Integration of Pricing and Production Scheduling Decisions: A Mathematical Model

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Abstract: In today's competitive manufacturing landscape, achieving operational efficiency and optimizing revenue generation are key objectives for make-to-order manufacturers. This paper presents a novel approach for integrating production scheduling and pricing decisions in a make-to-order manufacturing environment. We propose a comprehensive mathematical model that addresses the complex interplay between production scheduling and pricing strategies. By jointly optimizing these two critical aspects, manufacturers can enhance their competitiveness and profitability. The objective of the scheduling decisions is to minimize the total tardiness penalties of jobs on a non-preemptive parallel machine environment, a widely used measure of customer service. Pricing decisions on the other hand aim at maximizing the total revenue. A mixed integer linear programming model is formulated and an algorithm is developed based on the ε-constraint approach to conduct the experimental analysis. The algorithm aims to find the best solutions, from the decision-maker's perspective, by iteratively adjusting production schedules and pricing decisions.

1 INTRODUCTION

In the dynamic landscape of make-to-order manufacturing, effective coordination between production scheduling and pricing decisions is crucial for companies to optimize their operational efficiency and revenue generation. Traditionally, these two aspects have been treated as separate entities, leading to suboptimal outcomes and missed opportunities. However, recognizing the inherent interdependencies between production scheduling and pricing strategies, researchers and practitioners are increasingly focusing on integrating these decisions to gain a competitive advantage.

The integration of production scheduling and pricing decisions presents a complex challenge due to the dynamic nature of customer demands, evolving market conditions, and limited production resources. Manufacturers need to determine not only how to allocate production resources optimally but also how to set prices that maximize revenue and satisfy customer demand within specified delivery deadlines. By jointly optimizing these decisions, companies can enhance their ability to meet customer requirements while maximizing profitability.

To address this challenge, this paper proposes a comprehensive mathematical model based on mixed-integer linear programming that integrates production scheduling and pricing decisions in a make-to-order manufacturing environment. Our planning environment models the periodic ordering and scheduling decisions that are commonly employed in the industry (Russell and Taylor, 2006). In this context, orders are typically accepted on a timely basis, with no new jobs becoming available between consecutive orders. By formulating the problem as a mathematical optimization model, we aim to provide a systematic and rigorous approach for decision-making in this complex environment.

Furthermore, the principle of ε-constraint is considered in order to find the right balance between the two objective functions, that is minimizing the total tardiness penalty of orders and maximizing the total profit of the company. Leveraging mathematical programming techniques and optimization methods, the algorithm aims to find optimal solutions by iteratively adjusting the priority for one objective over the other depending on the company's strategy, i.e. increasing profit or improving customer satisfaction. By considering the dynamic nature of the environment, the algorithm enables companies to adapt and respond to

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changing market conditions and customer demands effectively.

To validate the effectiveness of the proposed mathematical model and algorithm, computational experiments were conducted using arbitrary chosen data and based on probability distributions. The implementation results confirm the effectiveness of the proposed model, while the proposed algorithm provides a decision-maker-based-system able to help a decision-maker determine the best compromise according to their perspective between the two objective functions and the executions time.

The remainder of this paper is organized as follows: Section 2 presents a brief literature review on the integration of pricing and production scheduling decisions in make-to-order manufacturing environment. The problem description along with the proposed mathematical model are presented in Section 3. Section 4 presents an illustrative example to represent the solution structure provided by the model. Then, computational experiments on the model using a proposed algorithm are conducted in Section 5. Finally, conclusions and future directions are presented in Section 6.

