, the clients can only be inserted into the routes departing from their assigned hub.

5 NUMERICAL EXPERIMENTS

5.1 Instance Generation

The numerical experiments have been conducted on the instances with up to 25 clients used in Wu et al. (2023). These instances are generated from Australia Post (AP) benchmark, and each instance is associated with 5 potential hubs. In AP benchmark, two types of capacities and fixed costs, tight (T) and loose (L), are included. Hence, for each instance, four types of

Table 1: Experiment Results.

Instance	F_{stc}	Ite	$Time$	Gap_{SAA}	Cov_{SAA}	Hub	F_{dtc}	Gap_{stc}
10-L-L	222384.42	2	152.98	0.78	2.42	5	245590.46	9.45
10-L-T	241054.01	2	139.45	1.97	3.05	2,4	264116.68	8.73
10-T-L	248461.71	2	153.27	1.09	2.10	4,5	302806.87	17.95
10-T-T	281902.80	2	188.32	1.32	2.38	2,3,4	342478.48	17.69
15-L-L	287050.82	2	243.53	1.71	4.01	4,5	307109.76	6.53
15-L-T	332315.13	3	342.98	1.87	2.12	1,3	377309.28	11.93
15-T-L	348108.70	2	240.57	1.71	3.68	1,2,5	371482.83	6.29
15-T-T	382314.72	2	251.31	2.56	3.32	1,4	424999.28	10.04
20-L-L	333865.76	2	516.37	1.74	2.49	4,5	364298.63	8.35
20-L-T	351368.10	3	718.51	1.62	3.65	3,4	409825.32	14.26
20-T-L	392668.70	2	588.52	2.56	3.35	3,4,5	438775.19	10.51
20-T-T	428306.20	2	595.87	1.19	2.21	1,2,4	435075.15	1.56
25-L-L	346804.95	2	1011.23	2.05	1.70	2,5	372620.88	6.93
25-L-T	372854.75	2	1403.59	1.54	2.58	2,3	406846.89	8.36
25-T-L	415943.60	2	1236.84	1.63	4.43	2,3,4	427874.59	2.79
25-T-T	441345.67	2	1238.92	2.41	1.88	2,3,4	487703.49	9.51

problems (i.e., LL, LT, TL, and TT) can be created. The used instances are named as *N-Q-F*, where $N \in \{10,15,20,25\}$ denotes the number of clients, and Q and F indicate the type of hub capacity and fixed cost (tight and loose), respectively. For example, 15-L-L means an instance with 15 clients, and its hub capacity and fixed cost are loose.

We have adjusted these instances and applied the proposed SAA framework to them. The main adjustments are:

(i) The flow d_{ij} of each pair of clients i and j $(j \neq i)$ is assumed to be subject to a uniform distribution $[0.6\hat{d}_{ij}, 1.4\hat{d}_{ij}]$, in which $\hat{d}_{ij}$ is the value provided by the generator.

(ii) Vehicle capacity and fixed cost were set as 850 and 3000 in all instances, respectively, ensuring that each client could be served by a single vehicle.

The SAA framework is corded in Java, and a PC with Intel i5-13600KF CPU and 32 GB RAM is used to conduct the experiments.

5.2 Computation Results

In this section, the stochastic model and deterministic model are compared. For the stochastic model, we employ the SAA framework ($N = 40$, $N' = 2000$, $M = 10$) for each instance. For the deterministic model, each instance is solved by the branch-and-price-and-cut algorithm used in Wu et al. (2023), in which the values of the random variables are set as their mathematical expectations. After solving the stochastic model and deterministic model, a new sample with size 2000 (called evaluation sample) is generated to compare their solutions' qualities. The comparison is concluded in Table 1. The definition of the notations in it is presented below:

- F_{stc}: the operation cost of the evaluation sample of the SAA framework.
- Ite: the number of SAA problems used to achieve sufficiently small gap and variance.
- $Time$: computational times (second) for the SAA framework.
- Gap_{SAA}: the SAA gaps.
- Cov_{SAA}: the coefficient of variation (COV) of the SAA approximator.
- F_{dtc}: the operation cost of the evaluation sample of the deterministic model.
- Gap_{stc}: the gap between F_{stc} and F_{dtc}.

