Multiverse: A Deep Learning 4X4 Sudoku Solver

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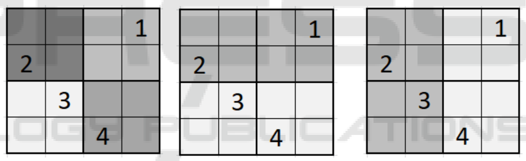
Abstract: This paper presents a novel deep learning-based approach to solving 4x4 Sudoku puzzles, by viewing Sudoku as a complex multi-level sequence completion problem. It introduces a neural network model, termed as ”Multiverse”, which comprises multiple parallel computational units, or ”verses”. Each unit is designed for sequence completion based on Long Short-Term Memory (LSTM) modules. The paper's novel perspective views Sudoku as a sequence completion task rather than a pure constraint satisfaction problem. The study generated its own dataset for 4x4 Sudoku puzzles and proposed variants of the Multiverse model for comparison and validation purposes. Comparative analysis shows that the proposed model is competitive with, and potentially superior to, state-of-the-art models. Notably, the proposed model was able to solve the puzzles in a single prediction, which offers promising avenues for further research on larger, more complex Sudoku puzzles.

1 INTRODUCTION

In this paper we propose a deep learning model for solution of 4X4 Sudoku puzzles and show that the model can solve over 99 percent of the puzzles provided to it in just a single prediction while state-of-the-art systems needed more prediction iterations to attain similar results.

The Sudoku puzzle was first introduced in the 1970s in a Dell magazine. It became fairly known throughout Japan. In the early 2000s, the puzzle started becoming extremely popular in Europe and then in the United States, igniting an interest not only in the form of competitions but also in the form of scientific research (see (Hayes, 2006) for a detailed background). The popularity of the puzzle caused a lot of research to be done regarding its logic-based properties and the diverse methods for solving it.

The Sudoku puzzle is composed of a square of cells with o^2 rows and o^2 columns for some natural constant o called the *order* of the puzzle. The square is subdivided into its o^2 primary $o \times o$ squares called *boxes* or *sub-squares*. The boxes divide the rows and columns into o sets of rows and o sets of columns called *row groups* and *column groups* respectively. The puzzle begins with some placement of values $1 \cdots o^2$ in some of the cells called *givens* or *hints*. The object of the puzzle is to fill the rest of the cells with values so that every row, column, and sub-square will have all the values from $1 \cdots o^2$. A *block* is either a row, a column or a sub-square interchangeably.



(a) Boxes. (b) Row Groups. (c) Column Groups.
Figure 1: Depiction of the Sudoku variables.

Sudoku is considered a logic based puzzle. In fact, there are many types of logic that must be combined to solve the hardest of puzzles, ranging from trial and error to deduction and from inference to elimination.

A Sudoku can, technically, have more than one correct solution. A *Well Posed* Sudoku is a puzzle that has only one correct solution. A puzzle can be easier to solve if there are redundant hints, namely givens that can be deduced from one or more other givens. A puzzle with no redundant hints is called *Locally Minimal* (Simonis, 2005). Figures 2 and 3 demonstrate examples of the types of puzzles and their solutions. The puzzle in Fig. 2a is locally minimal because removing any of the numbers will cause it to have more than one solution thus no longer being well posed. For example, removing the 2 will cause Fig. 3b to be a valid solution as well; removing the 1 will make Fig.

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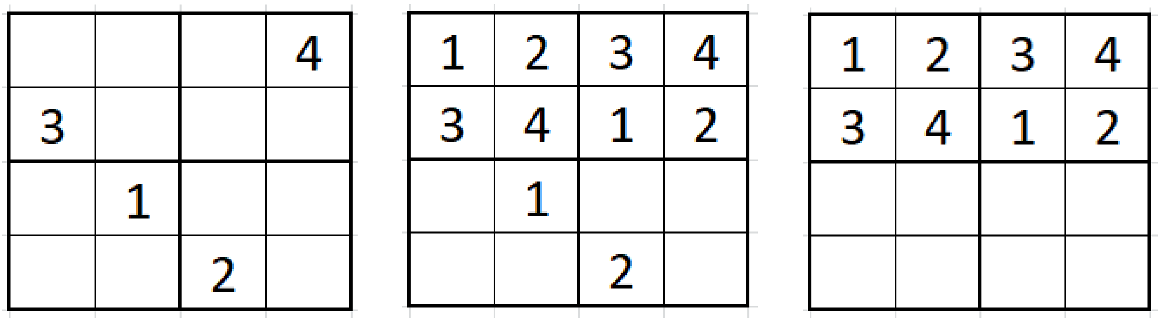

(a) Well posed, locally minimal. (b) Only well posed. (c) Not well posed (4 solutions).