2 LITERATURE REVIEW

The close interplay between operational aspects such as production planning and inventory policies, and marketing decisions including demand management and pricing strategies has long been acknowledged in practical contexts (Chen. and Hall, 2022). Consequently, it is essential to make coordinated marketing and production decisions to maximize overall efficiency and profitability throughout the supply chain. Extensive literature surveys were conducted by Eliashberg and Steinberg (1993), Chen and Simchi-Levi (2012), and Chen. and Hall (2022), showing considerable research attention devoted to Coordinated Pricing and Production Scheduling (CPPS) over the past decades. Among the vast array of research and advancements in this field, only a limited number of studies specifically addresses the detailed scheduling of individual orders. Nevertheless, as shown in (Chen. and Hall, 2022), many practical examples can state the pertinence of coordinating production scheduling and pricing decisions in make-to-order systems. Driven by this practical relevance, two categories of CPPS problems can be distinguished, including problems with a single period pricing, and problems with multiple periods pricing.

For the single-period pricing problem, orders prices are decided at the beginning of the scheduling horizon. Chen and Hall (2010) study the coordination of pricing and scheduling decisions in a make-to-order environment. Assuming knowledge of a deterministic non-increasing demand function, they study three objective functions for the scheduling problem, including the total work in progress, the total penalty for orders delivered late, and the total capacity usage while maximizing the total net profit of the company. Moreover, they assume that a single price is used for each product along with its respective demand over the entire scheduling horizon. They examine three degrees of coordinating pricing and scheduling decisions in order to conclude on the advantage of coordination in this context.

Liu et al. (2020) study the problem with a single machine environment, where the manufacturer receives order inquiries from customers and has to allocate a price for each enquiry. They consider a probability associated with the acceptance of the allocated price by a customer and aim at maximizing revenue while minimizing the total tardiness. To solve this problem, they propose an efficient heuristic after proving that the problem is NP-hard.

Lu et al. (2013)'s work includes modeling customer demand's uncertainty. They focus on minimizing the expected production cost based on the total weighted completion time. They design dynamic programming algorithms to solve the problem. Their study highlights the advantage of coordination in profit maximization.

Wang and Wang (2019) investigate the pricing and scheduling decisions coordination on a parallel machines environment, where the objective is to maximize revenue and minimize the total weighted tardiness of accepted orders. They propose a mixed integer linear programming model where products prices are decided at the beginning of the planning horizon.

For the multi-period pricing problem, Yue et al. (2019) study a particular problem motivated by the practical setting where a manufacturer makes multiple customized products from a common base product. They use dynamic pricing to match capacity with demand over a multi period planning horizon. Hence, at the beginning of each period, the price and the production schedule are decided for incoming orders on a single machine environment. They consider that due dates are equal to the end of each period, meaning that a common due date is fixed for orders arriving at the beginning of each period. They propose dynamic programming algorithms to solve three variants of the problem, including the total weighted completion time minimization, tardiness minimization with rejection and without rejection.

In comparison to the existing literature, our work makes a distinct contribution by introducing a novel perspective on the multi-period pricing problem within the context of coordinated marketing and production decisions. While the literature, exemplified by Yue et al. (2019), primarily considers a multi-period setting where orders are scheduled based on fixed end-of-period deadlines, our study extends this paradigm by allowing each order to possess a flexible and arbitrary deadline, thereby reflecting the complexities often encountered in real-world scenarios. This departure from traditional scheduling definitions permits a more accurate representation of practical situations, where orders might be processed across different periods depending on their individual deadlines. Furthermore, our proposed mathematical model is designed not only to provide optimal solutions for small instances but also serves as an initial step towards tackling larger-scale instances using heuristics and meta-heuristics.

3 PROBLEM DESCRIPTION

The problem under study involves scheduling orders in an identical parallel machine environment taking into account pricing decisions. The company proposes a set of products and a range of prices for each product and orders are received at specific arrival times set by the company. The model then decides the prices, the scheduling order and the machine assignment for customers orders, so as to maximize revenue and minimize production cost.

Following the Graham's notation (Graham, 1966), the scheduling problem can be denoted as $P_m|r_j|\sum w_j T_j$, where each order has a release date r_j at which it becomes available for scheduling, a deadline d_j at which it must be completed, and a weight w_j that refers to the penalty assigned to an order if delayed. Furthermore, the objective function is the total weighted tardiness of orders, expressed as $\sum w_j T_j$, where the tardiness of an order $T_j = \max\{c_j - dj, 0\}$, such as c_j refers to the completion time of the order. Finally, the scheduling environment is composed of m identical parallel machines. These notations are employed to describe the classical scheduling problem and will evolve in the following sections to accurately describe the specific problem under consideration, taking into account the pricing integration.