It can be concluded in Table 1 that the SAA framework dealt with the CSAHLRPSD adequately: Gap_{SAA} (1.74% on average and 2.56% in the worst case) and Cov_{SAA} (2.84% on average and 4.43% in the worst case) were small. Moreover, in 14 of 16 instances, two SAA replications are needed to reach the small-enough Gap_{SAA} and Cov_{SAA}, indicating that the sample size is chosen adequately. Furthermore, the column "*Time*" demonstrated that the SAA framework was able to solve the CSAHLRPSD in acceptable calculational times, and all instances were solved in less than 1500s. These computational times are acceptable as long-term decisions need to be determined only once for each network. Furthermore, for each service cycle, the short-term decisions can be determined in a very short time. Finally, one can find that considering stochastic factors can effectively cut down the cost: the average Gap_{stc} is 9.43%, while the best Gap_{stc} is 17.95%.

6 CONCLUSIONS

In this paper, we concentrated on the CSAHLRPSD problem. The aim of the problem is to design an intra-city express system in a practical environment. Therefore, capacitated hubs and vehicles were employed, and the flows were assumed to be stochastic. The problem was formulated as a multi-stage recourse model, and an SAA framework was introduced to solve the problem. In the framework, two variants of the ALNS algorithm were used to solve the SAA problem and to calculate the recourse cost. The proposed method was evaluated on the benchmark instances, proving that the SAA framework can solve the CSAHLRPSD in acceptable computational times and that considering stochastic factors can effectively decrease the operation cost (by 9.43% on average). Future studies include proposing more efficient algorithms to calculate the recourse cost and to apply the framework to more instances.

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REFERENCES

Alumur, S. A., Campbell, J. F., Contreras, I., Kara, B. Y., Marianov, V., & O'Kelly, M. E. (2021). Perspectives on modeling hub location problems. European Journal of Operational Research, 291(1), 1-17.

Bostel, N., Dejax, P., & Zhang, M. (2015). A model and a metaheuristic method for the hub location routing problem and application to postal services. 2015 International Conference on Industrial Engineering and Systems Management (pp. 1383-1389). IEEE.

Catanzaro, D., Gourdin, E., Labbé, M., & Özsoy, F. A. (2011). A branch-and-cut algorithm for the partitioning-hub location-routing problem. Computers & operations research, 38(2), 539-549.

Çetiner, S., Sepil, C., & Süral, H. (2010). Hubbing and routing in postal delivery systems. Annals of Operations research, 181(1), 109-124.

Danach, K., Gelareh, S., & Monemi, R. N. (2019). The capacitated single-allocation p-hub location routing problem: a Lagrangian relaxation and a hyper-heuristic approach. EURO Journal on Transportation and Logistics, 8(5), 597-631.

de Camargo, R. S., de Miranda, G., & Løkketangen, A. (2013). A new formulation and an exact approach for the many-to-many hub location-routing problem. Applied Mathematical Modelling, 37(12-13), 7465-7480.

Ebery, J., Krishnamoorthy, M., Ernst, A., & Boland, N. (2000). The capacitated multiple allocation hub location problem: Formulations and algorithms. European journal of operational research, 120(3), 614-631.

Ernst, A. T., & Krishnamoorthy, M. (1999). Solution algorithms for the capacitated single allocation hub location problem. Annals of operations Research, 86, 141-159.

Fontes, F. F. D. C., & Goncalves, G. (2021). A variable neighbourhood decomposition search approach applied to a global liner shipping network using a hub-and-spoke with sub-hub structure. International Journal of Production Research, 59(1), 30-46.

