Comprehensive Φ-Bonacci Index for Walking Ability Assessment in Paroxysmal Positional Vertigo: Role of Rehabilitation

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Abstract: Very recent research directions have been devoted to providing a theoretical foundation to the experimental evidence that human movements, such as walking, are able to induce time-harmonic motor patterns. The resulting findings have shown that such harmonic structures are characterized by the golden ratio occurring as the ratio of the durations of the walking gait sub-phases that compose generalized Fibonacci sequences. A new comprehensive gait index, named Φ-bonacci gait number, and a new related experimental conjecture – concerning the position of the foot relative to the tibia – have been concurrently proposed to capture the most reliable and objective (quantitive) outcome measures (and their distortions in pathological subjects) of recursivity, asymmetry, consistency, and self-similarity (harmonicity) of the gait cycle. This paper provides, for the first time, experimental results on healthy and pathological gaits – related to benign paroxysmal positional vertigo (BPPV) – that fully support the aforementioned theoretical derivations.

1 INTRODUCTION

Starting from the evidence that foot off reliably occurs at 60% to 62% of a physiological gait at a comfortable speed, it has been shown by (Iosa et al., 2013) that in (symmetrical and recursive) walking of healthy subjects – being described by four time intervals –, but not in pathological ones, the ratio between consecutive durations of swing and double support phases is close to the golden ratio ϕ value at a comfortable speed of 4 km/h (Cavagna and Margaria, 1966). Such an irrational number $\phi = (1+\sqrt{5})/2 \approx 1.618$ ($\phi^{-1} = \phi - 1$), which is related to Euclid's problem of cutting in a self-proportional way a given straight segment, captures self-similarity (harmonic) features in symmetric walking of healthy subjects (Iosa et al., 2019), and a reduction of the smooth, graceful, and melodic flow of movement in the altered gait of patients (Iosa et al., 2016) (see the related discussions in (Verrelli et al., 2021; Marino et al., 2020)). Furthermore, human walking naturally includes asymmetric and non-recursive components, especially in pathological cases, so at least eight (in place of four) time intervals have to be considered for left and right lower limbs, namely double support, swing, stance, and gait cycle (Dugan and Bhat, 2005). This has been actually done in (Verrelli et al., 2021), where the durations of those eight time intervals have been demonstrated to concern a newly defined composite gait cycle, which involves two specific couple of overlapping (left and right) gait cycles. Indeed, the dynamics-on-graph concepts- and generalized finite-length Fibonacci sequences- based analysis presented in (Verrelli et al., 2021) has generalized the one in (Iosa et al., 2013)[1], as much as the new quantitative index of (Verrelli et al., 2021) – namely, the Φ-bonacci gait number – has constituted the most straightforward generalization of the gait ratio in (Iosa et al., 2013) to the case in which non-{symmetric & recursive} components of walking (including the concept of double support consistency) occur. Furthermore, differently from the area of the Synchronicity Rectangle in (Marino et al., 2020), such a new index takes its minimum zero-value just when the strongest version of self-similarity, namely the enforced adjoint sym-

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[1]Differently from (Marino et al., 2020), no complex tools from linear algebra, associating special ϕ-dependent subspaces with a common temporal model for human walking gaits, are employed.

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metric self-similarity, occurs. It is able to unveil hidden time-harmonic and self-similar structures along the new direction toward a fractal human walking decomposition. The above index[2], has also innovatively involved a term relying on a new experimental conjecture that relies on the position of the foot relative to the tibia while opening new analysis and diagnosis perspectives on the internal analysis of the double support phase. The resulting theoretical approach thus moves along the direction of using temporal gait analyses to complement, in clinical or general performance evaluations the classical gait analyses including motion analysis, dynamic electromyography, force plate recordings, energy cost measurements or energetics, measurement of stride characteristics.

Now, even though the Φ-bonacci gait number is able to comprehensively capture the most reliable and objective (quantitive) outcome measures of recursivity, asymmetry, consistency, and self-similarity (harmonicity) of the gait cycle, however, up to this stage, only two different simplified versions of such an index have been tested: i) in (El Arayshi et al., 2022), concerning the distinction between patients affected by Ataxia Telangiectasia and their healthy counterparts, no internal analysis of the double support phase has been performed; in (Verrelli et al., 2021), concerning patients with highly asymmetric deficits (such as patients with hemiparetic stroke and patients with an alteration in gait ratio not always being accompanied by motor asymmetries [such as patients with quite symmetric symptoms due to Parkinson's Disease]), data concerning the adjoint gaits are neglected.

