Adaptive Direct Compensation of External Disturbances for MIMO Linear Systems with State-Delay

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Abstract: In the paper we propose a new method for compensation of external disturbances in MIMO linear systems with unmeasured and delayed state vector. A state observer is used to estimate the state vector, which used in another external disturbance observer. All these estimates are used in a control law to ensure asymptotic convergence of the system outputs to zero and boundedness of all the closed loop signals. Proposed method is based on the use of the internal model principle and the extended error adaptation algorithm. It is assumed that the disturbance is the output of an autonomous linear generator with unknown parameters. To focus on compensation of external disturbances, it is assumed that the system is stable and the delay is known constant. The performance of the obtained results is confirmed using computer simulation in MATLAB Simulink.

1 INTRODUCTION

The paper considers the problem of external disturbance compensation with a stationary and bounded amplitudes for a class of MIMO linear systems where the state vector is unmeasured and delayed. External disturbance rejection for automatic systems is one of the fundamental issues in control theory and has received significant attention from researchers over the years (Bodson and Douglas, 1996), (Nikiforov, 1996), (Marino et al., 2003). There are two methods commonly used for disturbance compensation: direct compensation and indirect compensation.

Indirect disturbance compensation is based on the identification of disturbance parameters, including amplitude, phase, frequency, and initial conditions (Francis and Wonham, 1975), (Nguyen et al., 2022), (Vlasov et al., 2018), (Vlasov et al., 2019). The advantage of this method is the independence of the controller and the identifier. This enabling developers to apply various control strategies. However, this method has a significant drawback, which is the requirement for regressor persistent excitation. Failure of this condition will result in incorrect identification of disturbance parameters (Narendra, 1989).

Direct disturbance compensation is another approach to overcome the issue of persistent excitation (Gerasimov et al., 2015), (Paramonov, 2018). This approach employs the state variables or output signal of the system to estimate the disturbance, which is then utilized to synthesize a controller that achieves the desired dynamics.

In practice most systems have a delay: an output delay (due to the sensor) or an input delay (due to the actuator) which adversely affects the performance of the system. This factor even may cause system instability. In (Fridman, 2014) author introduces various studies on different aspects of delayed system and control. In the (Chiasson and Loiseau, 2007) paper, authors offer illustrations of delay systems applicable to the domains of mechanical engineering, network control, and communication.

Several studies (Banas and Vacroux, 1970), (Göllmann et al., 2009), (Wu et al., 2019), (Sanz et al., 2016) have been conducted to reduce the impact of delay on the system. Combining the problem of disturbance compensation and suppressing the effect of delay on the system makes the problem more challenging (Pyrkin et al., 2015) (Paramonov, 2018),

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(Van Huan Bui, 2022), especially when the delay and the external disturbance are time varying (Liu et al., 2022). However, the published researches and works mainly focused on the delay in the control channel (Gerasimov et al., 2015), (Paramonov, 2018). A few works have considered systems with state delay (Fridman, 2014), (Gerasimov et al., 2019), (Kuperman and Zhong, 2011) which commonly occur in fluid flow models, communication networks and biological systems. To synthesize the control law and stabilize the state delayed system a sliding mode control design using LMI was presented in (Gouaisbaut et al., 2002). Dambrine M. and colleagues introduced feedback control of time-delayed systems with bounded control and state in (Dambrine et al., 1995). Most of the studies focus on SISO systems.

The presented method in the article offers the advantage of being able to compensate for external disturbances on the system even when no information about the disturbance is available (such as amplitude, phase, or initial value). By only requiring knowledge of the maximum number of harmonics, it is possible to provide a sufficiently large arbitrary value to ensure the algorithm's effectiveness. Secondly, the direct disturbance compensation algorithm does not require the identification of disturbance parameters, thereby eliminating the requirement for persistent excitation conditions. Finally, the method demonstrates fast convergence to the system's equilibrium state, regardless of the arbitrarily chosen initial values.

Time-delay is a frequent occurrence in various control systems, including aircraft, chemical or process control systems. In many cases, delay can contribute to instability, making the stability problem of systems with delay significant both in theory and practice. In order to analyze the problem in a more visually accessible manner, we make the assumption in this paper that the delay occurs only in the state variable and consider its maximum value. In practice, control systems may have input delays in the form of multidelay, but the approach to solving the problem remains essentially the same.

The paper is structured as follows: In Section I a succinct problem description is provided. Section II presents the mathematical problem statement with several assumptions. Section III details the construction of a full-order state observer. Section IV focuses on the development of an observer for external disturbance. The synthesis of the control law and adaptation algorithm are presented in Section V. The simulation results in MATLAB are presented in Section VI. Finally, Section VII provides our conclusions. To demonstrate the performance of the proposed method we conduct simulation in MATLAB Simulink.

2 PROBLEM STATEMENT

Let the mathematical model of a plant dynmics have the form:

$$\begin{cases} \dot{x}(t) = A_1 x(t) + A_2 x(t-\tau) + Bu(t) + Ef(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t), x(t-\tau) \in \mathbb{R}^n$ are unmeasured state vector; $u(t) \in \mathbb{R}^\alpha$ is the control signal vector; $y(t) \in \mathbb{R}^\beta$ is the system output; A_1, A_2, B, C, E are known constant matrices with an appropriate dimension; $f(t) \in \mathbb{R}^\gamma$ is an unmeasured bounded external disturbance, where γ is such that $\dim\{Ef(t)\} = n$.

