Pareto Front Approximation by Ant Colony Optimization

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Abstract: The Pareto frontier of multi-objective problem solutions denotes the unique exact solution to a problem with two or more equivalent objectives. Even when the number of problem solutions is finite, determining the precise Pareto frontier is a difficult task. Different metaheuristics can therefore provide a user with a decent approximation of the Pareto frontier in a reasonable amount of time, whereas the exact computational time-intensive approaches cannot. The acceptable computational time of metaheuristics counterbalances a solution's deviation from the Pareto frontier. This contribution describes one of a spectrum of metaheuristics implemented with the objective of locating non-dominated solutions to the public service system design problem involving two competing criteria. The metaheuristic minimizes the difference between the present set of non-dominated solutions and the Pareto front by applying the ant colony optimization principle. A series of numerical experiments with benchmarks for which the exact Pareto frontiers are known are used to evaluate the efficacy of the proposed metaheuristic. Even though the proposed method is applicable anywhere, the used dataset comes from an Emergency Medical Service system in Slovakia, which belongs to the generally known wide class of public service systems.

1 INTRODUCTION

One of those crucial application spheres where many advanced methodologies of operations research can frequently be met is the creation of various service systems. We are now able to address a wide range of challenging problems that were previously unsolvable due to huge advancements made in numerous technical disciplines, from hardware to cutting-edge software technology. One of such examples is the challenging problem of establishing a service system (Ahmadi-Javid et al, 2017, Avella, Sassano, Vasil'ev, 2007, Brotcorne, Laporte, Semet, 2003, Current, Daskin and Schilling, 2002). Let us go more specific about service systems.

Typically, a service system consists of a few components that have a structure, behavior (which may be characterized as a business process), and a purpose (people, facilities, tools, and/or software applications). The premise that a service system can be regarded as a work system that generates a certain type of specified services is an easier, but more constrained description (Doerner et al, 2005, Gendreau, Potvin, 2010, Gopal, 2013, Ingolfsson, Budge, Erkut, 2008, Jánošíková, 2007, Jánošíková, Žarnay, 2014).

In general, service systems can be split into two sizable categories. Public service systems are based on distinct presumptions, whereas private service systems are typically developed to deliver the maximum profit to their managers, founders, shareholders, and operators regardless of the number of clients served or equal access to given service.

Public service systems are designed to ensure that all locals will receive services, regardless of financial gain or loss. They are required by law to exist. These systems comprise many different things, such as state administration, emergency medical services (EMS), and many more (Ahmadi-Javid et al, 2017, Brotcorne, Laporte, Semet, 2003, Ingolfsson, Budge, Erkut, 2008, Marianov, Serra, 2002). The public service system design problem is a member of the family of location problems, which have been researched and successfully resolved by numerous authors (Avella, Sassano, Vasil'ev, 2007, Kvet, 2018).

Several factors must be taken into account while looking for a solving method for this class of huge location problems: The issues' combinatorial nature suggests that mathematical programming techniques, which have some specificities of their own, were used to solve them. The constraint of available resources is another significant factor. Additionally, the service

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must be concentrated in a number of service locations rather than being offered everywhere. Naturally, the customers who are served can travel to these centers or a team can go from a service center to the emergency location. As a result, the challenge in designing a public service system typically lies in determining the best network topology for service centers to meet a particular criterion. Based on the preliminary analyses given, the weighted p-median problem formulation is one of the modeling approaches that is most frequently employed.

The precise shape of the objective function and the modeling approach itself determine whether the problem can be solved. The so-called radial strategy can be used in place of the commonly used location-allocation form to tackle substantially bigger issue instances (Avella, Sassano, Vasil'ev, 2007, Kvet, 2018). The problem is significantly simpler to solve if the optimization criterion utilized has a min-sum form as opposed to one where the objective function has a min-max form with certain link-up constraints.