In this paper, we thus illustrate, for the first time in the literature, not only the effectiveness of the complete version of the aforementioned index in discriminating healthy subjects from pathological ones, but also its responsiveness in quantifying patients' improvements coming from rehabilitation. To this aim, we have recruited a cohort of patients with BPPV, i.e., a peripheral vestibular disorder leading to balance difficulties and increased fall risks (Zhang et al., 2021). Such BPPV patients suffer from transient vertigo and nystagmus, leading to balance impairments and increased fall risk, so their treatment typically involves a canalith reposition maneuver, practiced by expert physicians, and requires, at least, two weeks to have an appreciable effect. These features promote them as good candidates to be tested, before and after the repositioning maneuver, in order to show that the Φ-bonacci gait number, in its complete version, represents a meaningful index, capable of explicitly quantifying and detecting the recovery level and improvements due to rehabilitation. Experimental results confirm such a conjecture.

[2]Even though it can be naturally extended to even assess *gait index variability* along past walking gaits, this is however out of the focus of this paper.

2 MATERIALS AND METHODS

This section recalls the concept of composite gait cycle and the notions of recursivity, harmonicity, symmetry, and double support consistency as defined in (Verrelli et al., 2021). It also reports the mathematical expression of the Φ-bonacci gait number, in its complete and simplified versions. Methods are then described, along with the experimental setup and the data acquisition modality. Finally, the main features of the participants are introduced and the results coming from the statistical analysis are reported.

2.1 Φ-Bonacci Sequence-Based Indices

Consider a walking gait (Iosa et al., 2013; Verrelli et al., 2021) and let: GC stand for gait cycle; HS stand for heel-strike; TO stand for toe-off; r and l stand for right and left, respectively; adj stand for adjoint; ST stand for stance; SW stand for swing; DS stand for double support. In Figure 1, a comprehensive model of the composite gait cycle in (Verrelli et al., 2021), which involves two specific couples of overlapping gait cycles, namely the left and right gait cycles (GCl and GCr) and the adjoint right and left gait cycles (GCr_{adj} and GCl_{adj}), is shown. For the sake of clarity, STr, STl, SWr, and SWl represent the right and left stance phase durations and the right and left swing phase durations, respectively. Moreover, the durations DSr and DSl of the right and left double support phase satisfy DSr = DSx + DSy, DSl = DSy + DSz, with DSx, DSy, DSz being graphically defined in Figure 1. Accordingly, the equal partition of the double support sub-phases, i.e., DSx = DSy (and DSw = DSy, DSy = DSz in Figure 1) involves the concept of double support consistency. The same durations for the adjoint right and left gait cycles, denoted by STradj, STladj, SWradj, SWladj, DSradj, and DSladj, are reported in Figure 1. Now, Verrelli et al. (2021) have innovatively characterized the aforementioned composite gait cycle by means of a new mathematical and meaningful index, namely the Φ-bonacci gait number, which relies, in its self-similar kernel, on generalized finite-length Fibonacci sequences, exploiting the role of the golden ratio ϕ. Specifically, the complete version of such an index, here called $\mathcal{Y}_\phi$ and reported in (10) of (Verrelli et al., 2021), has relied on a new experimental conjecture concerning an extended fractal walking decomposition paying attention to the posi-