The following assumptions are accepted:

Assumption 1. *Matrix B has a full column rank and matrix C has a full row rank.*

Assumption 2. *System (1) is stable, i.e. the roots of the characteristic equation* $det(\mathbf{A_1} + \mathbf{A_2} e^{-\tau\lambda} - \lambda \mathbf{I}_{n\times n}) = 0$ *lie in the left half-plane.*

Assumption 3. *The external disturbance vector $f(t)$ can be represented as the output of a linear autonomous generator (Gerasimov et al., 2015):*

$$\begin{cases} \dot{w}(t) = \Gamma w(t) \\ f(t) = h^T w(t) \end{cases}$$

where the matrices Γ, h^T are unknown. The pair (Γ, h^T) is fully observable and the eigenvalues of the Γ lie on the imaginary axis.

Assumption 2 is proposed to address the compensation of external disturbances. In the case of an unstable system, it is possible to develop a control law:

$$u(t) = u_s(t) + u_c(t)$$

where $u_c(t)$ is compensating control component, $u_s(t)$ is stabilizing control component. In practice, if disturbance can be modeled as the output of a linear autonomous generator, the proposed method can be used for compensation. But in this paper for the convenience of readers we assume that the external disturbance $f(t)$ is a multi-harmonic signal. Without loss of generality, assuming that the external disturbance $f(t)$ is the sum of harmonics in the form:

$$f(t) = \sum_{j=0}^{p} R_j \sin(\omega_j t + \phi_j) + R_{0j}$$

where p is maximum number of harmonics; R_j are unknown amplitudes; ω_j are frequencies; ϕ_j are phases and R_{0j} are biases.

We must emphasize that from assumption 1 we know the dimension of the disturbance and the maximum number of harmonics. Therefore, dimension of the generator in assumption 3 is known. Assumption

1 guarantees a MIMO system with a given number of inputs and outputs. Assumption 3, the external disturbance can be considered as the output of a linear autonomous generator with unknown parameters but with a known limited number of harmonics.

The goal of this paper is as follows: it is necessary to construct a control law $u(t)$ such that ensures boundness of all signals in the closed loop system and convergence of the output signal $y(t)$ to zero when time tends to infinity:

$$\lim_{t\to\infty} \|y(t)\| = 0$$

Figure 1 illustrates the closed system schematic of the proposed approach.

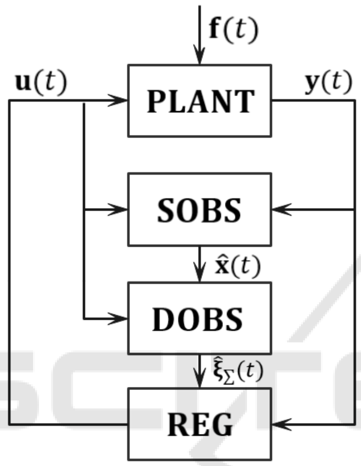

Figure 1: The structure of the closed-loop system scheme (SOBS is a full-order state observer, DOBS is an observer of external disturbances, $\hat{x}(t)$ is an estimate of the state vector, $\hat{\xi}_\Sigma(t)$ is an estimate of the regressor vector $\xi_\Sigma(t)$.

3 CONSTRUCTION OF A FULL-ORDER PLANT STATE OBSERVER

To develop an external disturbance observer, it is imperative to create a full-order state observer due to the unavailability of the state vector measurements. The state observer developed in (Chen and Patton, 2012) is utilized to construct a state observer with delay in the form:

$$\begin{cases} \dot{z}(t) = Ez(t) + Fz(t-\tau) + TBu(t) \\ +L_1 y(t) + L_2 y(t-\tau) \\ \hat{x}(t) = z(t) + Hy(t) \end{cases} \quad (2)$$

where $z(t), z(t-\tau) \in \mathbb{R}^n$ is a state vector of the full-order observer; $\hat{x}(t) \in \mathbb{R}^n$ is a state vector estimate; E, F, T, L_1, L_2, H are constant observer's matrices chosen to satisfy the system of equations (3).

Proposition 1. *The observer described by equation (2) is considered a UIO (Unknown Input Observer) for system (1) if and only if the following conditions are holds:*

$$\begin{cases} (I-HC)D = 0 \\ T = I - HC \\ E = A_1 - HCA_1 - L_{11}C \\ F = A_2 - HCA_2 - L_{21}C \\ L_{12} = EH \\ L_{22} = FH \\ L_1 = L_{11} + L_{12} \\ L_2 = L_{21} + L_{22} \\ \dot{e}_x(t) = Ee(t) + Fe(t-\tau) \text{ - is} \\ \text{asymptotically stable} \end{cases} \quad (3)$$

where I is a unit matrix with an appropriate dimension, $e_x(t) = x(t) - \hat{x}(t)$, $e_x(t-\tau) = x(t-\tau) - \hat{x}(t-\tau)$.

