Hybrid Quanvolutional Echo State Network for Time Series Prediction

Rebh Soltani[1][a], Emna Benmohamed[2][b] and Hela Ltifi[3][c]
[1] *Research Groups in Intelligent Machines, University of Sfax, National Engineering School of Sfax (ENIS), BP 1173, Sfax, 3038, Tunisia*
[2] *Department of Cyber Security, College of Engineering and Information Technology, Onaizah Colleges, P.O. Box 5371, Onaizah, K.S.A.*
[3] *Computer Science and Mathematics Department, Faculty of Sciences and Technology of Sidi Bouzid, University of Kairouan, Tunisia*
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Abstract: Quantum Machine Learning (QML) combines quantum physics with machine learning techniques to enhance algorithm performance. By leveraging the unique properties of quantum computing, such as superposition and entanglement, QML aims to solve complex problems beyond the capabilities of classical computing. In this study, we developed a hybrid model, the quantum convolutional Echo State Network, which incorporates QML principles into the Reservoir Computing framework. Evaluating its performance on benchmark time-series datasets, we observed improved results in terms of mean square error (MSE) and reduced time complexity compared to the classical Echo State Network (ESN). These findings highlight the potential of QML to advance time-series prediction and underscore the benefits of merging quantum and machine learning approaches.

1 INTRODUCTION

QML is the fusion of machine learning and quantum computing (Biamonte et al., 2017). The objective of QML is to harness the computational advantages offered by quantum computers, which can process data at exponentially faster rates than classical computers. By leveraging quantum systems, algorithms are developed to facilitate the continual improvement of computer programs over time. QML capitalizes on the inherent efficiency of quantum computers to address machine learning problems. Notably, the utilization of superposition quantum states enables the simultaneous analysis of multiple states, leading to substantial speedups. A key research focus lies in the design of networks known as quantum neural networks (Altaisky et al., 2014), where every element, including individual neurons and training algorithms, operates on a quantum computer.

In the 1990s, the field of quantum neural network research began to emerge, as evidenced by early publications (Fernández Pérez et al., 2022). However, in comparison to other areas of quantum machine learning discussed previously, neural networks in the quantum realm have not yet attained the same level of scientific maturity. This lack of progress can be attributed to the inherent non-linearity of neural network components conflicting with the linear nature of quantum physics (Schuld et al., 2015). To overcome this challenge, researchers have explored the application of a quantum mechanical framework to simulate the activation function of perceptrons. This approach involves incorporating specialized measurements and proposing non-linear quantum operators. Notably, Schuld (Schuld et al., 2015) has described a direct implementation of the activation function using the quantum phase estimation approach and a circuit-based model of quantum computation. These advancements offer promising avenues for bridging the gap between neural networks and quantum mechanics, thus unlocking new possibilities for quantum-inspired machine learning paradigms (Chen et al., 2022). Defining quantum neural networks remains a topic of ongoing debate and lacks a consensus, despite recent efforts to construct neural network versions that rely solely on principles from quantum mechanics. The primary obstacle in developing a quantum artificial neural network stems from the linear nature of quantum physics, whereas artificial neural networks necessitate non-linearity (Schuld et al., 2015)

[a] https://orcid.org/0000-0002-5644-2049
[b] https://orcid.org/0000-0002-3934-3962
[c] https://orcid.org/0000-0003-3953-1135

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(Cao et al., 2017) (Zhao et al., 2019). Consequently, many research papers have encountered challenges in reproducing essential features of traditional neural networks, contributing to the absence of successful replications in this regard.