Figure 2: Examples for well posed and locally minimal puzzles.

3d be a valid solution, and so on.

Sudoku can further be classified as a discrete constraint satisfaction problem (CSP). In particular, Sudoku is a special case of CSPs called an exact cover problem, where each constraint must be satisfied by only one variable in the solution assignment. Any algorithm that can solve discrete CSPs or exact cover problems will be able to provide a solution for a Sudoku, albeit usually in exponential run-time.

Sudoku can also be approached as a Machine Learning problem, thus described as a multi-class multi-label classification problem, meaning that a given puzzle has multiple values that need to be determined (namely the values of the unassigned cells) and each of those is one discrete value of a shared domain of integers. Therefore, there are various machine learning and deep learning methods that might be applicable to it. Unfortunately, a very high level of generalization is required for learning the implicit connections between the cells so simple methods are not good enough here (Palm et al., 2018). Some progress has been made in this avenue of research, particularly combining deep learning with other methods (Wang et al., 2019) or using networks with recurrent prediction steps (Palm et al., 2018; Yang et al., 2023).

In this study, we introduce a novel approach to tackle Sudoku solving by leveraging a neural network architecture. Our methodology is rooted in the notion that Sudoku puzzles share similarities with intricate sequential data completion problems. To address this, we devised a specialized sequential completion unit based on Long Short-Term Memory (LSTM) and interconnected multiple such units in a parallel fashion. Remarkably, our resultant model exhibits comparable competence to state-of-the-art models, while obviating the need for multiple prediction iterations. Notably, we demonstrate a substantial disparity between our model and existing approaches in terms of performance when such iterations are excluded.

Drawing upon the notion that Sudoku represents a complex sequential data completion task, we adopted bespoke deep learning techniques to tackle its solution. Recognizing that the puzzle's completion process entails a multi-dimensional sequence, our proposed model incorporates multiple parallel computational units. For the sake of simplicity, our investigation primarily focuses on training the model on 4x4 puzzles that exhibit local minimality. Through one-shot prediction, we achieved an impressive completion rate exceeding 99 percent, effectively showcasing the neural network's capacity to grasp the abstract relationships among the puzzle's cells.

To the best of our knowledge, even though Large Language Models (LLMs) have been used for Sudoku solving, there has not been previous research based on the premise that Sudoku is a sequential completion problem. We show that this approach is effective and justifiable.

2 RELATED WORKS

Since solution by logic based algorithms has proven intractable for difficult puzzles of high order, automated Sudoku solving has become the object of a large amount of highly varied research.

Simonis (Simonis, 2005) did a thorough job defining the basic constraints in Sudoku puzzles. He also showed that these constraints can be described in various ways, creating additional redundant constraints with which the puzzles can be solved more efficiently. In his paper, he compares 15 different strategies based on constraint propagation and showed that the well posed puzzles in his dataset can all be solved by an all-different hyperarc solver, if the execution starts with one shaving move.

Hunt et al. (Hunt et al., 2007) took this one step further by showing that the constraints can be realized in a binary matrix solvable by the DLX algorithm provided by Knuth (Knuth, 2000) and that many of the common logical solving techniques can be sum-

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

(a) The solution for Fig. 2.

1	2	3	4
3	4	1	2
4	1	2	3
2	3	4	1

(b) 2nd solution for Fig. 2c.

1	2	3	4
3	4	1	2
4	3	2	1
2	1	4	3

(c) 3rd solution for Fig. 2c.

1	2	3	4
3	4	1	2
2	3	4	1
4	1	2	3

(d) 4th solution for Fig. 2c.