The scheduling problem $P_m|r_j|\sum w_j T_j$ is NP-hard since even the problem without release dates $P_m||\sum w_j T_j$ is NP-hard, as proved by Koulamas (1994). The comparison between the latter and the studied problem reveals their similarity, with the only difference being the inclusion of the dynamic pricing part in the studied problem. As a result, it is evident that the studied problem is also NP-hard.

3.1 Assumptions

The mathematical model for the pricing and scheduling decisions coordination takes into account the following assumptions inherited from the standard parallel machine scheduling problem with release dates and due dates.

- Order preemption is not allowed, meaning that an order can not be interrupted once it starts its processing.
- Each order has a processing time.
- Each order has an arrival time and hence cannot start processing before.
- Each order has a due date, and if that due date is violated or not met, a penalty is incurred.
- Machines are identical.
- Each machine can process at most one order at a time.

3.2 Notations

The proposed mathematical model is based on the following notations for sets and indices:

- M: set of $\bar{m}$ identical parallel machines, with $M = \{1, ..., \bar{m}\}$.
- P: set of $\bar{p}$ products proposed by the company, with $P = \{1, ..., \bar{p}\}$.
- T: set of $\bar{t}$ orders arrival times, with $T = \{t_1, ..., \bar{t}\}$.
- L_i: set of $\bar{l}_i$ prices allowed for a product $i \in P$, with $L_i = \{1, ..., \bar{l}_i\}$.
- D_i: set of $\bar{d}_i$ orders of a product $i \in P$, with $D_i = \{1, ..., \bar{d}_i\}$, $D_{max} = \max_{i \in P} D_i$
- pt_i: processing time of a product i, with $i \in P$.
- w_i: tardiness penalty of a product $i \in P$.
- d_i^t: deadline of an order of product $i \in P$ made at time $t \in T$.
- f_i: decreasing demand function with respect to the product's price for a product $i \in P$.
- q_{il}: price of product $i \in P$ of index $l \in L_i$, such as $f_i(q_{il})$ is the associated demand.
- N: sufficiently large number.

3.3 Decision Variables

- T_{ij}^t: tardiness of an order j of a product i made at time t, with $j \in D_i$, $i \in P$, and $t \in T$.
- C_{ij}^t: completion time of an order j of product i made at time t, with $j \in D_i$, $i \in P$, and $t \in T$.
- x_{il}^t: binary decision variable, which indicates the price selected for a product order. This variable is equal to 1 if price q_{il} is fixed for product $i \in P$ at period t, with $l \in L_i$, and 0 otherwise.
- z_{ij}^t: binary decision variable, which indicates if an order of a product is made. This variable is equal to 1 if order $j \in D_i$ of product $i \in P$ is made at time t, and 0 otherwise.
- A_{ijt}^k: binary decision variable, which suggests the orders assignments to machines. This variable is equal to 1 if order $j \in D_i$ of product $i \in P$ made at time $t \in T$ is assigned to machine $k \in M$, and 0 otherwise.
- $y_{ijtuvt'}^k$: binary decision variable, which indicates the order of processed jobs. This variable is equal to 1 if, on machine $k \in M$, order $j \in D_i$ of product $i \in P$ made at time $t \in T$ is a direct predecessor of order $v \in D_u$ of product $u \in P$ made at time $t' \in T$, and 0 otherwise.

3.4 Mathematical Model

The mathematical model (P0) presented here for the problem of pricing and production scheduling was inspired from the Wang and Wang (2019)'s formulation. Their model considers that prices are decided at the beginning of the scheduling horizon and demand is then fixed consequently. The pricing integration part was adapted for this mathematical model while considering that pricing decisions are made dynamically at multiple order arrival times. Hence, the scheduling decision must be made taking into account the constraint regarding orders arrival times.