We draw attention to the fact that despite several studies on quantum neural networks (Gupta and Zia, 2001), (Purushothaman and Karayiannis, 1997) , (Bharti et al., 2022), and (Panella and Martinelli, 2011), no attempt has been made to explicitly model non-linearity on amplitudes, suggesting that no really effective quantum neural network has yet been presented. The Echo State Network (ESN) is a fundamental Recurrent Neural Network (RNN) model with a sparse reservoir and a straightforward linear output (Jaeger, 2002). It has found applications in various domains, including human activity recognition, clinical applications, distributed, embeddable and federated learning, and distributed intelligence applications (Sun et al., 2022) (Soltani et al., 2023). However, the ESN suffers from several fundamental challenges (Soltani et al., 2022). For instance, since the reservoir is randomly created prior to training, determining the hyperparameters of the dynamic reservoir typically involves experimentation (Liu et al., 2020). The analysis of reservoir hyperparameters is a complex task, and the initial connection and weight structure are unlikely to be in an optimal state. Consequently, creating a highly efficient reservoir that is specifically tailored to certain tasks becomes challenging. To tackle these challenges, various techniques have been proposed to address these issues. One technique introduced a deterministic reservoir topology that outperformed the standard ESN (Rodan and Tino, 2010). In an alternative approach, a highly-clustered ESN with a scale-free network structure was utilized (Deng and Zhang, 2006). In pattern recognition, leak integral neurons were employed to improve performance compared to conventional ESNs, utilizing filter neurons, delay variables, and readout (Jaeger et al., 2007). These methods offered various optimizations to the ESN model. The primary objective of this study is to assess the viability and potential benefits of utilizing a quantum echo state network. Our focus lies in integrating the dynamic reservoirs of ESNs with quantum computing to enhance the overall performance of the network and unlock unprecedented levels of efficiency. By leveraging the unique properties of quantum computing, we aim to explore novel avenues for improving the capabilities of echo state networks and achieving remarkable enhancements in their performance.

In this study, we delve into the realm of Quanvolutional Neural Networks (QNNs), a novel class of models. QNNs leverage certain strengths of quantum computation to enhance the capabilities of ESNs. A pivotal contribution of QNNs is the introduction of quantum convolutional (quanvolutional) layers, which augment the conventional ESN architecture. These quanvolutional layers operate in a manner akin to classical convolutional layers, generating feature maps through localized modifications of input data. In contrast, quantum circuit-based quanvolutional filters reshape specific subsets of data spatially to extract features from the input. We propose that leveraging features produced by random quanvolutional circuits can enhance the accuracy of machine learning models employed for time series forecasting.

This paper is structured as follows: Section 2 provides a comprehensive description of the ESN model. In Section 3, we delve into the suggested Approach, which combines quantum theory with the ESN, offering a detailed exploration of the proposed model. Section 4 is dedicated to the performance analysis and presentation of experimental results. Finally, Section 5 concludes the article by summarizing the key findings and conclusions.

2 ECHO STATE NETWORK

Echo state networks (Soltani et al., 2023) are a type of RNNs which has the ability to extract the time correlation of time series data(Bouazizi et al., 2022) (Bouazizi et al., 2023). For this reason, they are frequently employed to tackle time series forecasting tasks. As demonstrated in Fig. 1, ESN model uses a reservoir whose state and dynamics play the role of memory of past experiences, and from which we wish to extract the relevant information through the readout layer. Since just the weights in the readout layer may be trained, the training time is kept to a minimum. Considering that the network includes M input units,

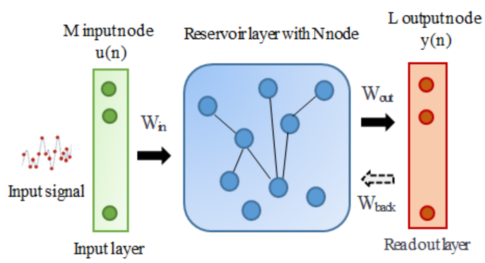

Figure 1: The structure of ESN.

N internal compute nodes (i.e. reservoir layer neurons), and L output units (as shown in Fig. 1). The

ESN can be modelled by :

$$x(n+1) = f(Wx(n) + W_{in}u(n+1) + W_{back}y(n)) \quad (1)$$

$$y(n+1) = W_{out}f_{out}(x(n+1), u(n+1), y(n)) \quad (2)$$

The internal state of ESN neurons is updating, as illustrated in equations (1) and (2). The hidden layer activation function is f. f is often a nonlinear function such as tanh or sigmoid. The reservoir's internal updating state is denoted by x(n), while the output of the linear function f_{out} is denoted by y(n). W_{in} represents the input weights from the input to the reservoir layer. The internal weights inside the reservoir are called W. W_{out} is the output weights from the reserve pool to the readout layer and is the only one that needs to be trained.W_{back} is the backward weights from the readout to the reservoir layer.

3 HYBRID QUANTUM ESN

This section first covers the basic concepts of quantum mechanics. The model is then proposed, followed by a full explanation.

