

Twisted Strategy Bolsters Minority Cooperator Populations

Akihiro Takahara¹ and Tomoko Sakiyama²^a

¹Information Systems Science, Graduate School of Science and Engineering, Soka University, Tokyo, Japan

²Department of Information Systems Science, Faculty of Science and Engineering, Soka University, Tokyo, Japan

Keywords: Spatial Prisoner's Dilemma, Populations, Minority.

Abstract: Defectors tend to survive in the spatial prisoner's dilemma. Thus, many studies have sought to keep the cooperator alive. Here, we aimed to enhance the survival of the cooperator by considering the memory length in the spatial prisoner's dilemma. In the proposed model, all players are assigned a memory length. Based on this memory length, players updated their strategies to those that were harder to choose in the past only when the score of each neighbor with the same strategy was high. This above strategy update rule therefore alleviates a disadvantageous situation for the player. In this paper, we focused on two cases where the cooperators were initially in the minority and observed their evolution over time. The results showed that the model eventually strives to maintain the cooperator population even when it was initially low.

1 INTRODUCTION

Cooperative behaviors are characteristic of several animals including humans (Smith and Price, 1973). Game theory presents the evolution of cooperation among defective players (Nowak and May 1992, Marko et al. 2022). In classical game theory, players have two different strategies: the cooperative strategy or the defector strategy. Defectors earn higher payoffs against the opponent if the opponent is cooperative. However, defectors earn a low payoff against the defector opponent (Doebeli and Hauert, 2004, Hauert and Doebeli, 2005). On the other hand, cooperators share payoffs with each other if they mutually interact with each other. Using the payoff matrix, classical game theory has revealed that cooperators cannot survive under some conditions (Doebeli and Hauert, 2004). To this end, many models have been developed for the sake of the evolution of cooperative players (Qin et al. 2018, Sakiyama and Arizono, 2019, Sakiyama, 2021).

Recently, we developed a spatial prisoner's dilemma (SPD) model called the twisted PD (TPD) model, where players considered the past occurrence of each strategy for themselves and sometimes ignored the classical strategy update rule (Takahara and Sakiyama, 2023). At that time, players adopted an unlikely strategy. As a result, the TPD model


outperformed the classical SPD. In fact, studies have revealed that introducing memory to players in the system facilitates cooperation (Danku et al. 2019, Deng et al. 2017, Javarone, 2016).

In this paper, we analyzed the flexibility of the TPD model by considering a situation where the cooperator population was a minority in the initial spatial distribution. In other words, most of the population was a defector. Under these conditions, a cooperative population developed in the TPD model over time.

2 METHODS

2.1 Simulation Environments

A 100×100 square lattice was formed. Players were placed in all cells and initially assigned a cooperator (C) or defector (D) strategy. There were two types of initial distributions of strategies: one where the value of initial density of defector r was set to 0.5, 0.9, 0.95, or 0.99 while a random uniform distribution was used for the players, and one where cooperators were placed on the center cell and its neighboring four cells in a fixed distribution, while the remaining players were defectors. We therefore assessed the

 <https://orcid.org/0000-0002-2687-7228>

performance of the model where cooperators were initially in the minority.

Payoff was set to $T = b$, $R = 1$, and $S = P = 0$ based on the payoff matrix shown in Table 1, where $T > R > P > S$. The parameter b that determines T was set to $1 < b < 2$ (Nowak & May, 1992). A player with strategy D received T if the neighboring player was assigned strategy C. A player with strategy C received S if the neighboring player was assigned strategy D. If both strategies were D, the player earned P . However, if both strategies were C, the player received R . We used the Neumann neighborhood and periodic boundary conditions. Individual players interacted with players above, below, and to the left, and right of them. Each trial was included 1000 time steps.

Table 1: Payoff matrix.

		neighbor	
		C	D
Player	C	$R(1)$	$S(0)$
	D	$T(b)$	$P(0)$

2.2 Model Description of SPD

The iteration was initiated after a strategy was assigned to each player, who compared neighborhoods, and strategies based on the payoff matrix and calculated a score. After completing this task, they compared score with their neighbors and memorized the strategy of the neighbor with the highest score. The strategy of each player was then synchronously updated to the learned strategy. However, the strategy was not updated if there were multiple nearby players with the same highest score but different strategies.