Proof. By differentiating $e_x(t)$ with (1) in time, we obtain a dynamic model of observation error:

$$\begin{aligned} &\dot{e}_x(t) = (A_1 - HCA_1 - L_{11}C)e_x(t) + [E \\ &-(A_1 - HCA_1 - L_{11}C)]z(t) + (A_2 - HCA_2 \\ &-L_{21}C)e_x(t-\tau) + [F - (A_2 - HCA_2 \\ &-L_{21}C)]z(t-\tau) + [L_{12} - (A_1 - HCA_1 - L_{11}C)H]y(t) \\ &+[T - (I - NC)]Bu(t) + [L_{22} - (A_2 - HCA_2 \\ &-L_{21}C)H]y(t-\tau) + (HC - I)Df(t). \end{aligned} \quad (4)$$

By substituting equation (3) into equation (4), we get:

$$\dot{e}_x(t) = Ee(t) + Fe(t-\tau)$$

Moreover, considering the final condition in the proposition (1), it can be concluded that $\dot{e}_x(t) \xrightarrow{t\to\infty} 0$ for all initial conditions. □

Proposition 2. *The necessary and sufficient condition for the system of equations (3) to have a solution are:*

1. *The rank of matrix CD is equal to the rank of matrix D.*
2. *The matrix pair $(C, \bar{A}_1)$ and $(C, \bar{A}_2)$ are detectable. where $\bar{A}_1 = A_1 - D[(CD)^T CD]^{-1}(CD)^T CA_1$ and $\bar{A}_2 = A_2 - D[(CD)^T CD]^{-1}(CD)^T CA_2$.*

Proof. The system of equations (3) has a solution if and only if the equation $(HC - I)D = 0$ has a solution, which can be represented as $HCD = D$ or alternatively $(CD)^T H^T = D^T$. Matrix D^T is a member of the spectral space of matrix $(CD)^T$, resulting in:

$$rank(D^T) \leqslant rank((CD)^T) \Rightarrow rank(D) \leqslant rank(CD)$$

On the other hand,

$$rank(CD) \leqslant \min\{rank(C), rank(D)\} \leqslant rank(D)$$

It is evident that the solution $H = D[(CD)^T CD]^{-1}(CD)^T$ exists if and only if $rank(CD) = rank(D)$. It follows that:

$$E = A_1 - HCA_1 - L_{11}C = \bar{A}_1 - L_{11}C$$

$$F = A_2 - HCA_2 - L_{21}C = \bar{A}_2 - L_{21}C$$

according to condition (3) the matrix pair $(C, \bar{A}_1)$ and $(C, \bar{A}_2)$ are detectable, which allows to choose the matrix L_{11} , L_{21} such that $\dot{e}_x(t) \xrightarrow{t\to\infty} 0$, i.e. $\hat{x}(t) \xrightarrow{t\to\infty} x(t)$. □

Consider Lyapunov function in the form:

$$V(e,t) = e^T(t)Pe(t) + \int_{t-\tau}^{t} e^T(s)Qe(s)ds$$

where P and Q are two symmetric positive definite matrices. The derivative of the Lyapunov function with respect to time can be denoted as:

$$\dot{V} = e^T(t)[E^TP + PE + Q]e(t) + e^T(t)PFe(t-\tau)$$
$$+e^T(t-\tau)F^TPe(t) - e^T(t-\tau)Qe^T(t-\tau),$$

$$\dot{V} = \begin{bmatrix} e^T(t) & e^T(t-\tau) \end{bmatrix} W \begin{bmatrix} e(t) \\ e(t-\tau) \end{bmatrix}.$$

where $W = \begin{bmatrix} E^TP + PE + Q & PF \\ F^TP & -Q \end{bmatrix}$. In order to $\dot{V} < 0$ matrix W must be negative.

Remark 1. *To determine the matrices E and F that satisfy the final condition of (3), one can utilize the LTI toolbox in MATLAB. This will enable us to obtain the matrices* L_1 *and* L_2*.*

Remark 2. *If the matrix D does not have a full column rank, we can break the matrix D into* $D = D_1D_2$*. Where* D_1 *has a full column rank and* $D_2 f(t)$ *is considered as a new external disturbance.*

4 CONSTRUCTION OF AN EXTERNAL DISTURBANCE OBSERVER

4.1 Parameterization of External Disturbances

The output of an autonomous linear generator (Nikiforov, 1996), (Gerasimov et al., 2015) is utilized to represent the external disturbances:

$$\begin{cases} \dot{\xi}(t) = G_\Sigma \xi_\Sigma(t) + L_\Sigma f(t) \\ f(t) = \theta_\Sigma^T \xi_\Sigma(t) \end{cases} \quad (5)$$

where $\xi_\Sigma(t) = \begin{bmatrix} \xi_1 & \xi_2 & \dots & \xi_q \end{bmatrix}^T \in \mathbb{R}^q$ is a regressor, $G_\Sigma = \begin{bmatrix} G_1 & 0 & 0 & 0 \\ 0 & G_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & G_\gamma \end{bmatrix}$, G_i are Hurwitz matrices; $L_\Sigma = \begin{bmatrix} L_1 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & L_\gamma \end{bmatrix}$, L_i are constant vectors; $\theta_\Sigma^T \in \mathbb{R}^{\gamma \times q}$ is a vector of unknown constant parameters that depend on the disturbance parameters. The pairs$(G_i, L_i)$are chosen arbitrarily to ensure that each pair (G_i, L_i) is fully controllable.