Figure 3: Solutions for puzzles in Fig. 2.

marized in a singe theorem based on that matrix. The advantages of using DLX is that the algorithm is faster than other backtracking algorithms and also provides the number of possible solutions thus giving indication if a puzzle is well posed or not. The disadvantage in using DLX is that since it is a backtracking algorithm its run-time is exponential by nature.

Another related approach can be found in the works of Weber (Weber, 2005), Ist et al. (Ist et al., 2006) and Posthoff and Steinbach (Posthoff and Steinbach, 2010) who all model Sudoku as a SAT problem and use various SAT solvers to solve it. This method utilizes powerful existing systems but requires explicit definition of the rules and layout of each puzzle, a number of clauses which could expand exponentially with more complex puzzles. In our study we favoured machine learning because it obviates the need of explicit description of the logic problem.

As mentioned in the introduction, deep learning is also a good candidate for Sudoku solving. The advantage of machine learning systems in general and deep learning systems in particular is that they can generalize a direct solution for a problem without having to use algorithms of exponential or worse runtimes to solve particular instances of the problem. Once trained, such a model could be significantly better even than the DLX algorithm.

Park (Park, 2018) provides a model based on a standard convolutional neural network that can solve a Sudoku in a sequence of interdependent steps. It solved 70 percent of the puzzles it was tested on, using a loop that predicted the value of the one highest probability cell in each iteration.

Palm et al. (Palm et al., 2018) created a graph neural network that solves problems that require multiple iterations of interdependent steps and showed that it solved more than 96 percent of the Sudokus presented to it, including very hard ones as well, although the solution in this method had to be done in a sequence of interdependent steps. While the results achieved are noteworthy, it's important to acknowledge that they necessitated both an understanding of the problem structure coded manually and a series of predictive steps towards a solution. In contrast, our study eliminates the need for prior knowledge of the problem's architecture and successfully resolves the puzzles in a single prediction step.

Wang et al. (Wang et al., 2019) combined a SAT solver with neural networks to add logical learning methods that can overcome the difficulty traditional neural networks have with global constraints in discrete logical relationships. Using that combination they succeeded in attaining a 98.3 percent completion rate without any hand coded knowledge of the problem structure. This approach differs from our approach in that it requires the use of a SAT solver to complement the prediction provided by the network. Moreover, Chang et al. (Chang et al., 2020) showed that the good results presented by Wang et al. (Wang et al., 2019) are limited to easy puzzles and the results are significantly worse than those of Palm et al. (Palm et al., 2018) when trying to solve hard puzzles.

Mehta (Mehta, 2021) created a reward system for a Q-agent and achieved 7 percent full puzzle completion rate in easy Sudokus and 2.1 and 1.2 percent win rates in medium and hard Sudokus respectively all with no rules provided. She did this unaware of Poloziuk and Smrkovska (Poloziuk and Smrkovska, 2020) who tested more complex Q-based agents and Monte-Carlo Tree Search (MCTS) algorithms and came to the conclusion that they require too much computation power to be used reasonably to solve Sudoku. With MCTS they performed a small number of experiments achieving 35-46 percent accuracy and called it a success, even though the results are fairly low, considering that in each experiment the training took them days to perform.

Du et al. (Du et al., 2021) used a Multi Layered Perceptron (MLP) to solve order 2 puzzles. They created a small 4 layer dense neural network and performed their prediction stage by stage each time filling in only the one highest probability cell. Their dataset included puzzles none of which were locally minimal - all missing between 4 and 10 values. Their model solved more than 99 percent of the puzzles tested. However, the vast majority of the puzzles tested were not locally minimal and the number of prediction iterations they required were equal to the number of missing values. In our study, we not only performed experiments on locally minimal puzzles with 10, 11, or 12 missing values - the hardest well posed order 2 puzzles - and attained a completion rate greater than 99 percent, but did so in a single prediction step, albeit with a more intricate model.