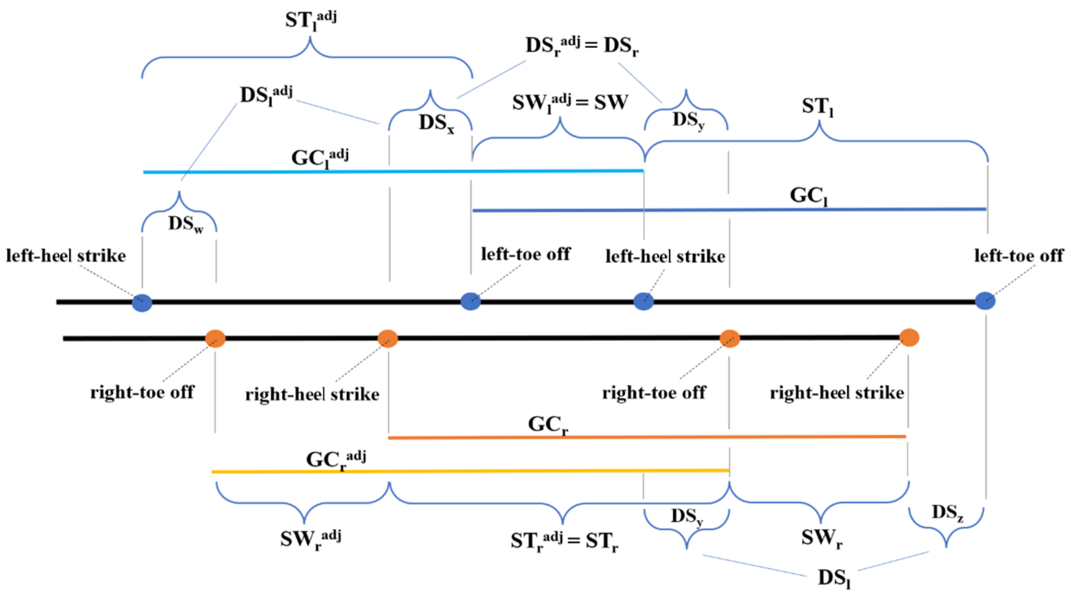

Figure 1: *Composite gait cycle*: right and left gait cycles and adjoint right and left gait cycles.

tion of the foot relative to the tibia. It turns out to constitute the most natural generalization, to the non-{symmetric & recursive} walking case, of the corresponding gait ratio $|SW/DS - \phi|$ defined in (Iosa et al., 2013; Iosa et al., 2016) for symmetric walking, while it simply incorporates a weighted modification of the index $= |\Delta SW|/\overline{SW}$ in (Błażkiewicz et al., 2014), evaluated at both the gait and the adjoint gait. The corresponding two simplified versions, here called s1-Φ-bonacci gait number $\mathcal{G}_\phi[s1]$ and s2-Φ-bonacci gait number $\mathcal{G}_\phi[s2]$, have appeared in (6) of El Arayshi et al. (2022) and in (11) of Verrelli et al. (2021), respectively. They are reported in Table I, where λ, δ, μ^{adj}, λ^{adj}, ν^{conj} are positive weights[3] and the normalized quantity is given by:

$$\left(\frac{\xi_n}{\xi_d} - \xi_v\right)^2_n = \left(\frac{\xi_n}{\xi_d}\right)^{-1}\left(\frac{\xi_n}{\xi_d} - \xi_v\right)^2 \qquad (1)$$

in terms of positive reals ξ_n, ξ_d, ξ_v (where n generically stands for numerator, d stands for denominator, v stands for value), whereas the positive real numbers z_1, z_2, z_3 denote the time distances the corresponding left or right heel-strikes and toe-off instants of the three time instants representing the three instants of minimum angular positions (with negative signs) of the (left and right) feet relative to the tibias (with a 90 degrees-angle between foot and tibia being plotted at 0-degrees).

Values close to 0 for the above complete or simplified indices describe different levels of recursivity, self-similarity (harmonicity), swing symmetry, and double support consistency, depending on the level of terms neglected in the index computation. Values far from 0 typically refer to pathological gaits.

Remark 2.1. *The conjecture used in the above indices (see (Verrelli et al., 2021)) extends the ideas underlying a fractal approach to the double support sub-phases within the gait. It is inspired by the experimental results reported in (Novacheck, 1998) showing that physiological symmetric walking is not only characterized by a stance duration being close to 62% of gait cycle duration, a swing duration being close to 38% of gait cycle duration, a double support duration being consequently close to 24% of gait cycle duration, but also by an instant of minimum angular position (with negative sign) of the foot relative to the tibia (with a 90 degrees-angle between foot and tibia being plotted at 0-degrees) occurring at about 7% of gait cycle duration in each double support sub-phase (with 5% as percentage for the complementary interval duration). It may thus be interestingly recognized that the structure of a Fibonacci sequence (with fixed point ϕ) appears in the sequence:* $5 \times 2 = 10$ $(1/\phi^5 \approx 9.018)$; $7 \times 2 = 14$ $(1/\phi^4 \approx 14.591)$; 24 $(1/\phi^3 \approx 23.608)$; 38 $(1/\phi^2 \approx 38.198)$; $62 (1/\phi \approx 61.804)$; 100.