4.2 An External Disturbance Observer

Base on the disturbance observer presented in (Nikiforov, 1996), we propose a modified observer:

$$\begin{cases} \hat{\xi}_\Sigma(t) = \varphi_\Sigma(t) + Q_\Sigma x(t) \\ \dot{\varphi}_\Sigma(t) = G_\Sigma[\varphi_\Sigma(t) + Qx(t)] - Q_\Sigma A_1 x(t) \\ -Q_\Sigma A_2 x(t-\tau) - Q_\Sigma Bu(t) \end{cases} \quad (6)$$

where $\hat{\xi}_\Sigma(t) = \begin{bmatrix} \hat{\xi}_1 & \hat{\xi}_2 & \dots & \hat{\xi}_q \end{bmatrix}^T \in \mathbb{R}^q$ is an estimate of vector $\xi_\Sigma(t)$; $\varphi_\Sigma(t) = \begin{bmatrix} \varphi_1 & \varphi_2 & \dots & \varphi_q \end{bmatrix}^T \in \mathbb{R}^q$ is an auxiliary vector used in the disturbance observer; matrix $Q_\Sigma = \begin{bmatrix} Q_1 & Q_2 & \dots & Q_\gamma \end{bmatrix}^T \in \mathbb{R}^{q \times n}$ satisfies:

$$Q_i D = L_{0i}$$

where $i = \overline{1,\gamma}$ is the number of the observer part corresponding to the external disturbance and the matrix L_{0i}:

$$L_{0i} = [0_{qi}, \dots, 0_{qi}, L_i, 0_{qi}, \dots, 0_{qi}]$$

where vector L_i as the $i-th$ column, and 0_{qi} is the q_i-dimensional zero vector. This means that:

$$L_\Sigma = \begin{bmatrix} L_{01} & L_{02} & \dots & L_{0\gamma} \end{bmatrix}^T$$

Here $e_\xi(t) = \xi_\Sigma(t) - \hat{\xi}_\Sigma(t)$ denotes the observation error. The time derivative of $e_\xi(t)$ with respect to (5) and (6) can be expressed as:

$$\dot{e}_\xi(t) = G_\Sigma e_\xi(t) + \underbrace{(L_\Sigma - N_\Sigma E)}_{0} f(t)$$

According to the assumption the matrix G_Σ is Hurwitz. This implies that the observation error e_ξ asymptotically converges zero. This is equivalent to $\hat{\xi}_\Sigma(t) \xrightarrow{t\to\infty} \xi_\Sigma(t)$.

Furthermore, by solving equation (5), we obtain:

$$\dot{\xi}_\Sigma(t) = (G_\Sigma + L_\Sigma \theta_\Sigma^T)\xi_\Sigma(t)$$

$$\Rightarrow \xi_\Sigma(t-\tau) = P^{-1}\xi_\Sigma(t)$$

where $P = \exp(G_\Sigma + L_\Sigma \theta_\Sigma^T)$ with the initial values set to zero.

As a result, the external disturbance can be represented as:

$$f(t) = \theta_\Sigma^T \hat{\xi}_\Sigma(t) + \upsilon$$

where υ is an exponentially decaying function.

5 SYNTHESIS OF THE CONTROL LAW AND ADAPTATION ALGORITHM

To compensate external disturbances we create a controller based on the research (Marino and Tomei, 2003). By utilizing the transformation matrix J we convert the external disturbance coordinates into the coordinate frame of the plant. The parametric tracking error of the plant state is then expressed as:

$$e(t) = x(t) - J\xi_\Sigma(t) \tag{7}$$

Upon taking the derivative of equation (7) and taking into account equations (1) and (5), we can obtain the error dynamics:

$$\dot{e}(t) = A_1 e(t) + A_2 e(t-\tau) + [A_1 J + A_2 M P^{-1} \\ -J(Q_\Sigma + L_\Sigma \theta_\Sigma) + D\theta_\Sigma^T]\xi_\Sigma(t) + Bu(t)$$

and the output signal:

$$y(t) = C^T e(t) + C^T J \xi_\Sigma(t)$$

There always exists a pair of matrices J and ψ that is a solution of the following system of equations:

$$\begin{cases} A_1 J + A_2 J P^{-1} - J(Q_\Sigma + L_\Sigma \theta_\Sigma^T) = B\psi_\Sigma^T - D\theta_\Sigma^T \\ C^T J = 0 \end{cases}$$

The system of equations also known as the Francis or regulator equations has at least one solution as stated in (Francis and Wonham, 1975). The matrix $\psi_\Sigma^T = [\psi_1, \psi_2 \dots \psi_\alpha]^T \in \mathbb{R}^{\alpha \times q}$.