Gelareh, S., & Nickel, S. (2011). Hub location problems in transportation networks. Transportation Research Part E: Logistics and Transportation Review, 47(6), 1092-1111.

Ghaffarinasab, N. (2022). Stochastic hub location problems with Bernoulli demands. Computers & Operations Research, 145, 105851.

Hernandez, F., Gendreau, M., Jabali, O., & Rei, W. (2019). A local branching matheuristic for the multi-vehicle routing problem with stochastic demands. Journal of Heuristics, 25, 215-245.

Hu, Q. M., Hu, S., Wang, J., & Li, X. (2021). Stochastic single allocation hub location problems with balanced utilisation of hub capacities. Transportation Research Part B: Methodological, 153, 204-227.

Karimi, H. (2018). The capacitated hub covering location-routing problem for simultaneous pickup and delivery systems. Computers & Industrial Engineering, 116, 47-58.

Kartal, Z., Hasgul, S., & Ernst, A. T. (2017). Single allocation p-hub median location and routing problem with simultaneous pick-up and delivery. Transportation

Research Part E: Logistics and Transportation Review, 108, 141-159.

Kleywegt, A. J., Shapiro, A., & Homem-de-Mello, T. (2002). The sample average approximation method for stochastic discrete optimisation. SIAM Journal on optimisation, 12(2), 479-502.

Laporte, G., Louveaux, F. V., & Van Hamme, L. (2002). An integer L-shaped algorithm for the capacitated vehicle routing problem with stochastic demands. Operations Research, 50(3), 415-423.

Li, X., & Zhang, K. (2018). A sample average approximation approach for supply chain network design with facility disruptions. Computers & Industrial Engineering, 126, 243-251.

Mohammadi, M., Razmi, J., & Tavakkoli-Moghaddam, R. (2013). Multi-objective invasive weed optimisation for stochastic green hub location routing problem with simultaneous pick-ups and deliveries. Economic Computation & Economic Cybernetics Studies & Research, 47(3).

Nagy, G., & Salhi, S. (1998). The many-to-many location-routing problem. Top, 6(2), 261-275.

Pandiri, V., & Singh, A. (2021). A simple hyper-heuristic approach for a variant of many-to-many hub location-routing problem. Journal of Heuristics, 27(5), 791-868.

Ratli, M., Urošević, D., El Cadi, A. A., Brimberg, J., Mladenović, N., & Todosijević, R. (2020). An efficient heuristic for a hub location routing problem. Optimisation Letters, 1-20.

Rodríguez-Martín, I., Salazar-González, J. J., & Yaman, H. (2014). A branch-and-cut algorithm for the hub location and routing problem. Computers & Operations Research, 50, 161-174.

Ropke, S., & Pisinger, D. (2006). An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. Transportation science, 40(4), 455-472.

Sun, J. U. (2013). An integrated hub location and multi-depot vehicle routing problem. Applied Mechanics and Materials, 409, 1188-1192.

Sun, J. U. (2015). An endosymbiotic evolutionary algorithm for the hub location-routing problem. Mathematical Problems in Engineering, 2015.

Wu, Y., Qureshi, A. G., & Yamada, T. (2022). Adaptive large neighborhood decomposition search algorithm for multi-allocation hub location routing problem. European Journal of Operational Research, 302(3), 1113-1127.

Wu, Y., Qureshi, A. G., Yamada, T., & Yu, S. (2023). Branch-and-price-and-cut algorithm for the capacitated single allocation hub location routeing problem. Journal of the Operational Research Society, 1-13.

Yang, X., Bostel, N., & Dejax, P. (2019). A MILP model and memetic algorithm for the hub location and routing problem with distinct collection and delivery tours. Computers & Industrial Engineering, 135, 105-119.

Zhao, L., Wang, X., Stoeter, J., Sun, Y., Li, H., Hu, Q., & Li, M. (2019). Path optimization model for intra-city express delivery in combination with subway system and ground transportation. Sustainability, 11(3), 758.