2.3 Model Description of the TPD Model

Here, we describe the twisted Prisoner’s Dilemma model (TPD model) (Takahara & Sakiyama, 2023), where every player is assigned a length of memory of value p that was constant between trials. After the score was calculated, each player reflected on his or her previous strategy. The length of the past considered is from t (current) to $t - p$, and the number of experienced cooperative strategies was recorded in the parameter $Count_c$.

If neighboring players had the same strategy while the player had a lower score, the player updated their strategy using one of the two following probabilities:

The player will update its strategy to C with the following probability:

$$1 - (Count_c)/p$$

The player will update its strategy to D with the following probability:

$$(Count_c)/p$$

If the above conditions were not satisfied, the rule of the SPD model was applied for the strategy update.

The strategy of each player synchronously updated. In this model, the strategy update rule to use the values of p that was different from the SPD model rules was not executed until t was greater than p . The proposed model was based on the following concept: the player changes their behavior when their score is lower than that of neighbors who have the same strategy.

3 RESULTS

3.1 Defector Density

First, the r was set to 0.5, 0.9, 0.95, and 0.99, whereas p was fixed at 10. The defector density over 1000 time steps was calculated by averaging 10 trials. The results are shown in Figure 1. We found that an initially large defector populations did not affect the evolution of cooperators, though cooperators did not survive if r was set to 0.99. This is perhaps because not enough cooperators are placed, and they cannot interact with each other.

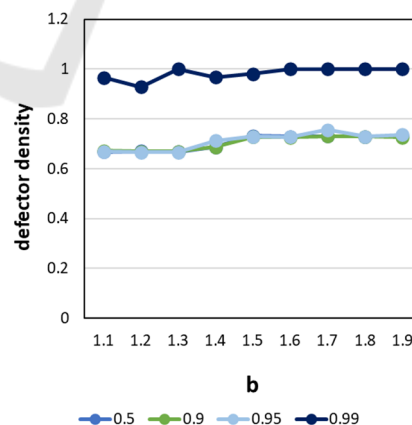


Figure 1: Defector density for various values of r (0.5, 0.9, 0.95, 0.99).

Next, we switched the initial distribution of players to the second condition, where each cooperator was placed on the center cell and its

neighboring four cells. The remaining players were defectors. Results were compared with $r = 0.99$ as shown in Figure 2, where the defector density was lower than $r = 0.99$ and remained around 0.80 for any value of b . Although the initial density was much higher than $r = 0.99$ in the density with a fixed initial distribution, the defector did not increase as much as $r = 0.99$.

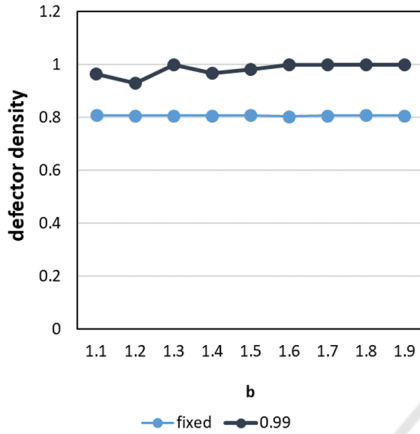


Figure 2: Defector density of the two initial distributions.

3.2 Spatial Distribution

Next, we compared the spatial distribution of the fixed initial distribution between the TPD model and the SPD model. Here, we set the parameter $b = 1.9$.

As shown in Figure 3, the distribution of cooperators in the TPD model was spread out from the center at $t = 10$ but sparse at $t = 1000$. However, the distribution remained constant over time in the SPD model.

In both models, the score of the player at the center was 4 at $t = 1$, and the neighbors of the centered player adopted strategy C. Their strategy did not change because the player in the neighborhood with the highest score is the one in the center. In the TPD model, this process is repeated until $t = 9$ according to the SPD model rule where $p = 10$. Therefore, the distribution of strategies did not change until $t = 9$.

However, neighbors of the centered player considered previous strategies and followed an unusual update rule because their own strategy earned a score lower than that of the centered player, even though their strategies were the same at $t = 10$. As a result, they adopted strategy D according to the rules of the TPD model.