3.1 Quantum

(Qubit): Conventional information is represented by one or more bits. A bit can take the values 0, 1 or true, false. A qubit can take on values of both $|0\rangle$ and $|1\rangle$ said in quantum superposition (Gupta and Zia, 2001). This is in contrast to analog circuits which take values between 0 and 1. The algebraic part of qubits is better understood using Dirac notation. The $|0\rangle$ states and $|1\rangle$ states are algebraically equivalent to the 0 and 1 of a classical bit. The phenomena of quantum superposition is represented by the linear combinations of both $|0\rangle$ and $|1\rangle$. As a result, various potential states emerge:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (3)$$

where α and β are complex values that make up a unit norm vector:

$$\|\alpha\|^2 + \|\beta\|^2 = 1 \quad (4)$$

The coefficients α and β reflect the probability amplitude or for making shorter amplitude in a quantitative sense. The probability of measuring $|0\rangle$ is given by the squared norm, while the probability of measuring $|1\rangle$ is given by the squared norm.The squared norm of an amplitude reflects a probability (Biamonte et al., 2017), hence $\|\alpha\|^2 + \|\beta\|^2 = 1$.

4 HYBRID QUANVOLUTIONAL ESN MODEL

The idea behind this approach is that the quanvolution filter can extract more relevant features from the input data, which can improve the performance of the ESN. Additionally, the quanvolution filter allows including quantum mechanical properties, such as superposition and entanglement, which could enhance the representational power of the ESN. Fig 2 presents in details our proposed method.

1) The input data is transformed into a quantum state, which is a mathematical representation of the data in the quantum mechanical system (equation 3). This can be done using techniques such as quantum state encoding, which maps classical data to a quantum state.

$$i_t = e(u(t)) = e^{iu(t)} = cos(u(t)) + isin(u(t)) \quad (5)$$

where u(t) is the input data at time t. 2) The quantum state is then passed through a quantum random circuit that applies a random sequence of quantum gates to the qubit.

$$O_t = g(i_t) \quad (6)$$

3) Next, the output of the random quantum circuit is passed through a measurement gate.

$$Z_t = d(O_t) = < O_t > \quad (7)$$

4) The output of the quanvolutional filter is a new input that contains the relevant features extracted from the input data. The new input is then used in the recurrent part of the ESN. the following equation can describe the state dynamic of our model:

$$x(n+1) = f(Wx(n) + W_{in}Z_t + W_{back}y(n)) \quad (8)$$

5) The output of the ESN is then generated.

$$y(n+1) = W_{out}f_{out}(x(n+1), u(n+1), y(n)) \quad (9)$$

By incorporating the quanvolution filter and its associated quantum computations, this approach aims to extract relevant features from the input data and integrate them into the ESN framework, potentially leading to improved performance in time series forecasting tasks.

5 EXPERIMENTATIONS AND RESULTS

5.1 Experimental Environment

The experiments conducted in this research employed the PennyLane quantum machine learning library.

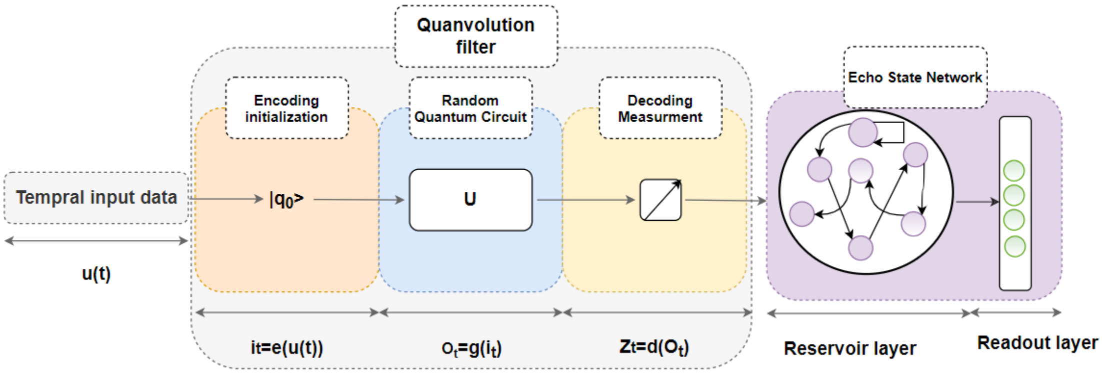

Figure 2: Hybrid Quanvolutional Echo State Network.

The code utilized a local qubit simulator designated as "default.qubit" in PennyLane, enabling quantum computations on a single qubit. This setup allowed for executing gate-model instructions with flexibility in gate width, circuit depth, and fidelity. The experiments intentionally omitted the incorporation of noise models to assess the effectiveness of the ideal universal quantum computational model compared to classical computational models. Future experimentation could involve investigating the impact of noise or utilizing alternative quantum hardware or simulators.