The dynamics of the error model can be obtained in the following form:

$$\begin{cases} \dot{e}(t) = A_1 e(t) + A_2 e(t-\tau) + B[\psi_\Sigma^T \xi_\Sigma(t) + u(t)] \\ y(t) = C^T e(t) \end{cases} \tag{8}$$

Thus, the output signal vector can be reformulated as:

$$y(t) = W(s)[\psi_\Sigma^T \xi_\Sigma(t)] + W(s)[u(t)]$$

Alternatively, it can be written as:

$$y(t) = W(s)[\xi^T{}_\Sigma(t)]\psi_\Sigma + W(s)[u(t)] \tag{9}$$

where $W(s) = C^T(sI - A_1 - A_2 e^{-\tau s})^{-1} B$ is matrix function. $W(s)[\xi_\Sigma^T] =$

$$\begin{bmatrix} W_{11}(s)[\xi_\Sigma^T] & W_{12}(s)[\xi_\Sigma^T] & \cdots & W_{1\alpha}(s)[\xi_\Sigma^T] \\ W_{21}(s)[\xi_\Sigma^T] & W_{22}(s)[\xi_\Sigma^T] & \cdots & W_{2\alpha}(s)[\xi_\Sigma^T] \\ \vdots & \vdots & \ddots & \vdots \\ W_{\beta 1}(s)[\xi_\Sigma^T] & \cdots & \cdots & W_{\beta\alpha}(s)[\xi_\Sigma^T] \end{bmatrix}$$

It should be noted that if ψ_Σ^T is a vector it can be placed outside the brackets while maintaining the vector and its position unchanged in $\psi_\Sigma^T W(s)[\xi_\Sigma]$. In the context of the paper, due to ψ_Σ^T being a matrix, it is not permissible to exclude ψ_Σ^T from the transfer function in the conventional manner. To remove ψ_Σ^T from the transfer function, the following procedure must be executed:

$$W(s)[\psi_\Sigma^T \xi_\Sigma] = [W_{i1}(s)[\xi_\Sigma^T] + W_{i2}(s)[\xi_\Sigma^T] + \\ \dots + W_{i\alpha}(s)[\xi_\Sigma^T]]\psi_\Sigma$$

Finally, we obtain

$$W(s)[\psi_\Sigma^T \xi_\Sigma] = W(s)[\xi_\Sigma^T]\psi_\Sigma$$

From the disturbance observer (6) $\hat{\xi}_\Sigma(t)$ asymptotically converges to $\xi_\Sigma(t)$, hence the control law for the closed-loop system can be selected as:

$$u(t) = -\hat{\psi}_\Sigma^T \hat{\xi}_\Sigma(t). \tag{10}$$

5.1 Gradient Algorithm of Adaptation

We define the swapping term:

$$\varsigma = W(s)[\hat{\psi}_\Sigma^T \hat{\xi}_\Sigma] - W(s)[\hat{\xi}_\Sigma^T]\hat{\psi}_\Sigma$$

and the augmented error:

$$\bar{y} = y + \varsigma$$

Considering the characteristics of linear systems and the constant value of ψ_Σ, we can deduce the following.

$$\bar{y} = W(s)[(\psi_\Sigma - \hat{\psi}_\Sigma)^T \hat{\xi}_\Sigma] + W(s)[\hat{\psi}_\Sigma^T \hat{\xi}_\Sigma] \\ -W(s)[\hat{\xi}_\Sigma^T]\hat{\psi}_\Sigma + \upsilon$$

$$\bar{y} = W(s)[(\psi_\Sigma - \hat{\psi}_\Sigma)^T \hat{\xi}_\Sigma] + W(s)[\hat{\psi}_\Sigma^T \hat{\xi}_\Sigma] - \\ W(s)[\hat{\xi}_\Sigma^T]\hat{\psi}_\Sigma + \upsilon$$

The system output error can be obtained by utilizing equations (8) and (9):

$$\bar{y}(t) = y(t) - W(s)[u(t)] - W(s)[\hat{\xi}_\Sigma^T(t)]\hat{\psi}_\Sigma$$

$$\bar{y}(t) = W(s)[\hat{\xi}_\Sigma^T(t)]\tilde{\psi}_\Sigma \tag{11}$$

where $\tilde{\psi}_\Sigma^T = \psi_\Sigma^T - \hat{\psi}_\Sigma^T$.

Based on equation (11), we can synthesize a standard adaptation algorithm:

$$\dot{\hat{\psi}}_\Sigma = \mu W(s)[\hat{\xi}_\Sigma^T(t)]\bar{y}(t) \tag{12}$$

with the adaptive coefficient $\mu > 0$.

Proposition 3. *Let assumptions 1-3 hold. The control law (10) combined with the observer (2), (6) and the adaptation algorithm (12) ensures in the closed-loop system (1):*

1. *The boundedness of all signals in the closed-loop system.*
2. *Output signal* $\lim_{t\to\infty} \|y(t)\| = 0$.