Yang et al. (Yang et al., 2023) trained a generative pre-trained transformer (GPT) based model with the dataset used by Palm et al. (Palm et al., 2018) and tested it with iterative predictions. They had superior results, solving more than 99 percent of the puzzles, although when restricted to the hardest puzzles achieved a 96.7 percent completion rate. However, Yang et al. (Yang et al., 2023) required a sequence of prediction steps to reach the solution and did not provide a solution in one end-to-end prediction. They needed 32 prediction iterations to achieve their results in order 3 puzzles. The baseline for our study is the model used by Yang et al. (Yang et al., 2023) modified for order 2 puzzles. We show that our model has competitively good results in less prediction iterations than required by their model.

As can be seen above, the significant results so far have been achieved only in systems that integrate sequences of interdependent prediction stages. In this paper we propose a model that achieves competent results in a single prediction stage.

3 DEEP LEARNING METHODS

Our paper introduces a deep learning approach specifically targeted at tackling order 2 Sudoku puzzles. These are 4x4 puzzles that, while smaller in scale compared to the standard order 3 Sudokus, present a unique appeal for scientific investigation. Given their relative simplicity, both in terms of representation and analysis, focusing our research on order 2 puzzles enables more rapid training and facilitates quicker attainment of results.

We consider Sudoku to be akin to a sophisticated, multi-layered sequence completion problem. With this perspective, we developed a deep learning neural network that leverages LSTM modules designed for sequence completion. This approach has yielded results that are on par with current leading models.

While our demonstrated results are limited to order 2 puzzles, we maintain the belief that these puzzles are sufficiently complex to serve as a sound foundation for creating a successful model for higher-order Sudoku problems. The importance of studying 4x4 puzzles lies in the opportunity they provide to build, test and refine models that could be efficiently scaled to more intricate Sudoku variants. This makes them an essential stepping stone in advancing deep learning methodologies for solving larger and more complex problems.

Table 1: Well Posed Order 2 Puzzle Count.

H	WP	LM	H	WP	LM
4	25728	25728	10	2204928	0
5	284160	58368	11	1239552	0
6	1041408	1536	12	522624	0
7	2141184	0	13	161280	0
8	2961024	0	14	34560	0
9	2958336	0			

H - The number of hints in the puzzle.
WP - Well Posed, LM - Locally Minimal.

In this section we bring the technical information of our methods, in particular the data composition and the structure of our models.

3.1 Datasets

Since most of the research into Sudoku was on order 3 puzzles, we did not find an existing dataset of order 2 puzzles so we created our own.

There exist exactly 288 unique order 2 solved Sudoku boards. Those boards represent 85632 puzzles which are both well posed and locally minimal, each containing only 4, 5, or 6 hints. It is possible to create a larger number of well posed puzzles by adding more hints, although those puzzles are not locally minimal. Figures 2 and 3 demonstrate examples for the various possible types of puzzles and their solutions. Table 1 shows the full number of well posed puzzles with 4 to 14 hints and how many of them are locally minimal.

Our primary dataset consists of all 85632 well posed and locally minimal order 2 puzzles. Details on the generation process of the puzzles is provided in the appendix. The training of our models was performed on a subset of 77069 puzzles using 9-fold cross validation (We divided the puzzles into 10 subsets and left one out of the process).

The puzzles and their solutions are composed of strings of digits, where missing values are denoted as zeros. Since the values are discrete and categorical, we one-hot encoded them into five digit binary vectors in order to make processing easier.

3.2 Machine Learning Models

Below we describe the models used in this study. The first section describes our main model, a neural network architecture we call the Multiverse, which is composed of a set of parallel computational units. The next two sections describe variants of the Multiverse model that we used for our convergence study and our ablation study. These studies show that all the parts of each computational unit are important for the results attained and the results are competent with state-of-the-art models. The last section describes the model we used as a baseline for our study.

Multiverse Model

In the model description below, we use the following variables. o is the order of the puzzle ($o = 2$ for all the tests in this study). r is the size of the range of values possible in a given puzzle ($r = 5$ for all the tests in this study, to include the 4 possible values and zero). s is the length of a side of a given puzzle ($s = o^2$ always). a is the number of cells in a given puzzle ($a = s^2$ always).