The mathematical model (P0) is formulated as indicated through equations (1) to (15):

$$Minimize \quad \sum_{t\in T}\sum_{i\in P}\Big(\sum_{j\in D_i} w_i T_{ij}^t - \sum_{l\in L_i} q_{il} f_i(q_{il}) \times x_{il}^t\Big) \tag{1}$$

Subject to:

$$\sum_{l\in L_i} x_{il}^t = 1, \forall t \in T, i \in P \tag{2}$$

$$\sum_{j\in D_i} z_{ij}^t = \sum_{l\in L_i} f_i(q_{il}) \times x_{il}^t, \forall i \in P, t \in T \tag{3}$$

$$\sum_{l\in L_i} f_i(q_{il}) \times x_{il}^t - j \geq D_{max} \times (z_{ij}^t - 1), \forall i \in P, j \in D_i, t \in T \tag{4}$$

$$\sum_{k\in M} A_{ijt}^k = z_{ij}^t, \forall i \in P, j \in D_i, t \in T \tag{5}$$

$$\sum_{u\in P}\sum_{v\in D_u}\sum_{t'\in T} y_{ijtuvt'}^k \leq A_{ijt}^k, \forall i \in P, j \in D_i, t \in T\ , k \in M \tag{6}$$

$$\sum_{i\in P}\sum_{j\in D_i}\sum_{t\in T} y_{ijtuvt'}^k \leq A_{uvt'}^k, \forall u \in P, v \in D_u, t' \in T\ , k \in M \tag{7}$$

$$\sum_{i\in P}\sum_{u\in P}\sum_{j\in D_i}\sum_{v\in D_u}\sum_{t'\in T}\sum_{t\in T} y_{ijtuvt'}^k = \sum_{u\in P}\sum_{v\in D_u}\sum_{t'\in T} A_{uvt'}^k - 1, \forall k \in M \tag{8}$$

$$C_{uv}^{t'} \geq C_{ij}^t + pt_u - N \times (3 - y_{ijtuvt'}^k - A_{ijt}^k - A_{uvt'}^k), \forall i,u \in P, v \in D_u, j \in D_i, t \in T, t' \in T, k \in M \tag{9}$$

$$C_{ij}^t \geq t + pt_i - N \times (1 - z_{ij}^t), \forall i \in P, j \in D_i, t \in T \tag{10}$$

$$T_{ij}^t \geq C_{ij}^t - d_i^t, \forall i \in P, j \in D_i, t \in T \tag{11}$$

$$T_{ij}^t \geq 0, \forall i \in P, j \in D_i, t \in T \tag{12}$$

$$C_{ij}^t \leq N \times z_{ij}^t, \forall i \in P, j \in D_i, t \in T \tag{13}$$

$$C_{ij}^t \geq 0, \forall i \in P, j \in D_i, t \in T \tag{14}$$

$$x_{il}^t, z_{ij}^t, A_{ijt}^k, y_{ijtuvt'}^k \in \{0,1\}, \forall i,u \in P, j \in D_i, v \in D_u, l \in L_i, t, t' \in T, k \in M \tag{15}$$

The objective function (1) minimizes the total tardiness penalty of orders while maximizing the total revenue.

Constraint (2) verifies that a single price is chosen for each product ordered at a time $t \in T$.

Constraint (3) fixes the total number of orders to be scheduled for each product at each arrival time $t \in T$ to be equal to the total demand made.

Constraint (4) sets $z_{ij}^t = 0$ in case $\sum_{l\in L_i} f_i(q_{il}) \times x_{il}^t - j \geq 0$, i.e. the actual demand is less than the maximum demand. On the other hand, in case $z_{ij}^t = 1$ the inequality remains valid.