2.2 Subjects, Data Acquisition, and Experimental Protocol

Patients affected by BPPV ($n = 7$, age $= 55.6 \pm 5.3$) were asked to perform 20-meters-walking tests in a

[3] Such weights play the role of gains. They can be freely chosen by the user, in accordance with the specific analysis requirements.

Table 1: Mathematical expressions for the Φ-bonacci gait number and its simplified versions.

$\mathcal{Y}_\Phi = \sqrt{\left(\frac{\mathrm{SW}_l}{\mathrm{DS}_r}-\phi\right)_{\mathrm{n}}^2 + \left(\frac{\mathrm{SW}_r}{\mathrm{DS}_l}-\phi\right)_{\mathrm{n}}^2 + \mu^{\mathrm{adj}}\left(\frac{\mathrm{SW}_r^{\mathrm{adj}}}{\mathrm{DS}_l^{\mathrm{adj}}}-\phi\right)_{\mathrm{n}}^2} + \lambda\sqrt{\left(\frac{\mathrm{SW}_r}{\mathrm{SW}_l}-1\right)_{\mathrm{n}}^2 + \lambda^{\mathrm{adj}}\left(\frac{\mathrm{SW}_r^{\mathrm{adj}}}{\mathrm{SW}_r}-1\right)_{\mathrm{n}}^2} + \nu^{\mathrm{conj}}\sqrt{\left(\frac{\mathrm{DS}_r}{z_1+z_2}-\phi\right)_{\mathrm{n}}^2 + \left(\frac{\mathrm{DS}_l}{z_2+z_3}-\phi\right)_{\mathrm{n}}^2} + \delta\sqrt{\left(\frac{\mathrm{DS}_x}{\mathrm{DS}_y}-1\right)_{\mathrm{n}}^2}$
$\mathcal{Y}_\Phi[s1] = \sqrt{\left(\frac{\mathrm{SW}_l}{\mathrm{DS}_r}-\phi\right)_{\mathrm{n}}^2 + \left(\frac{\mathrm{SW}_r}{\mathrm{DS}_l}-\phi\right)_{\mathrm{n}}^2 + \mu^{\mathrm{adj}}\left(\frac{\mathrm{SW}_r^{\mathrm{adj}}}{\mathrm{DS}_l^{\mathrm{adj}}}-\phi\right)_{\mathrm{n}}^2} + \lambda\sqrt{\left(\frac{\mathrm{SW}_r}{\mathrm{SW}_l}-1\right)_{\mathrm{n}}^2 + \lambda^{\mathrm{adj}}\left(\frac{\mathrm{SW}_r^{\mathrm{adj}}}{\mathrm{SW}_r}-1\right)_{\mathrm{n}}^2} + \delta\sqrt{\left(\frac{\mathrm{DS}_x}{\mathrm{DS}_y}-1\right)_{\mathrm{n}}^2}$
$\mathcal{Y}_\Phi[s2] = \sqrt{\left(\frac{\mathrm{SW}_l}{\mathrm{DS}_r}-\phi\right)_{\mathrm{n}}^2 + \left(\frac{\mathrm{SW}_r}{\mathrm{DS}_l}-\phi\right)_{\mathrm{n}}^2} + \lambda\sqrt{\left(\frac{\mathrm{SW}_r}{\mathrm{SW}_l}-1\right)_{\mathrm{n}}^2} + \delta\sqrt{\left(\frac{\mathrm{DS}_x}{\mathrm{DS}_y}-1\right)_{\mathrm{n}}^2}$