Sudoku in many respects is a sequence completion problem. Each row must be completed with a permutation of the values in the range. Each column and box must be completed likewise. Each value must have a location in each row forming a sequence of value-in-row location indices that requires completion. Similarly, such sequences are also formed by values in columns and boxes. There may exist more aspects of Sudoku puzzles that can also be viewed as completion problems but that is subject for a separate study.

Our initial sequence completion unit (below referred to as a "verse") is the following sequence of deep learning layers, with reshape layers between them where necessary:

1. Conv1D, where: input size = (a, r), output size = (a, r), filters = r, kernel size = 1, strides = 1
2. Dense, where: input size = $(a \times r)$, output size = $(a \times r)$
3. Bidirectional LSTM, where: input size = (a, r), output size = $(a, 2 \times r)$, return sequences is set to true.
4. Bidirectional LSTM, where: input size = $(a, 2 \times r)$, output size = $(a, 2 \times r)$, return sequences is set to true.

Our complete model is composed of a number of parallel verses with their output concatenated into a dense softmax termination layer (hence the name "Multiverse"). A Multiverse model for order 2 puzzles with 6 parallel verses (M6) is depicted in Figure 4. The motivation for this architecture is that the combination of the convolutional and the dense layers provide a basic embedding feature that allows for different interpretations for the parallel verses. The Bidirectional LSTM is a good sequencing modeler when

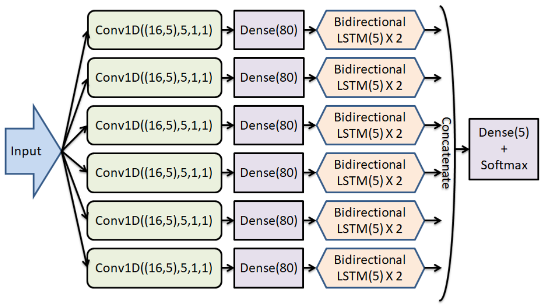

Figure 4: An order 2 Multiverse model with 6 parallel verse modules.

the direction is unimportant. Unlike NLP problems, Sudoku data is discrete and all-different removing the semantic related aspects from the problem, so other techniques that have greater effect when used on text related problems are not required here.

Convergence Study Models

Some of the configurations of the baseline model surpassed a 99 percent completion rate, albeit with more than one prediction, so we performed a study testing the number of verses required to attain such prestigious results in one-shot predictions. This study was performed on Multiverse models with 6 (See Fig. 4), 10 and 12 parallel verses (called M6, M10, and M12 respectively). M6 was our first robust model with significantly good results. We based most of our study around that model, but also tested the more powerful M10 and M12 models to show that results achieved by the baseline are attainable by our model as well. The results themselves will be detailed in Section 4.

Ablation Study Models

We performed our ablation study on the following incomplete Multiverse models with 6 verses:

- M6 - 6 complete verses.
- No Conv - 6 verses that have no Conv1D layers.
- No Dense - 6 verses that have no Dense layers.
- No LSTM - 6 verses that have no LSTM layers.
- One LSTM - 6 verses that have only the first LSTM layer.
- M5 - Only 5 verses, all complete.

Results will be detailed in Section 4.

Baseline Model

As a baseline we modified the transformer based model provided by Yang et al. (Yang et al., 2023) to fit our order 2 data. The model is based on a MinGPT module (Karpathy, 2020) set to the input of a Sudoku puzzle. MinGPT performs the computations on sparse categorical data. Therefore, the input is not one-hot encoded, rather left in numerical format with one change: all the values in the solution that correspond to hints were changed to -100. We maintained this data format in our modified model.

For a convincing comparison with our model we ran tests on the baseline model with a variety of settings for the following parameters:

- Recurrences - The number of prediction iterations. We refer to this parameter by the name *Iterations* so as not to confuse with RNN.
- Heads - The number of transformer heads used.
- Embedding - The size of the puzzle embedding.

The outcomes will be elaborated upon in Section 4.

4 RESULTS

This section contains the experiments performed in our study and the results they yielded. The first experiment recorded highlights the effectiveness of our approach by yielding an impressive completion rate greater than 99 percent in a single prediction iteration. The second experiment determines the necessity of all the components of our model by showing the vast improvement each component contributes to the results. Finally, we present a baseline experiment resulting with a favorable comparison with state-of-the-art models. All the training and the testing was performed using Google Colaboratory.