Constraint (5) states that each order of a product made at an arrival time $t \in T$ is assigned to a single machine.
Constraints (6) and (7) verify that if an order is assigned to a machine, then it is succeeded and preceded by at most one order.
Constraint (8) fixes the number of precedence relationships on each machine.
Constraints (9) and (10) set the completion time of each order.
Constraints (11) and (12) set the tardiness of each product.
Constraint (13) sets the completion times of unrequested orders to zero. Constraints (14) and (15) define the domains of the decision variables.

4 ILLUSTRATIVE EXAMPLE

In order to illustrate the solution structure, we consider the following instance: A manufacturer has two types of products and two parallel machines. Orders are received at three different arrival times, $T = \{2, 5, 17\}$, and a deadline is associated with each product's order associated with an arrival time. Inspired by Yalaoui (2012), the deadlines are generated following the uniform distribution $\mathcal{U}[t + pt_i, t + pt_i + ((\sum_i pt_i)/2)]$ with i $\in$ P and t $\in$ T. Table 1 presents processing times, tardiness penalties, deadlines, and the set of prices and demands for each product.

Table 1: Parameters of the illustrative example.

Product	pt_i	w_i	d_i^t/Time	Price/Demand
1	4	2	6 / 2	8 / 1
			10 / 5	7 / 2
			27 /17	6 / 3
				5 / 4
2	8	1	14 / 2	5 / 1
			18 / 5	4 / 2
			30 / 17	3 / 3
				2 / 4

The mathematical model was implemented in IBM CPLEX solver version 22.1.1.0, on a Intel(R) Core(TM) i5-8350U CPU @ 1.70 GHz of 8 GB RAM machine.

An optimal solution for the input described in Table 1 was found in 648 seconds. The optimal objective value obtained is -57. Moreover, the selected price for the first product at times 2 and 5 is 7 with two orders each, while at $t = 17$ the selected price of this product is 6 with three orders. Moreover, for the second product, the selected price is 5 for each of the arrival times, resulting in a total of three orders.

As presented in Table 2, the selected price for the first product at times 2 and 5 is 7 with two orders each, and at $t = 17$ the price of this product is 6 with three orders. But for the second product, the selected price is 5 with the associated demand at all the arrival times.

Table 2: The selected price and demand for each product and arrival time.

Product	Time	Price/Demand
1	2	7 / 2
	5	7 / 2
	17	6 / 3
2	2	5 / 1
	5	5 / 1
	17	5 / 1

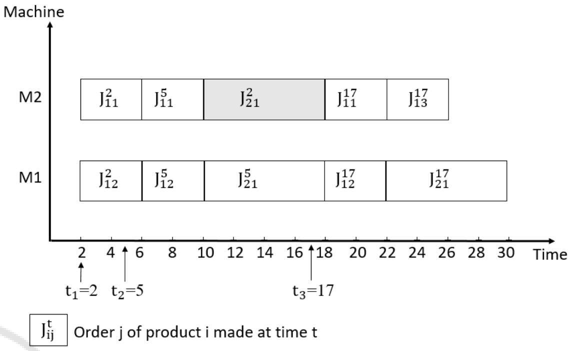

Figure 1: Scheduling Process for Orders in Two Parallel Machines.

Figure 1 displays the solution scheduling structure. The order colored in grey is considered late as its completion time exceeds the due date. In this case, the total tardiness penalty is equal to 4 and the actual benefit is 61.

5 EXPERIMENTAL STUDY

The proposed model aims at minimizing the tardiness penalty of the orders to improve service quality and maximizing the total benefit of the company at the same time. In order to find the best compromise between these two objectives, the ε-constraint method Chankong and Haimes (2008) is considered. It consists in optimizing one of the objective functions by considering the other objective as an additional constraint. For the studied problem, we consider the minimization of the tardiness penalty, while considering the following additional constraint on account of benefit maximization:

$$\sum_{i \in P} \sum_{t \in T} \sum_{l \in L_i} q_{il} f_i(q_{il}) \times x_{il}^t \geq \alpha \times MaxBenefit \quad (16)$$