hallway, at their comfortable speed, before and after the canalith repositioning maneuver. For comparison, age-matched healthy control subjects (HCS) ($n = 6$, age = 59.4 ± 7.3) were asked to perform, at their comfortable speed, a single 20-meter-walking test. HCS were asked about their history of dizziness, lightheadedness, balance problems, and other symptoms similar to the ones exhibited by the BPPV patients in the past six months. All the subjects were required to wear the wearable sensors of the motion capture system Movit System G1 (Captiks, Rome, Italy) for movement capture and analysis (see (El Arayshi et al., 2022)). The Movit System G1 provides accelerometer, gyroscope, magnetometer, quaternion, and barometer synced data and is composed of 13-DOF wireless light-weight wearable small inertial devices and an USB wireless receiver (Costantini et al., 2018, Ricci et al., 2019b, Ricci et al., 2019a). The Motion Studio & Motion Analyzer software, to be used in conjunction with the aforementioned data acquisition system, is then respectively used to collect the movement data from each sensor and fuse/process sensors data to generate the .csv files of the consecutive time instants of the HS and TO for the left and right foot, in addition to the anatomical angles, which are used to obtain information on the position of the foot (left and right) relative to the tibia.

2.3 Data and Statistical Analysis

The temporal gait parameters, obtained from Motion Analyzer from the raw sensors data fusion and processing operations, were loaded on a custom-made MATLAB (The Mathworks, Natick, MA, USA) algorithm that recognizes the time instants of HS and TO for the left and right foot belonging to the (left and right) gait cycle and the adjoint gait cycle subphases and involved in the computation of the Φ-bonacci gait number (all the weights therein were set equal to 1). The developed MATLAB algorithm automatically allows for the computation of the values of $\mathcal{Y}_\phi$, $\mathcal{Y}_\phi$[s1], and $\mathcal{Y}_\phi$[s2] corresponding to all the composite gait cycles detected in the full walking event for each subject. To reduce the effect of measurement errors of the sensors and possible transient behaviors of the gait, the algorithm just considered the composite gait cycles belonging to the middle of the walking event, along with the immediately previous and subsequent gait cycles. Statistical analyses and data visualization were performed using GraphPad Prism Software Version 9. The reported results involve the average of the indices computed over the three gait cycles belonging to the middle of the walking event for each subject. Since data do not follow a normal distribution (evaluated by the Kolmogorov- Smirnov normality test), a non-parametric Mann-Whitney U test was used to find differences between the mean values of each pair of samples. Statistical analyses were considered significant in all cases when the p-values were less than 0.05. Moreover, for the evaluation of the indices as diagnostic test receivers, operating characteristic curve (ROC) analyses [Wilson/Brown method] were performed; index accuracy was measured by the area under the ROC curve (AUC). If the AUC value was greater than 0.5, the test was considered significant. We considered the Youden index with likelihood ratio LR= Sensitivity / (1 - Specificity) > 2 as a criterion for choosing the optimal threshold value for the ROC curve, namely the threshold value c for which the quantity Sensitivity + Specificity - 1, evaluated at c, is the largest one.

3 RESULTS AND DISCUSSION

Up to now, studies have shown how gait phases are a reliable and valid measure for the assessment of subjects' walking ability. In addition, gait phase changes have been reported in patients affected by several neu-

Pre-Treatment									**Post-Treatment**								
$\mathcal{Y}_{\Phi[s1]}$			$\mathcal{Y}_{\Phi}$			$\mathcal{Y}_{\Phi[s2]}$			$\mathcal{Y}_{\Phi[s1]}$			$\mathcal{Y}_{\Phi}$			$\mathcal{Y}_{\Phi[s2]}$		
0.56	**0.50**	0.63	1.17	**1.12**	1.33	0.46	**0.41**	0.61	0.24	**0.20**	0.24	0.30	**0.24**	0.25	0.20	**0.16**	0.21
0.80	**0.76**	0.81	1.64	**1.67**	1.74	0.64	**0.67**	0.68	0.78	**0.67**	0.73	0.72	**0.77**	0.78	0.66	**0.59**	0.62
1.61	**1.63**	2.36	3.11	**2.85**	4.58	1.47	**1.19**	0.42	0.86	**0.61**	0.71	1.67	**1.77**	1.85	0.81	**0.61**	0.68
0.57	**0.53**	0.46	2.85	**3.11**	4.58	0.48	**0.43**	0.43	0.34	**0.27**	0.29	0.55	**0.37**	0.52	0.31	**0.25**	0.26
0.11	**0.19**	0.08	0.51	**0.53**	0.66	0.13	**0.17**	0.07	0.13	**0.12**	0.10	0.21	**0.15**	0.31	0.09	**0.12**	0.10
0.81	**0.62**	0.49	0.85	**0.67**	0.64	0.69	**0.53**	0.38	0.15	**0.09**	0.22	0.47	**0.36**	0.49	0.10	**0.08**	0.21
1.15	**1.01**	1.16	2.38	**2.21**	2.32	0.94	**0.95**	1.11	0.77	**0.55**	0.62	0.93	**0.75**	0.83	0.53	**0.48**	0.43