Proof. Let us denote the gradient function of $\hat{\psi}_\Sigma$ as$\nabla\bar{J}(\hat{\psi}_\Sigma)$ with cost function $\bar{J}(\hat{\psi}_\Sigma) = \frac{1}{2}\bar{y}^2$. The gradient descent of $\hat{\psi}_\Sigma$ has the form:

$$\dot{\hat{\psi}}_\Sigma = -\mu\nabla\bar{J}(\hat{\psi}_\Sigma)$$

Taking the derivative of $\bar{J}(\hat{\psi}_\Sigma)$ with respect to $\hat{\psi}_\Sigma$ we obtain:

$$\frac{\partial\bar{J}(\hat{\psi}_\Sigma)}{\partial\hat{\psi}_\Sigma} = \bar{y}\frac{\partial\bar{y}}{\partial\hat{\psi}_\Sigma}; \frac{\partial\bar{y}}{\partial\hat{\psi}_\Sigma} = -W(s)[\hat{\xi}_\Sigma^T(t)]$$

$$\Rightarrow \dot{\hat{\psi}}_\Sigma = \mu W(s)[\hat{\xi}_\Sigma^T]\bar{y}$$

From equation (12) derive:

$$\dot{\tilde{\psi}}_\Sigma = -\mu\Delta\Delta^T\tilde{\psi}_\Sigma \tag{13}$$

where $\Delta = W(s)[\hat{\xi}_\Sigma(t)]$, $\Delta^T = W(s)[\hat{\xi}_\Sigma^T(t)]$.

Select function Lyapunov as follows:

$$V(\tilde{\psi}_\Sigma) = \frac{1}{2\mu}\tilde{\psi}_\Sigma^T\tilde{\psi}_\Sigma$$

Subsequently, taking the derivative of $V(\tilde{\psi}_\Sigma)$ with respect to time using equation (12) yields the following form:

$$\dot{V} = \tilde{\psi}_\Sigma^T\dot{\tilde{\psi}}_\Sigma = -\tilde{\psi}_\Sigma^T\Delta\bar{y} = -\bar{y}^2 \le 0$$

□

5.2 Adaptation Algorithm with Memory Regressor Extension

Within this subsection, our proposal involves the utilization of the adaptive algorithm MRE, which was introduced in (Nikiforov and Gerasimov, 2022), to accelerate the achievement of system equilibrium (12). By utilizing the extended error method, we obtain:

$$\hat{y}(t) = y(t) + W(s)[\hat{\xi}_\Sigma^T(t)]\hat{\psi}_\Sigma$$

$$\Rightarrow \hat{y}(t) = W(s)[\hat{\xi}_\Sigma^T(t)]\psi_\Sigma$$

By multiplying both sides by Δ and subsequently applying the transfer function $H(s)$to both sides, we obtain:

$$Y = \Omega\psi_\Sigma \tag{14}$$

where $H(s) = \frac{1}{\alpha s+\beta}, \alpha > 0$ is asymptotically stable and minimal phase transfer function; $Y = H(s)[\Delta\hat{y}]$; $\Omega = H(s)[\Delta\Delta^T]$.

Based on equation (14), we can synthesize an alternative adaptation algorithm:

$$\dot{\hat{\psi}}_\Sigma = \mu(Y - \Omega\hat{\psi}_\Sigma) \tag{15}$$

with the adaptive coefficient $\mu > 0$.

Proposition 4. *Let assumptions 1-3 hold. The control law (10) combined with the observer (2), (6) and the adaptation algorithm (15) ensures in the closed-loop system (1):*

1. *The boundedness of all signals in the closed-loop system.*
2. *Output signal* $\lim_{t\to\infty} \|y(t)\| = 0$.

Proof. From equation (14) we can get an extended output error:

$$\varepsilon_Y = Y - \Omega\hat{\psi}_\Sigma \tag{16}$$

Substituting (14) in (16) we obtain:

$$\varepsilon_Y = Y - \Omega\hat{\psi}_\Sigma = H(s)[\Delta\Delta^T\tilde{\psi}_\Sigma] + H(s)[\Delta\Delta^T\hat{\psi}_\Sigma] - \Omega\hat{\psi}_\Sigma \; \varepsilon_Y = \Omega\tilde{\psi}_\Sigma$$

Using the gradient descent algorithm for the cost function $\bar{J}(\hat{\psi}_\Sigma) = \frac{1}{2}|\varepsilon_Y(\hat{\psi}_\Sigma)|$ we have:

$$\dot{\hat{\psi}}_\Sigma = -\mu\nabla\bar{J}(\hat{\psi}_\Sigma) = -\mu\varepsilon_Y(\hat{\psi}_\Sigma)\frac{\partial\varepsilon_Y(\hat{\psi}_\Sigma)}{\partial\hat{\psi}_\Sigma}$$

$$= \mu\Omega(Y - \Omega\hat{\psi}_\Sigma)$$

Now the parameter error of the closed-loop system described by equation:

$$\dot{\tilde{\psi}}_\Sigma = -\mu\Omega^2\tilde{\psi}_\Sigma$$

It is easy to see that if $\mu > 0$ then $\tilde{\psi}_\Sigma \xrightarrow{t\to\infty} 0$. Therefore, it can be concluded that if the control system satisfies the abovementioned assumptions 1-3, then given the conditions of system (1) and external disturbances, the selection of the adaptation coefficient $\mu > 0$ guarantees the boundedness of all signals in the closed-loop system and the attainment of the target condition: $\lim_{t\to\infty} \|y(t)\| = 0$ □