The second drawback is brought on by the limitation that only one target can be optimized. Large public service systems, like an EMS, are known to be complex systems with a variety of conflicting demands made by various stakeholder groups (Arroyo et al, 2010, Grygar, Fabricius, 2019, Janáček, Fabricius, 2021). Consequently, the primary focus of this research study is on creating multi-objective service systems. Only two opposing aims will be considered for the sake of simplicity.

The primary scientific contribution of this study is to bring a new heuristic approach to address the issue of constructing two-objective service systems. A so-called Pareto frontier of service system designs must be built since a multi-criteria optimization presumes providing a condensed set of options from which the final system design is to be selected. It takes a lot of time to obtain the entire Pareto frontier (Arroyo et al, 2010, Grygar, Fabricius, 2019, Janáček, Fabricius, 2021). Therefore, from a practical standpoint, the creation of effective heuristics is required. The quality of the set of solutions that is produced is examined and empirically confirmed on a dataset from the actual world in this study.

The structure of this article takes the following form. Section 2 is devoted to the mathematical formulation of the problem and the conflicting criteria explanation. The notion of a Pareto frontier is discussed together with the method for different Pareto sets comparison. Section 3 provides the readers with the principle of Pareto frontier approximation by gradual refinement. In the fourth section, we describe the ant colony optimization and all the ideas behind suggested solving algorithm. The fifth section contains the results of performed experiments with real-world data and finally, the last section is devoted to concluding remarks.

2 NON-DOMINATED PUBLIC SERVICE SYSTEM DESIGNS

It is vital to clarify the Pareto frontier and give the readers a mathematical description of the problem before going into detail about the incremental refinement strategy itself. Let us focus on the problem specification, now.

Finding the stations from which ambulances are sent to demand sites is a difficulty in EMS design. For the mathematical formulation of the problem, suppose that a finite set I of candidates is given. The candidates are often selected from network components that meet specific criteria for EMS station location. The resulting selection of service center locations must include exactly p entries (p is an integer less than or equal to the cardinality of I) in order for the given aim to take on its best value due to several personal, technological, or other constraints. A zero-one variable, $y_i \in \{0, 1\}$, which equals one if a center should be placed at $i \in I$ and zero otherwise, will be used to simulate the choice of where to place a service center (EMS station). A vector $\mathbf{y}$ of location variables y_i can therefore be used to define any solution to the corresponding p-location issue. The following expression (1) can describe the basic model.

$$\min\left\{ f(\mathbf{y}) : y_i \in \{0, 1\}, i \in I, \sum_{i \in I} y_i = p \right\} \qquad (1)$$

If one wanted to make the *EMS* system design problem more general, there could be added at least one extra objective. Under the assumption that m denotes the cardinality of the set I and n denotes the cardinality of the set J, the former model (1) may be rewritten into the form of (2).

$$\min\left\{ f_1(\mathbf{y}), f_2(\mathbf{y}) : \mathbf{y} \in \{0, 1\}^m, \sum_{i=1}^{m} y_i = p \right\} \qquad (2)$$

As mentioned in the paper's introduction, combining two criteria might provide a variety of difficulties for the decision-making process, especially when the aims are incompatible. We can only concentrate on two objective functions, $f_1(\mathbf{y})$ and $f_2(\mathbf{y})$, which will be referred to as so-called system and fair criteria, respectively. To formulate them in a mathematical way, several notations are necessary to be introduced.

Let J represent the set of locations of system users (service recipients). Analogically, let the symbol I stand for the set of candidates for facility locations. The sets I and J can be equivalent. The number of unique users located at j from J will be represented by the value of b_j. The quantity of anticipated demands during a specific period is one possible interpretation of this nonnegative integer coefficient. It can be understood as the weight of location j, though. According to the possible center location i, the disutility for a patient located at j will be indicated as d_{ij}. Despite the benefits of integer values, the value of d_{ij} need not be an integer. It is important to remember that service requests come in at random; therefore, the most nearby station need not be used to cover the current demand that has emerged anywhere. From a mathematical standpoint, it is assumed that r nearest located centers participate in offering the service to users, and q_k signifies the possibility that the k-th nearest center is the one that is closest and easily accessible at the time the demand occurs. To complete the formulations of the system and fair objective functions, let the function min_k return the k-th smallest element from the list in the function's parameter. The system criterion $f_1(\mathbf{y})$ optimizes the average distance between system users and the closest available center. It can be formulated by (3). Optimization of the average distance may be achieved by minimization of the sum of distances.