Figure 2: Φ-bonacci gait number $\mathcal{Y}_\phi$ and its simplified versions $\mathcal{Y}_\phi$[s1], $\mathcal{Y}_\phi$[s2], computed for each subject ($n = 7$, each row) with BPPV before and after the patient treatment and evaluated at three subsequent stages $(i-1, i, i+1)$ [i = gait cycle in the middle of the walking event (bold values)].

Healthy Control Subjects								
$\mathcal{Y}_{\Phi[s1]}$			$\mathcal{Y}_{\Phi}$			$\mathcal{Y}_{\Phi[s2]}$		
0.21	**0.28**	0.48	0.35	**0.52**	0.43	0.20	**0.24**	0.45
0.43	**0.45**	0.53	0.51	**0.62**	0.44	0.36	**0.32**	0.41
0.32	**0.51**	0.55	0.43	**0.61**	0.64	0.27	**0.43**	0.47
0.40	**0.42**	0.51	0.42	**0.49**	0.50	0.20	**0.34**	0.50
0.43	**0.50**	0.58	0.53	**0.58**	0.55	0.31	**0.45**	0.53

Figure 3: Φ-bonacci gait number $\mathcal{Y}_\phi$ and its simplified versions $\mathcal{Y}_\phi$[s1], $\mathcal{Y}_\phi$[s2], computed for each healthy subject ($n = 5$, each row) and evaluated at three subsequent stages $(i-1, i, i+1)$ [i = gait cycle in the middle of the walking event (bold values)].

rodegenerative diseases or disorders after rehabilitative interventions (Jonsdottir et al., 2020; Leone et al., 2018). In light of those findings, we were encouraged to investigate the capability of the new indices reported in the previous section to be successfully used in clinical settings and thus, to exhibit responsiveness to (even small) changes coming from rehabilitation (see (Verrelli et al., 2021) for a comparison with classical indices such as Mean Gait Ratio MGR and Symmetry Index SI).

The values of $\mathcal{Y}_\phi$, $\mathcal{Y}_\phi$[s1], $\mathcal{Y}_\phi$[s2] – computed for each HCS and each BPPV patient before and after the canalith repositioning maneuver and evaluated at the gait cycle (i) in the middle of the walking event and at the immediately previous ($i-1$) and subsequent one ($i+1$) – appear in the tables belonging to Figures 2 and 3. Figures 4 and 5 report the corresponding mean ± SD. They assess a certain level of stability of the measurement, for both the HCS and the patients. For the first time in the literature, data of Figure 6-8 illustrate how the $\mathcal{Y}_\phi$ [largely better than its simplified versions $\mathcal{Y}_\phi$[s1], $\mathcal{Y}_\phi$[s2]] is able to distinguish healthy subjects from pathological ones better than the corresponding simplified indices. In particular, the ROC analysis and the AUC being obtained using $\mathcal{Y}_\phi$ observed in BPPV patients and controls shows that such a comprehensive index – when used as a diagnostic test – displays a great accuracy (in terms of sensitivity & specificity, as well as of p-value, AUC and Likelihood Ratio) in distinguishing between BPPV patients and healthy subjects: ROC curve related to $\mathcal{Y}_\phi$ for the evaluation of the index as a