To construct the quantum filter in this implementation, a single qubit is considered. The size of the input for the quanvolutional filters determines the number of qubits required for the circuit. In this particular case, a 1-by-1 quanvolutional filter is employed, resulting in a circuit with 1 qubit (n = 1).

Random 1-qubit gates are generated for the quanvolutional filter, selected from the gate set [X(θ), Y(θ), Z(θ), U(θ), P, T, H]. The value of θ represents a randomly determined rotational parameter, and the target qubit for each gate is also selected randomly.

We applied the first training-test split to all datasets. We further deduced a divide into training (75 %) and validation (25 %) for model selection. Bayesian Optimization was used to tweak the hyper-parameter values on the validation set. We investigated the following configurations in our studies using Hybrid quanvolutional ESN: the number of units [5, 500] in the reservoir layer, input scaling [0.01, 1.5], and spectral radius [0.01, 1.5] for the most part. We employed a single dense layer trained using adam optimizer as the output readout, with a learning rate of 1e-5 , for the Mackey-Glass (MG) dataset (Mackey and Glass, 1977), the model was trained for 100 epochs. Similarly, for the NARMA-10 dataset, the model underwent 50 epochs of training. Following model selection, we trained our models using the chosen hyper-parametrameter on the whole training set, evaluated their performance on the test set. We carried out the same trials with ESN for comparison. In both instances, we used the same model selection and performance evaluation procedure, looking at the identical values for the training algorithm's hyper-parameters and the number of recurrent units.

The model's performance was assessed by measuring the mean square error and computation time on the test set. The experiments were conducted in a Python environment, utilizing a computer equipped with an i5 CPU and 8.0 GB RAM. The results of the quanvolutional filter applied to ESN method were evaluated on the Mackey-Glass (Zhao et al., 2019) and NARMA-10 (Connor et al., 1991) datasets which are a classic example of a chaotic system and is often used as a benchmark for testing the performance of numerical methods for solving differential equations.

5.2 Mackey-Glass Time Series Prediction

For the prediction of chaotic series, the Mackey-Glass' chaotic series is a widely used test (Fig. 2). The following is MG's mathematical model:

$$\frac{dx(t)}{dt} = \frac{ax(t-\tau)y}{1+x^n(t-\tau)} + bx(t) \tag{10}$$

The key parameters are set as a=0.2, b=-0.1, τ=17, n=10. The results obtained on the Mackey-Glass datasets are presented in Table 1, Table 2, Figure 3 and Figure 4.

Quanvolutional ESN model is computationally more efficient and faster in processing the data compared to the ESN model.

Based on the provided results on Table 1 Fig 3 and 4, the Hybrid Quanvolutional ESN model outperformed the ESN model in terms of both predictive accuracy and computational efficiency. The Hybrid Quanvolutional ESN model achieved a lower MSE, indicating better predictive performance on

Table 1: Results on Mackey-Glass dataset.

Model	MSE	Time Complexity (second)
ESN	9.96e-05	116.74
Hybrid Quanvolutional ESN	5.05e-05	106.58

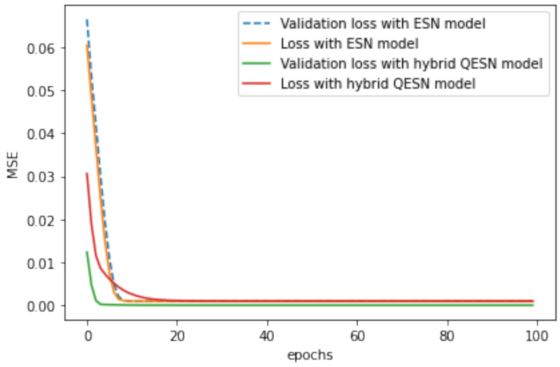

Figure 3: Evolution of training and validation loss during train with MG dataset.

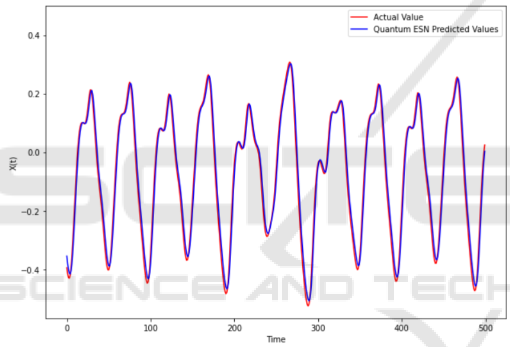

Figure 4: Real output and predicted output generated by hybrid Qanvolutional ESN on MG dataset.

the Mackey-Glass dataset. Additionally, the Hybrid Quanvolutional ESN model demonstrated a faster processing time, making it more efficient in handling the dataset.