$$f_1(\boldsymbol{y}) = \sum_{j=1}^{n} b_j \sum_{k=1}^{r} q_k \min_k \left\{ d_{ij} : i \in \{1, ..., m\},\ y_i = 1 \right\} \tag{3}$$

If we wanted to evaluate the average distance $AvgDist$, it could be done in the following way described by (4).

$$AvgDist = \frac{f_1(\boldsymbol{y})}{\sum_{j=1}^{n} b_j} \tag{4}$$

The number of users whose distance from the closest facility exceeds the radius D is expressed by the *fair* objective function value $f_2(\mathbf{y})$, which was developed in accordance with formula (5). To provide certain level of fairness (Bertsimas, Farias, Trichakis, 2011, Buzna, Kohání, Janáček, 2013).

$$\begin{gathered} f_2(\boldsymbol{y}) = \sum_{j=1}^{n} b_j \max\{0, sm\} \\ sm = sign\left(\min\left\{ d_{ij} : i \in I, y_i = 1 \right\} - D \right) \end{gathered} \tag{5}$$

There is no doubt that the criteria (3) and (5) are in direct conflict. It suggests that improving one would inevitably make the other worse. The aforementioned goal conflict can be resolved by creating a full Pareto frontier of solutions, or at least, its approximation. In other words, rather than one final system design, a specific small subset of options are presented. Naturally, in order to select one of the available options and arrive at the resulting system design, politics, negotiation, and experts must be involved. Let us focus on Pareto frontier, now.

A Pareto frontier is often made up of a few solutions that satisfy non-dominance for each pair of its members. No matter what form a feasible solution P takes, it may be evaluated using the two criteria $f_1(P)$ and $f_2(P)$ in the bi-criteria optimization. The non-dominance can be explained by the following: A solution P is referred to as a non-dominated solution if $[f_1(P), f_2(P)] \neq [f_1(R), f_2(R)]$ matches the inequality $f_1(P) < f_1(R)$ or $f_2(P) < f_2(R)$. Then, a straightforward explanation of the Pareto frontier is given in the following Figure. 1.

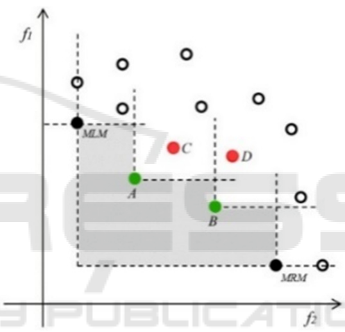

Figure 1: Explanation of the Pareto frontier.

The red solutions do not belong to the Pareto frontier because of being dominated by the green ones. Since there are no solutions dominating the green elements, both of them must be included into the Pareto frontier (Grygar, Fabricius, 2019, Janáček, Fabricius, 2021).

Sometimes, symbols *MLM* and *MRM* are used to denote the most left and the most right members of the Pareto frontier. These bordering solutions can be computed easily making use of a simple mathematical model solvable usually in a short time. We will concentrate our efforts on creating a good approximation of the Pareto frontier because it is a challenge to obtain the entire Pareto set. The sequence of *noNDSS* (non-negative integer value) solutions $\boldsymbol{y}^1$, …, $\boldsymbol{y}^{noNDSS}$ ordered according to increasing values of f_2 will be used to represent the approximate set of non-dominated solutions (*NDSS*). The bordering solutions $\boldsymbol{y}^1$ and $\boldsymbol{y}^{noNDSS}$ must be found to be very close to the most left and the most right solutions of the Pareto frontier in terms of the values of $f_1(\mathbf{y})$ and $f_2(\mathbf{y})$, in order to achieve a useful approximation. Under these presumptions, so-called area *A*(*NDSS*) computed in accordance with expression (6), can be used to assess the accuracy of the *NDSS* as the Pareto frontier approximation. The size of the grey polygon in Figure 1 is represented by the *A*(*NDSS*) to make it easier to comprehend.