Pre-Treatment			Post-Treatment		
$\mathcal{Y}_{\Phi[s1]}$	$\mathcal{Y}_{\Phi}$	$\mathcal{Y}_{\Phi[s2]}$	$\mathcal{Y}_{\Phi[s1]}$	$\mathcal{Y}_{\Phi}$	$\mathcal{Y}_{\Phi[s2]}$
0.58 ± 0.04	1.21 ± 0.10	0.49 ± 0.10	0.22 ± 0.02	0.26 ± 0.03	0.19 ± 0.03
0.80 ± 0.005	1.68 ± 0.05	0.66 ± 0.02	0.72 ± 0.05	0.76 ± 0.03	0.62 ± 0.04
1.86 ± 0.43	3.51 ± 0.93	1.02 ± 0.54	0.72 ± 0.13	1.76 ± 0.09	0.70 ± 0.10
0.53 ± 0.06	3.52 ± 0.94	0.44 ± 0.02	0.30 ± 0.04	0.48 ± 0.09	0.27 ± 0.03
0.10 ± 0.01	0.56 ± 0.08	0.12 ± 0.05	0.12 ± 0.02	0.22 ± 0.08	0.10 ± 0.02
0.70 ± 0.18	0.72 ± 0.11	0.53 ± 0.15	0.15 ± 0.06	0.44 ± 0.07	0.13 ± 0.07
1.15 ± 0.005	2.30 ± 0.08	1.01 ± 0.09	0.65 ± 0.11	0.84 ± 0.09	0.48 ± 0.05

Figure 4: Mean ± SD of Φ-bonacci gait number $\mathcal{Y}_{\phi}$ and its simplified versions $\mathcal{Y}_{\phi}[s1]$, $\mathcal{Y}_{\phi}[s2]$, for data appearing in Figure 2.

Healthy Control Subjects		
$\mathcal{Y}_{\Phi[s1]}$	$\mathcal{Y}_{\Phi}$	$\mathcal{Y}_{\Phi[s2]}$
0.32 ± 0.13	0.43 ± 0.08	0.30 ± 0.13
0.47 ± 0.05	0.52 ± 0.09	0.37 ± 0.04
0.46 ± 0.12	0.56 ± 0.11	0.39 ± 0.10
0.44 ± 0.06	0.47 ± 0.04	0.34 ± 0.15
0.50 ± 0.07	0.55 ± 0.02	0.43 ± 0.11

Figure 5: Mean ± SD of Φ-bonacci gait number $\mathcal{Y}_{\phi}$ and its simplified versions $\mathcal{Y}_{\phi}[s1]$, $\mathcal{Y}_{\phi}[s2]$, for data appearing in Figure 3.

diagnostic test. AUC=1.000; 95% confidence interval AUC=[1.000, 1.0000]; p-value AUC =0.0045; cut-off value ROC=0.5577; Sensitivity at cut-off value=100%; 95% CI=[64.57%,100.0%]; Specificity at cut-off value=80%, 95% CI=[37.55%,98.97%]; Likelihood Ratio (LR)=5.00. Such accurate results are not achieved when the simplified versions of index $\mathcal{Y}_{\phi}$ are used, instead (see Figures 7 and 8). Quantification of the recovery level of each BPPV patient after the canalith repositioning maneuver can be taken from Figure 4. Specifically, $\mathcal{Y}_{\phi}$, $\mathcal{Y}_{\phi}[s1]$, and $\mathcal{Y}_{\phi}[s2]$ values show a reduction for each patient in post-treatment condition, highlighting a different recovery level of healthy harmonic and symmetric components among the patients and thus, point out the indices ability to detect and quantify individual improvements due to rehabilitation. In particular, the percentage-variation $((post.treatment.value - pre.treatment.value)/pre.treatment.value) * 100\%$ for $\mathcal{Y}_{\phi}$ of each patient reads: Patient 1: -78%, Patient 2: -55%, Patient 3: -50%, Patient 4: -86%, Patient 5: -61%, Patient 6: -39%, Patient 7: -63%.

4 CONCLUSIONS

The experimental results on healthy and pathological gaits related to BPPV of Section 3 illustrate how the new comprehensive Φ-bonacci gait number of Table I (better than its simplified versions) is able to capture the most reliable and objective (quantitive) outcome measures of recursivity, asymmetry, consistency, and self-similarity (harmonicity) of the gait cycle (and their distortions in pathological subjects), as well as of rehabilitation-based recovery effects.