The metric used in all the tests is the completion rate, namely the percent of puzzles completed correctly out of the grand total.

4.1 Convergence Study

With the intent of maximizing the completion rate of order 2 Sudoku puzzles, our study has produced compelling findings. The performance of our model, demonstrated in Table 2 and Figure 5, underscores the strength of our research approach. Our 12 verse model successfully achieves an impressive completion rate of over 99 percent within just 22 epochs, all within a solitary prediction iteration.

This highlights the ability of our model to effectively tackle nearly all order 2 puzzles, demanding only a single predictive step for solution. This robust performance does not only exemplify the model's precision, but also its proficiency in rapidly deriving solutions.

Table 2: Convergence Study Results.

Model	No. Epochs	Completion Rate
M6	100	92 percent
M10	28	97 percent
M12	**22**	**>99 percent**

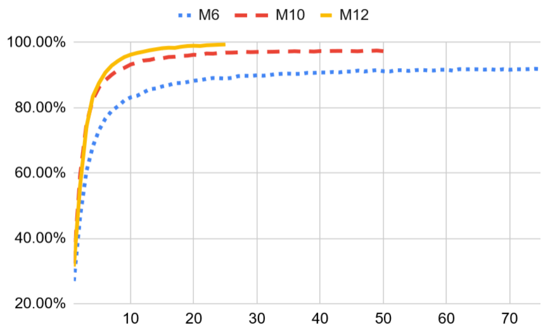

Figure 5: Completion percentages of M6, M10 and M12 models by number of training epochs.

4.2 Ablation Study

In the ablation study our intent is to show that all the parts of the model are required and important for the results achieved.

Table 3 shows the results of the ablation study. When using complete verses, the completion rates are higher than when using partial verses, as illustrated by the *M5* completion rate which is higher than all the completion rates attained by incomplete M6 variants. As far as the components of each verse are concerned, The LSTM layers and the dense layer have the greatest effect on the results as shown by the significantly low completion rates attained by the *No LSTM* and the *No Dense* models, while the convolutional layer also has an important role to play as shown by the relatively low completion rates attained by the *No Conv* model.

Table 3: Ablation Study Results.

Model	Epochs		
	5	10	15
M12	**87.63**	**96.25**	**98.21**
M10	**86.3**	**93.25**	**95.14**
M6	**73.19**	**83.18**	**86.53**
No Conv	49.22	68.49	76.11
No Dense	1.69	5.73	9.34
No LSTM	9.5	9.26	9.23
One LSTM	66.01	77.53	81.43
M5	67.04	78.41	82.4

Results are in percent of completion rate.

4.3 Baseline

We now compare our method with the state-of-the art model. To do so we made use of a modified version of the model used be Yang et al. (Yang et al., 2023).

Table 4 shows the results of the tests done on the baseline model over 15 epochs. The original order 3 model had Embedding set to 128 and Heads set to 4. Since the order 2 problems are simpler than order 3 problems we tried using less complex models without significant results. Since our M6 model might be comparable with a 6 head transformer model, we also tested some models that have 6 heads.

Table 4: Baseline Results.

Embedding	Heads	Iterations	
		1	2
16	2	0	0
16	4	0	0.0011
32	2	0.5189	24.41
32	4	0.8822	41.96
128	2	13.17	89.8
128	**4**	**25.06**	**>99**
24	6	2.42	5.28
96	**6**	**27.06**	**>99**

All tests were performed over 15 epochs.

As can be seen, none of the tests reached significant results when Iterations was set to 1. In fact, we continued the 128 embedding 4 heads model up to 200 epochs (the number of epochs used for order 3 in (Yang et al., 2023)) and the results were still only 37.19 percent completion and rising *really* slowly in contrast with the 98.21 percent achieved by M12 in just 15 epochs of training and one prediction iteration, and the 86.53 percent and 82.4 percent achieved by M6 and M5 respectively (See Table 3).