Even though players outside of the region of interest described above adopted strategy D, their scores were lower than the player whose strategy was D and who neighbors the player in C, so those players

were likely to change their strategy to C. As a result, a diamond-like shape formed, and repeated many times; a constant number of players with strategy C survived at $t = 1000$. However, in the SPD model, the strategy distribution maintained its shape and did not deviate from the initial distribution even at $t = 1000$. Therefore, the cross-like shape in the spatial distribution of the TPD model during early stages contributed to cooperator survival.

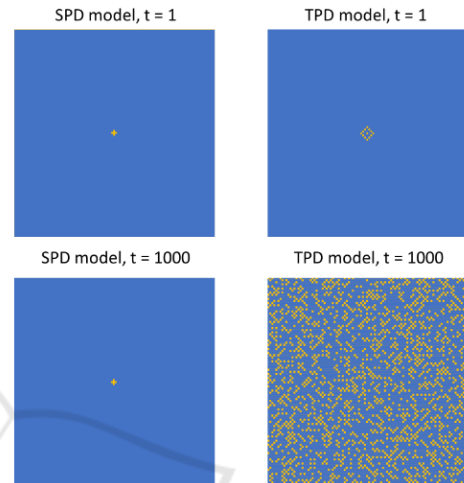


Figure 3: Spatial distribution of fixed initial density in two models.

4 CONCLUSIONS

In this paper, we evaluated the TPD model in two cases where cooperators were in the minority of the population. In the first case, players of each strategy were randomly distributed according to the defector density parameter r . As we considered cooperative populations as a minority group, the parameter r had high values. We found that cooperators could evolve despite their low initial density. A fixed distribution was used in the second case, where only five players in the center of the system adopted cooperative strategies while others were initially defectors. However, the number of cooperators increased over time. Interestingly, the initial number of cooperators in the second condition was lower than that of the first condition with $r = 0.99$, and the final cooperator population in the former was higher than the latter, suggesting that the initial placement of cooperators influences outcomes.

REFERENCES

- Danku, Z., Perc, M., Szolnoki, A. (2019). Knowing the past improves cooperation in the future. *Scientific Reports*. 9, 262.
- Deng, Z., Ma, C., Mao, X., Wang, S., Niu, Z., Gao, L. (2017). Historical payoff promotes cooperation in the prisoner's dilemma game. *Chaos, Solitons & Fractals*. 104, 1–5.
- Doebeli, M., Hauert, C. (2005). Models of cooperation based on the Prisoner's Dilemma and the Snowdrift game. *Ecology Letters*. 8, 748–766.
- Hauert, C., Doebeli, M. (2004). Spatial structure often inhibits the evolution of cooperation in the snowdrift game. *Nature*. 428, 643–646.
- Javarone, M. A. (2016). Statistical physics of the spatial prisoner's dilemma with memory-aware agents. *The European Physical Journal B*. 89, 1–6.
- Jusup, M., Holme, P., Kanazawa, K., Takayasu, M., Romić, I., Wang, Z., Geček, S., Lipić, T., Podobnik, B., Wang, L., Luo, W., Klanjšček, T., Fan, J., Boccaletti, S., Perc, M. (2022). Social physics. *Physics Reports*. 948, 1–148.
- Smith, J. M., Price, G. R. (1973). The logic of animal conflict. *Nature*. 246, 15–18.
- Nowak, M. A., May, R. M. (1992). Evolutionary games and spatial chaos. *Nature*. 359, 826–829.
- Qin, J., Chen, Y., Fu, W., Kang, Y., Perc, M. (2018). Neighborhood diversity promotes cooperation in social dilemmas. *IEEE Access*. 6, 5003–5009.
- Sakiyama, T. (2021). A power law network in an evolutionary hawk–dove game. *Chaos, Solitons & Fractals*. 146, 110932.
- Sakiyama, T., Arizono, I. (2019). An adaptive replacement of the rule update triggers the cooperative evolution in the Hawk–Dove game. *Chaos, Solitons & Fractals*. 121, 59–62.
- Takahara, A., Sakiyama, T. (2023). Twisted strategy may enhance the evolution of cooperation in spatial prisoner's dilemma. Submitted.