Such as, $\alpha \in [0, 1]$ represents a coefficient fixed by the decision-maker indicating the minimum amount of benefit they are expecting to receive, while $MaxBenefit$ denotes the maximum benefit that can be obtained using the following equation:

$$MaxBenefit = \bar{t} \times \sum_{i \in P} \max_{l \in L_i} (q_{il} \times f_i(q_{il})) \quad (17)$$

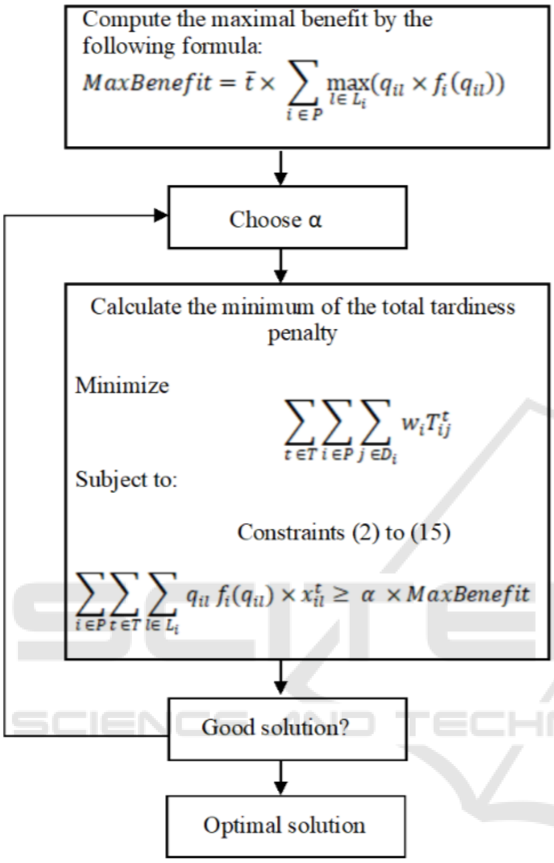

Figure 2: Resolution algorithm.

The ε-constraint based algorithm is depicted in Figure 2. The algorithm starts by calculating the maximum benefit following Equation (17). The α coefficient is then fixed by the decision-maker. Next, the model for the total tardiness penalty (TTP) is solved, subject to the same constraints of problem (P0), in addition to Inequality (16). Finally, the optimal solution associated with the chosen value of α is obtained and the decision-maker decides whether to accept the solution or choose another value for α.

By applying this algorithm to the illustrative example presented in the previous section, the maximum benefit obtained following Equation (17) is 87. Table 3 presents the results for different values of α and a limited execution time.

It must be noted that $\alpha = O.7$ refers to the optimal solution obtained by implementing the initial model (P0) where the objective functions are expressed as a single objective function, this result was obtained in 648 seconds, as stated earlier. However, using the ε-constraint algorithm, for $\alpha = 0.7$ the result is obtained within 15 seconds. Furthermore, Figure 3 displays the Pareto front of non-dominated solutions for α ranging from 0.1 to 0.7, with solutions for $\alpha = 0.8$ to 1.0 representing the best outcomes obtained after an extended execution time of 1800 seconds. As depicted in the figure, the two objectives, namely minimizing TTP and maximizing benefit, exhibit a conflict, signifying that optimizing one objective comes at the expense of the other.

Table 3: Implementation results of the algorithm on the illustrative example.

α	Orders	Delayed orders	TTP	GAP(%)	Time(s)
0.1	6	0	0	0	0.7
0.2	6	0	0	0	0.8
0.3	6	0	0	0	0.8
0.4	6	0	0	0	0.8
0.5	8	0	0	0	1.3
0.6	9	0	0	0	3.6
0.7	10	1	4	0	15
0.8	13	5	34	70.16	> 1800
0.9	16	7	128	100	> 1800
1	21	14	312	98	> 1800
0.7	10	1	4	0	15

Figure 4 presents the mean computational time of the algorithm for each alpha value over 10 arbitrary generated instances, taking into account two products, with four possible prices and demands each. The processing times and the tardiness penalties for these instances are generated following a uniform distribution $\mathcal{U}[1,8]$. Moreover, three arrival times for the incoming orders are considered, and the deadlines are generated following the same uniform distribution as previously used in the illustrative example.