6 SIMULATION RESULTS

As stated above, the algorithm presented in the paper can be effectively employed in diverse domains, including *chemical process control, the quadruple tank benchmark, and linear manipulator systems*. To simplify the algorithm to make it easier for readers to understand, we considered a third-order system as an illustrative example:

$$\begin{cases} \dot{x}(t) = A_1x(t) + A_2x(t-\tau) + Bu(t) + Df(t) \\ y(t) = Cx(t) \end{cases}$$

with matrices $A_1 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ -4 & -5 & -6 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ -1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and initial conditions $x(0) = [1;-1;0]$. External disturbances $f(t) = \begin{bmatrix} 5\sin(t) \\ 1+5\sin(2t) \end{bmatrix}$ are described by the output of autonomous linear generators with matrices: $G_1 = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$, $L_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $G_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$, $L_2 = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$.

To design a full-order state observer (2), the matrix was chosen as follows: $L_{11} = \begin{bmatrix} 2.778 & -0.111 \\ -1.333 & 0.333 \\ 0.889 & 0.444 \end{bmatrix}$, $L_{21} = \begin{bmatrix} 12.579 & -1.618 \\ 16.049 & -15.900 \\ -16.379 & 15.660 \end{bmatrix}$,

$$E = \begin{bmatrix} -12.579 & 1.618 & 3.237 \\ -16.050 & 14.900 & 32.800 \\ 16.378 & -15.160 & -31.820 \end{bmatrix},$$

$$F = \begin{bmatrix} -2.779 & 0.111 & 0.222 \\ 1.333 & -0.333 & -0.167 \\ -0.900 & -0.444 & -1.139 \end{bmatrix}.$$

An observer of the external disturbances (6) was constructed using the following matrices: $Q_1 = \begin{bmatrix} 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$, $Q_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -6 & 0 & 6 \end{bmatrix}$.

Figure 2 shows the transients using the standard adaptation algorithm (12) with an adaptation gain $\mu = 5$: output signal vector $y(t)$ (a); estimation errors of the state vector $e_x(t)$ (b); estimates of the tunable parameter vector $\hat{\psi}_\Sigma$ (c); control signal vector $u(t)$ (d).

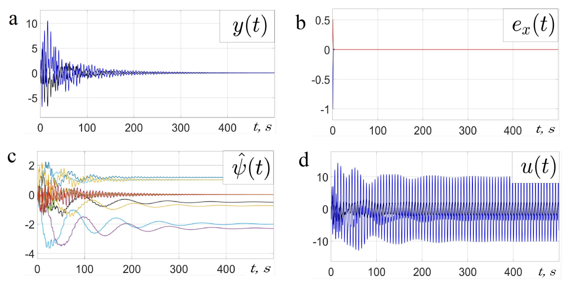

Figure 2: Graphs of transients of the standard adaptation algorithm (12) with adaptation coefficient $\mu = 5$: output signal vector $y(t)$ (a); estimation errors of the state vector $e_x(t)$ (b); estimates of the tunable parameter vector $\hat{\psi}_\Sigma$ (c); control signal vector $u(t)$ (d).

Figure 3 shows the transients of the alternative adaptation algorithm (15) with $H(s) = \frac{1}{s+1}$ and adaptation gain $\mu = 25$: output signal vector $y(t)$ (a); estimation errors of the state vector $e_x(t)$ (b); estimates of the tunable parameter vector $\hat{\psi}_\Sigma$ (c); control signal vector $u(t)$ (d).

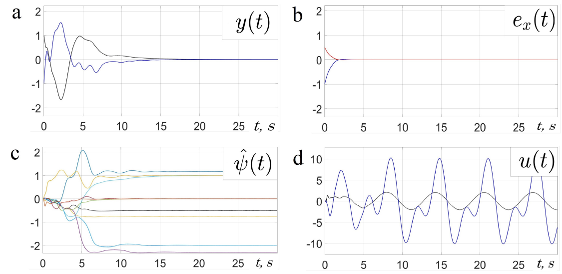

Figure 3: Graphs of transients of the standard adaptation algorithm (15) with adaptation coefficient $\mu = 25$: output signal vector $y(t)$ (a); estimation errors of the state vector $e_x(t)$ (b); estimates of the tunable parameter vector $\hat{\psi}_\Sigma$ (c); control signal vector $u(t)$ (d).

The simulation results show that using the standard adaptation algorithm (12) leads to the long transient time. Alternative adaptation algorithm (15) provides significantly decreased transient time. After 5 seconds of simulation, the state vector of the plant converges to the true value $x(t)$, which allows us to conclude that it works correctly. Analysis of simulation results has proven the applicability of the proposed method.

7 CONCLUSIONS

In this paper, a novel approach for compensation of external unknown disturbances in linear MIMO systems with state delay has been proposed. The approach outperforms existing techniques in terms of the time required for the system to attain equilibrium state, which is relatively short (15s). However, the main limitation of this method lies in the requirement of being able to manipulate the coefficient matrix $D = D_1 D_2$ of disturbance such that the resulting $rank(CD)$ is equal to $rank(D)$. The simulation results in the MATLAB environment have demonstrated the effectiveness of this method. In the future, this method could be further developed to address problems in uncertain and state-delayed systems, as well as control channel delay.