The results show that the quantum convolutional filter applied to ESN method achieved a better performance in terms of MSE and time complexity on both datasets, compared to the classical ESN. This indicates that the quantum convolutional filter can extract more relevant features from the input data, which can improve the performance of the ESN. In summary, the proposed method shows a promising performance on time series prediction problems, and it is worth to further investigate this approach on other datasets and problems.

5.3 NARMA-10 Time Series Prediction

one of the most popular tasks, known as NARMA-10 for NARMA defined in (Connor et al., 1991), which entails modeling the following order 10 systems:

$$y(t+1) = c_1 y(t) + c_2 y(t) \sum_{i=0}^{k-1} y(t-i) + c_3 x(t-(k-1)) x(t) + c_4 \tag{11}$$

The important variables are set to k=10, c1=0.3, c2=0.05, c3, and c4=0.1. The results obtained on the NARMA-10 datasets are presented in Table 1, Table 2, Figure 3 and Figure 4.

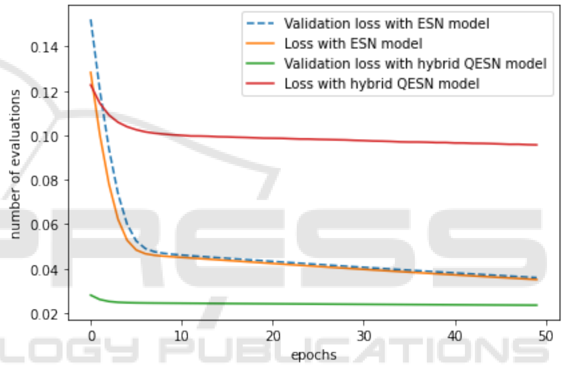

Figure 5: Evolution of training and validation loss during train with NARMA-10 dataset.

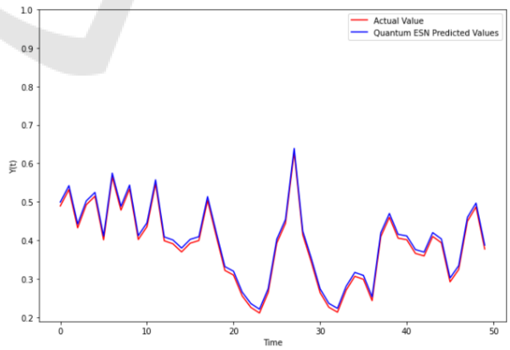

Figure 6: Real output and predicted output generated by hybrid Qanvolutional ESN on NARMA-dataset.

Based on the provided results on Table 3, Fig 5 and 6, the Hybrid Quanvolutional ESN model outperforms the ESN model in terms of both predictive accuracy and computational efficiency on the NARMA-10 dataset. The Hybrid Quanvolutional ESN model achieved a lower MSE, indicating superior predic-

Table 2: Results on Narma-10 dataset.

Model	MSE	Time Complexity (second)
ESN	0.04	677.48
Hybrid Quanvolutional ESN	0.02	479.52

tive performance. Additionally, the Hybrid Quanvolutional ESN model demonstrated a faster processing time, making it more efficient in handling the dataset. In conclusion, the Hybrid Quanvolutional ESN model shows superiority over the ESN model for both MG and NARMA-10 datasets. It achieves better predictive accuracy and is more computationally efficient. These results suggest that the Hybrid Quanvolutional ESN model could be a favorable choice for modeling and predicting time series. However, it is important to note that further analysis and experimentation may be necessary to confirm these findings and explore the models' generalizability to other datasets or scenarios.

6 CONCLUSIONS

In this research, we suggested a new method for time-series prediction using quanvolution filter applied to ESN. On the Mackey-Glass and NARMA datasets, we assessed the method's performance and contrasted the outcomes with those obtained using traditional ESN. The results shown that, when compared to the traditional ESN, the suggested technique outperformed it in terms of MSE and time complexity on both datasets. This suggests that by extracting more relevant features from the input data, the quantum convolutional filter may enhance the effectiveness of the ESN. It is crucial to note that the findings are still in the research stage and that more studies are required to prove the method's efficacy across a range of issues and datasets.

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