The results show that the algorithm quickly provides an optimal solution for the associated coefficient α between 0.1 and 0.7. However, for α greater than 0.7, no optimal solution is obtained even after half an hour of execution. In fact, for higher values of α, the algorithm prioritizes fulfilling orders to achieve minimal benefit, which in turn prolongs the scheduling process on the parallel machines.

6 CONCLUSION

This research contributes to the field by providing a comprehensive framework for integrating production scheduling and pricing decisions in a make-to-order manufacturing environment. By considering the inter-

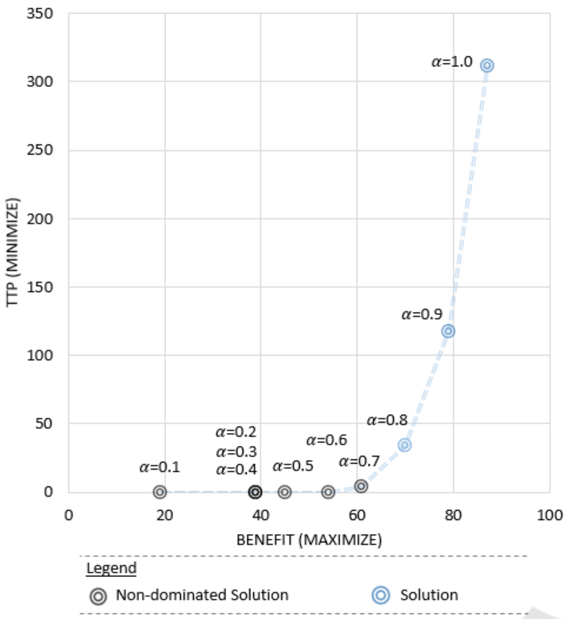

Figure 3: Pareto front of non-dominated solutions for α values (0.1 to 0.7) and best solutions (0.8 to 1.0) after a fixed execution time.

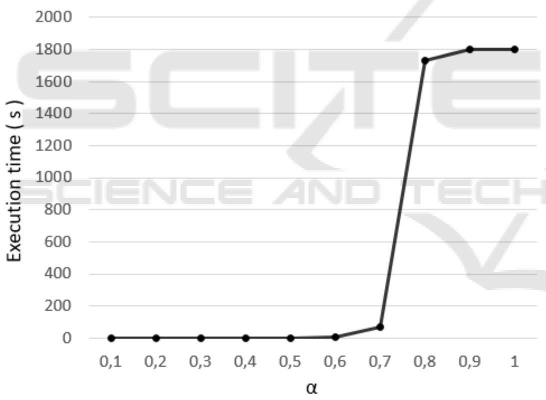

Figure 4: Mean Computational time of 10 arbitrary instances with respect to different values of the coefficient α.

dependencies between these decisions, manufacturers can achieve better coordination, improve customer satisfaction, and enhance their overall competitiveness in a dynamic market environment. In this paper, a new mathematical model is proposed for pricing and scheduling coordination on a parallel machine environment where arrival times are associated with customers orders. A computational study is presented to confirm the effectiveness of the proposed mixed integer linear programming model. Using the ε-constraint principle to conduct the experimental analysis, we provide insight for decision-makers to find the best compromise between revenue maximization and improving customer service. In future work, it is important to focus on developing efficient algorithms that can effectively address large-scale industrial instances. In this regard, a promising resolution method lies in the application of Benders decomposition, specifically tailored for tackling the multi-objective nature of the integrated problem. Additionally, drawing inspiration from the well-established literature on the mono-objective scheduling problem, a method based on Tabu Search shows promise. As evidenced by Lara et al. (2016), Tabu Search has demonstrated effectiveness in addressing the basic scheduling problem. Extending its application to the integrated pricing and scheduling context could yield valuable insights and solutions. Finally, more complex machine environments can be studied in the future.

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