With 2 iterations the 128 embedding 4 heads model and the 96 embedding 6 heads model both achieved >99 percent completion. Even though these results are excellent, it must be noted both models required 2 prediction iterations and the results attained in just one iteration were very poor. This is in contrast with our convergence study in which we showed that the M12 model can achieve a similar >99 percent completion rate with just one iteration.

5 CONCLUSION

In this paper we approached the Sudoku problem as a sophisticated sequence completion problem and described models for solving 4X4 Sudoku puzzles.

To the best of our knowledge, previous studies have refrained from using regular end-to-end deep learning for Sudoku solving because of the implicit logical connections necessary for the solution. In our research, we show that when addressing the problem as a multi-dimensional sequence completion problem it is possible to reach competent results even with only a single prediction.

On order 2 puzzles, our M12 model successfully reached an over 99 percent completion rate with a single prediction, a result which required another prediction iteration by the baseline model. The implication of this result is that not only our model is capable of solving Sudoku puzzles despite their complexity, but may prove to have greater competence in this field than the state-of-the-art models.

Future studies should focus on trying to scale upward to order 3 puzzles, to see if the existing datasets used in other studies are enough to reach significant results in less prediction iterations. The described model is easily scalable to higher-order Sudoku puzzles by adjusting the value of o and adding more verses. However, two key challenges need to be addressed: the availability of data, as higher-order puzzles have a vast number of possible solutions making it difficult to compute and store them, and the resource requirements - including significant memory drain and longer training times when scaling up to higher-order puzzles. These issues taken into account, it would be interesting to see if competitive results could still be reached with less predictions.

Future research could also attempt to solve well posed exact cover problems using the Mutiverse model. Although the DLX algorithm is capable of solving such problems, deep learning could reduce the prediction process to polynomial runtime. The binary constraint matrix that describes an exact cover problem could be approached as an extractive sequence summary problem, where only those variable assignments that are relevant for the solution are extracted from the partially complete list of possible assignments that remain after applying the givens of the problem. The factors contributing to our model's strong performance in sequence completion may also be applicable to extractive sequence summary.

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APPENDIX

Order 2 Puzzle Dataset Generation

The generation process of locally minimal well posed order 2 puzzles and their solutions was a 3 stage process. The first stage entailed creating a file containing all the 288 unique solutions for order 2 puzzles (*solutions*). We also needed to create a file with all the 65534 possible 16 bit binary strings except for an all-0 string which is an empty puzzle and obviously not well posed and an all-1 string which is a completed solution (*bitmasks*). The *bitmasks* file was created in descending order of the amount of zeroes in the strings. In the final stage, for each solution in *solutions* we iterated over *bitmasks* and filtered the well posed and locally minimal puzzles resulting with a file with all the 85632 well posed and locally minimal puzzles and their respective solutions (*puzzles*).

Stage 1: Order 2 Solutions

Since there are only 288 unique solutions for the order 2 Sudoku problem, this stage is very straightforward. We simply took an empty puzzle, applied to it the DLX algorithm, and saved to a file all the resulting solutions.

Stage 2: Binary Strings

Creation of the *bitmasks* file was done with array manipulation. We created empty arrays and added to them indices of locations where a bit in a mask should be '1', by gradually appending to the arrays more arrays that are based on them and have more indices added to them. After that we scanned the arrays and transformed them into bit strings.

The algorithm runs in $\Theta(2^{16})$ for order 2 puzzles, and if modified for order o puzzles its runtime is $\Theta(2^{area})$, where $area = side^2$ and $side = o^2$. It uses a similar amount of memory.

Stage 3: Order 2 Puzzles

This stage approaches the binary strings in *bitmasks* as binary masks. We consider a mask a to *cover* another mask b if for every i where $b[i] =$'1', also $a[i] =$'1'.

For each solution in *solutions* we iterate over the binary masks in *bitmasks* and for each mask m we check the following conditions:

- Has a puzzle already been added whose mask is covered by m (and the refore the puzzle is not locally minimal)?
- Is the corresponding puzzle well posed?

If the first condition is false and the second is true we add the corresponding puzzle and its solution to *puzzles*.

In order to check if a puzzle is well posed we apply to it the DLX algorithm and test if the number of solutions is equal to 1.