$$A(NDSS) = \sum_{k=1}^{noNDSS-1} dif_{k1} dif_{k2}$$
$$dif_{k1} = f_1(\mathbf{y}^k) - f_1(\mathbf{y}^{noNDSS}) \quad (6)$$
$$dif_{k2} = f_2(\mathbf{y}^{k+1}) - f_2(\mathbf{y}^k)$$

Whenever the Pareto frontier is needed to be approximated by a set *NDSS*, the quality of the approximation must be evaluated. For this purpose, so-called *gap* may be used. The *gap* can be defined as follows: Under the assumption that *PF* denotes the original complete Pareto frontier and *NDSS* denotes its approximation, the *gap* can be evaluated in percentage according to the expression (7).

$$gap = 100 * \frac{A(NDSS) - A(PF)}{A(PF)} \quad (7)$$

3 GRADUAL REFINEMENT SCHEME

Many alternative methods, some of which may also be based on the decrementing neighbourhood search algorithm, can be used to generate a Pareto frontier or at least a good approximation of one. The schema of its gradual refining is one of the approaches that might be used in the development of the *NDSS* (Janáček, Kvet, 2022a, Janáček, Kvet, 2022b, Janáček, Kvet, 2022c, Kvet, Janáček, 2022).

The two-element initial *NDSS* of the most left and the most right bordering solutions of the Pareto frontier serves as the basis for the process. These bordering solutions are simple to compute, and getting them usually doesn't take too much effort. The previously mentioned refining process is repeated. This indicates that the results of processing one round produce a set of *NDSS* data that may be utilized as an input set for the subsequent inspection procedure. The results of a process that is repeated more than once may have different outcomes since the inbuilt decrementing algorithm may employ random actions. As a result, the inner cycle is nested inside a time-controlled cycle, which keeps repeating the inner cycle until the time limit is reached. In other words, *NDSS* refining continues until a time limit prevents algorithm performance.

The gradual refinement approach processes the input *NDSS* solutions iteratively one by one. The elements of *NDSS* are expected to form a sequence $\boldsymbol{y}^1$, …, $\boldsymbol{y}^{noNDSS}$. If any solution $\boldsymbol{y}^k$ is processed, then a neighborhood search algorithm may be applied to find candidates for *NDSS* updating. As *NDSS* can change during one run of the algorithm, the solution corresponding with the *k*-th position may also change. If it happens, the algorithm is applied once again to this new solution $\boldsymbol{y}^k$, otherwise the following solution y^{k+1} is processed. If $k = noNDSS$-1, the refinement process terminates (Janáček, Kvet, 2022b, Kvet, Janáček, 2022).

4 ANT COLONY OPTIMIZATION

The ant colony optimization algorithms imitate in general cooperation of ant colony members in searching food which is distributed in nodes of a network of possible ants' moves. In most of implementations, an ant is an agent, who searches for an improving solution in a finite set of problem solutions equipped with a topology given by system of neighbourhoods. The ant chooses its inspected way in the network which vertices are represented by individual solutions and which edges correspond to passible moves from one solution to a neighbouring one. As the set of feasible solution is too vast to be able to record real path in the associated network, only some attributes of the path inspection are taken into account. Performance of an ant starting at a given solution will be determined by the neighbourhood searching strategy and buy swap operations, which implement moves between the network vertices. The searching strategy is determined by a combination of two parameters *thr* and *maxNos*, where the first one gives minimal improvement to consider a move admissible and the second parameter gives the number of admissible moves, from which the best one is realized. The swap operation replaces one location of a current solution by a location which is not included in the solution.