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REFERENCES

Banas, J. and Vacroux, A. (1970). Optimal piecewise constant control of continuous time systems with time-varying delay. *Automatica*, 6(6):809–811.

Bodson, M. and Douglas, S. C. (1996). Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency. *IFAC Proceedings Volumes*, 29(1):5168–5173.

Chen, J. and Patton, R. J. (2012). *Robust model-based fault diagnosis for dynamic systems*, volume 3. Springer Science & Business Media.

Chiasson, J. and Loiseau, J. J. (2007). *Applications of time delay systems*, volume 352. Springer.

Dambrine, M., Richard, J.-P., Borne, P., et al. (1995). Feedback control of time-delay systems with bounded control and state. *Mathematical Problems in Engineering*, 1(1):77–87.

Francis, B. A. and Wonham, W. M. (1975). The internal model principle for linear multivariable regulators. *Applied mathematics and optimization*, 2(2):170–194.

Fridman, E. (2014). *Introduction to time-delay systems: Analysis and control*. Springer.

Gerasimov, D. N., Nikiforov, V. O., and Paramonov, A. V. (2015). Adaptive disturbance compensation in delayed linear systems: internal model approach. In *2015 IEEE Conference on Control Applications (CCA)*, pages 1692–1696. IEEE.

Gerasimov, D. N., Nikiforov, V. O., and Paramonov, Alexei V, M. A. (2019). Adaptive tracking of multiharmonic signal in linear system with state delay. *Journal of Instrument Engineering*, 62(9):772–781.

Göllmann, L., Kern, D., and Maurer, H. (2009). Optimal control problems with delays in state and control variables subject to mixed control–state constraints. *Optimal Control Applications and Methods*, 30(4):341–365.

Gouaisbaut, F., Dambrine, M., and Richard, J.-P. (2002). Robust control of delay systems: a sliding mode control design via lmi. *Systems & control letters*, 46(4):219–230.

Kuperman, A. and Zhong, Q.-C. (2011). Robust control of uncertain nonlinear systems with state delays based on an uncertainty and disturbance estimator. *International Journal of Robust and Nonlinear Control*, 21(1):79–92.

Liu, C., Loxton, R., Teo, K. L., and Wang, S. (2022). Optimal state-delay control in nonlinear dynamic systems. *Automatica*, 135:109981.

Marino, R., Santosuosso, G. L., and Tomei, P. (2003). Robust adaptive compensation of biased sinusoidal disturbances with unknown frequency. *Automatica*, 39(10):1755–1761.

Marino, R. and Tomei, P. (2003). Output regulation for linear systems via adaptive internal model. *IEEE Transactions on Automatic Control*, 48(12):2199–2202.

Narendra, K. S. (1989). Annaswamy. stable adaptive systems. *Information And System Sciences. Prentice-Hall*.

Nguyen, T. K., Vlasov, S. M., Skobeleva, A. V., and Pyrkin, A. A. (2022). Adaptive parameter estimation of deterministic signals. In *2022 8th International Conference on Control, Decision and Information Technologies (CoDIT)*, volume 1, pages 1291–1296. IEEE.

Nikiforov, V. (1996). Adaptive servocompensation of input disturbances. *IFAC Proceedings Volumes*, 29(1):5114–5119.

Nikiforov, V. and Gerasimov, D. (2022). *Adaptive regulation: reference tracking and disturbance rejection*, volume 491. Springer Nature.

Paramonov, A. (2018). Adaptive robust disturbance compensation in linear systems with delay. *Nauchno-Tekhnicheskii Vestnik Informatsionnykh Tekhnologii, Mekhaniki i Optiki*, 18(3):384.

Pyrkin, A. A., Bobtsov, A. A., Nikiforov, V. O., Kolyubin, S. A., Vedyakov, A. A., Borisov, O. I., and Gromov, V. S. (2015). Compensation of polyharmonic disturbance of state and output of a linear plant with delay in the control channel. *Automation and Remote Control*, 76:2124–2142.

Sanz, R., Garcia, P., and Albertos, P. (2016). Enhanced disturbance rejection for a predictor-based control of lti systems with input delay. *Automatica*, 72:205–208.

Van Huan Bui, M. A. A. (2022). Compensation of output external disturbances for a class of linear systems with control delay. *Journal Scientific and Technical Of Information Technologies, Mechanics and Optics*, 146(6):1072.

Vlasov, S. M., Kirsanova, A. S., Dobriborsci, D., Borisov, O. I., Gromov, V. S., Pyrkin, A. A., Maltsev, M. V., and Semenev, A. N. (2018). Output adaptive controller design for robotic vessel with parametric and functional uncertainties. In *2018 26th Mediterranean Conference on Control and Automation (MED)*, pages 547–552. IEEE.

Vlasov, S. M., Margun, A. A., Kirsanova, A. S., and Vakhvianova, P. (2019). Adaptive controller for uncertain multi-agent system under disturbances. In *ICINCO (2)*, pages 198–205.

Wu, D., Bai, Y., and Yu, C. (2019). A new computational approach for optimal control problems with multiple time-delay. *Automatica*, 101:388–395.