Instead of recording the inspected path in detail, we reduce the path description to the set of location which have been subsequently included into the starting solution. These recorded entries will be considered in the phase of laying pheromone. A detailed description of the ant's search and pheromone laying follows.

The ant starts with a given feasible solution $\boldsymbol{y}$ of the *p* locations saved in the list *P* of the locations selected from the set I of all possible service center locations.

Based on thickness of the pheromone layer, the ant randomly choices the strategy given by a pair of the parameters *thr* and *maxNos*.

Following the chosen strategy, the ant searches the neighbourhood of the current solution while constantly updating the *NDSS* and evaluating each found admissible swap operation applied to the exchange of location $i \in P$ for $j \in I\text{-}P$ from the point of move to a new current solution.

The admissible operation is characterized by a decrement Idec *of the A*(*NDSS*) caused by its performing. The inserted location j has its pheromone layer $F(j)$ and the resulting fitness value is $Dec*F(j)$. The ant's decision on the best swap for the move to the new solution is performed according to the result of a sequence of comparisons. The recently appointed swap operation with fitness value *MFit* is compared to fitness *CFit* of a candidate and it is decided on update according to a random trial with probability $CFit/(CFit+MFit)$ in favor of the candidate. The ant's search finishes, when either whole neighbourhood is inspected or *ImaxNos* candidates are evaluated.

The ant's search terminates with failing of finding an admissible candidate for the move to a new current solution. The difference between the $A(NDSS)$ before the ant's search and $A(NDSS)$ after the search denoted by *Dec* is used to update the pheromone layer $F(s)$ of the chosen strategy s and all inserted locations j using the following formula (8).

$$F(s) = F(s) + \frac{Dec}{InitArea} \tag{8}$$

The final pheromone adjustment is performed with a pheromone layer of each object according to the expression $F(s) = (1 - \rho)*F(s)$, where ρ is an evaporating coefficient.

5 NUMERICAL EXPERIMENTS

Suggested ant colony optimization for Pareto fro frontier nt approximation was explained in previous sections of this contribution. To verify its efficiency and accuracy, several computational analyses needed to be performed. The content of this section is aimed at the results of performed numerical experiments. Let us concentrate on available software tools and technical parameters of used machine, first.

All computational experiments reported in this study were performed making use of Java programming language within the NetBeans development kit. The algorithms were run on a common notebook equipped with the 11th Gen Intel® Core™ i7 1165G7 2.8 GHz CPU and 40 GB RAM.

After having introduced necessary software and hardware tools for this computational study, let us describe solved problems and their most important characteristics.

As far as the set of used problem instances is concerned, we took the benchmarks from our previous research (Grygar, Fabricius, 2019, Janáček, Fabricius, 2021, Janáček, Kvet, 2020 , Janáček, Kvet, 2021, Janáček, Kvet, 2022a, Janáček, Kvet, 2022b, Janáček, Kvet, 2022c, Kvet, Janáček, 2022). Mentioned dataset represents the existing EMS system operated by private agencies - service providers in eight autonomous higher territorial units in Slovakia. The list of problem instances covers the regions of Bratislava (BA), Banská Bystrica (BB), Košice (KE), Nitra (NR), Prešov (PO), Trenčín (TN), Trnava (TT) and Žilina (ZA). It must be noted that all network nodes represent both the set of candidates for service center locating and the set of clients being provided with urgent healthcare service as well. But generally, the sets of candidate locations and system users' locations may differ. The number of users b_j located in each node j from J were taken from the official Slovak statistical analysis and the values were rounded up to hundreds.

As the objective function $f_1(\mathbf{y})$ follows the concept of so-called generalized disutility (Grygar, Fabricius, 2019, Janáček, Fabricius, 2021, Kvet, 2014), the parameter r was set to 3. The associated probability coefficients q_k were set so that q_1 = 77.063, q_2 = 16.476 and $q_3 = 100 - q_1 - q_2$. These values correspond also to the data used in our previous research to make the results of different methods comparable. More details about the parameter settings suitable for the objective function (3) can be found in (Jankovič, 2014).

Parameter D used in the fair objective (5) was set to the value of 10.

The following summary reported in Table 1 brings the overview of used benchmarks. The structure of the table is designed so that each row corresponds to one solved problem. The first column of the table is used to identify the instance by the abbreviation of the region. The second column denoted by $|I|$ reports the cardinality of the set I. In other words, there are reported the numbers of candidates for service center locating, from which exactly p elements are to be chosen. The values of p are reported in the third column of the table. The right part of Table 1 is used to summarize the most important characteristics of the complete Pareto fronierts. The column denoted by *NoS* gives the number of solutions creating the entire Pareto set. In the column denoted by *A*(*PF*) we provide the readers with the size of the polygon defined by the Pareto frontier elements as suggested by the expression (6).

Table 1: Benchmarks sizes and the exact Pareto frontiers characteristics.

Region	|*I*|	*p*	*NoS*	*A*(*PF*)
BA	87	14	34	569039
BB	515	36	229	1002681
KE	460	32	262	1295594
NR	350	27	106	736846
PO	664	32	271	956103
TN	276	21	98	829155
TT	249	18	64	814351
ZA	315	29	97	407293

The achieved results are summarized in Table 2 and Table 3 which take the same structure. Since many heuristic methods may perform a random trial or they need generating random numbers, we have performed ten runs of the algorithm for each benchmark, and we report the average values of all studied parameters. While Table 2 contains the results of experiments with the first four datasets, Table 3 summarizes the second half of problem instances. Each column of the tables corresponds to one solved problem and each row is used to one studied characteristic.

Table 2: Results of ant colony optimization– part 1.

Region	BA	BB	NR	KE
CT [s]	300.0	320.0	315.5	302.4
noNDSS	33.0	206.6	250.6	102.1
Gap [%]	1.47	1.03	2.21	0.72
NoTR	1227.7	1.9	1.5	9.1
NoTOR	4205.1	8.9	10.8	52.2

Table 3: Results of ant colony optimization – part 2.

Region	PO	TN	TT	ZA
CT [s]	357.6	301.3	300.5	303.0
noNDSS	263.5	94.4	62.2	92.4
Gap [%]	2.30	0.72	0.07	0.30
NoTR	1.8	32.0	56.0	15.8
NoTOR	5.0	110.3	220.5	64.5

It must be noted that the computational process was limited to five minutes of processing. Let the symbol *CT* [s] denote the average computational time. The second studied feature consists in the average number of found solutions, which approximate the original Pareto frontier. This result is denoted by *noNDSS*. Since we do not consider it useful to report the exact values of areas computed according to (6), we evaluate the quality of approximation by gap computed by (7). The column denoted by *NoTR* reports the number of time runs and *NoTOR* denotes the number of outer runs.

6 CONCLUSIONS

Bi-criteria optimization is crucial when the specifics of a problem do not permit the use of a simple model that minimizes only one objective function. A typical example is the design of emergency medical services. Operations researchers and other experts have concentrated their efforts on the development of heuristic methods that can approximate the optimal Pareto frontier in a much shorter amount of time, as the search for the optimal Pareto frontier has proven to be an arduous endeavor. We provided numerical experiment results for evaluating the quality of the proposed algorithms. Based on the reported results, it can be concluded that the proposed algorithm substantially extends the state-of-the-art tools for solving specific location problems involving the optimization of two contradictory objectives.

Future research may concentrate on new advanced algorithms that generate a close approximation of the Pareto frontier or on modifying certain existing techniques to obtain more precise results. Application of self-learning methods to parameter adjustment represents a fruitful research direction.

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