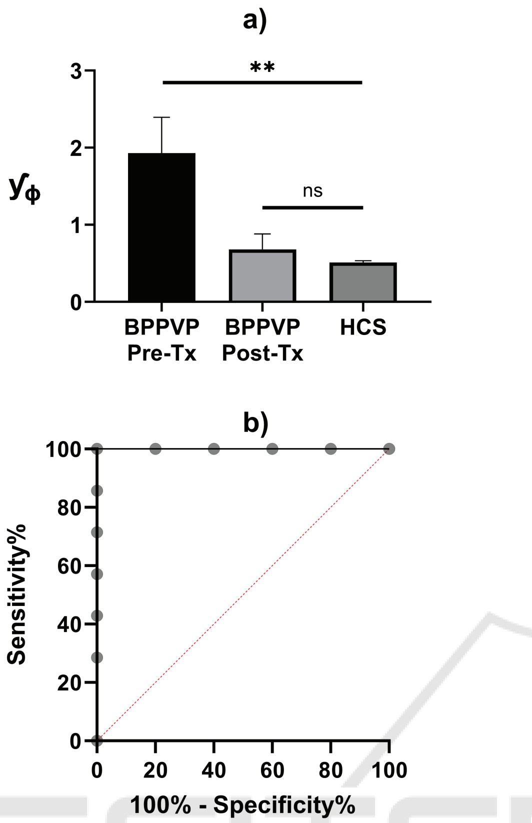

Figure 6: (a) mean ± SEM of the average $\mathcal{Y}_\phi$ values computed for BPPV patients (before and after the treatment) and healthy controls subjects (HCS): $n = 7$ (BPPV patients), $n = 5$ (HCS). p-values are calculated using the Mann-Whitney U test, $** : p < 0.01$, ns: not significant. b) ROC curve related to $\mathcal{Y}_\phi$ for the evaluation of the index as a diagnostic test. AUC=1.000; 95% confidence interval AUC=[1.000, 1.0000]; p-value AUC = 0.0045; cut-off value ROC=0.5577; Sensitivity at cut-off value=100%; 95% CI=[64.57%,100.0%]; Specificity at cut-off value=80%, 95% CI=[37.55%, 98.97%]; Likelihood Ratio (LR)=5.00.

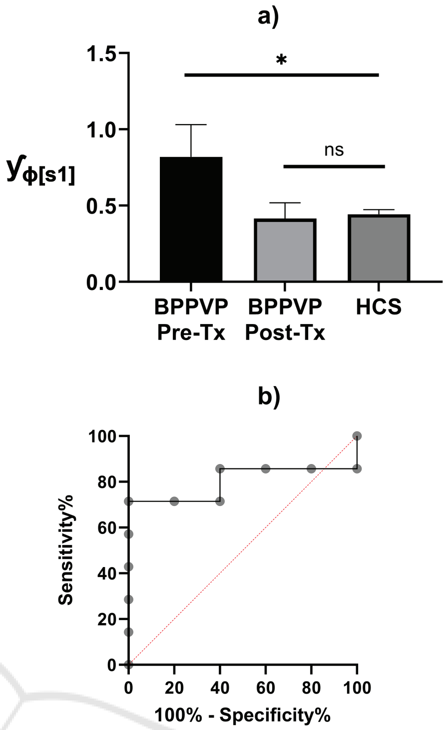

Figure 7: (a) mean ± SEM of the average $\mathcal{Y}_\phi$[s1] values computed for BPPV patients (before and after the treatment) and healthy controls subjects (HCS): $n = 7$ (BPPV patients), $n = 5$ (HCS). p-values are calculated using the Mann-Whitney U test, $* : p \in (0.01, 0.05)$, ns: not significant. b) ROC curve related to $\mathcal{Y}_\phi$[s1] for the evaluation of the index as a diagnostic test. AUC=0.8000; 95% confidence interval AUC=[0.5244, 1.0000]; p-value AUC=0.0882; cut-off value ROC=0.5577; Sensitivity at cut-off value=71.43%; 95% CI=[35.89%,94.92%]; Specificity at cut-off value=80.00%; 95% CI=[37.55%,98.97%]; Likelihood Ratio (LR)=3.571.

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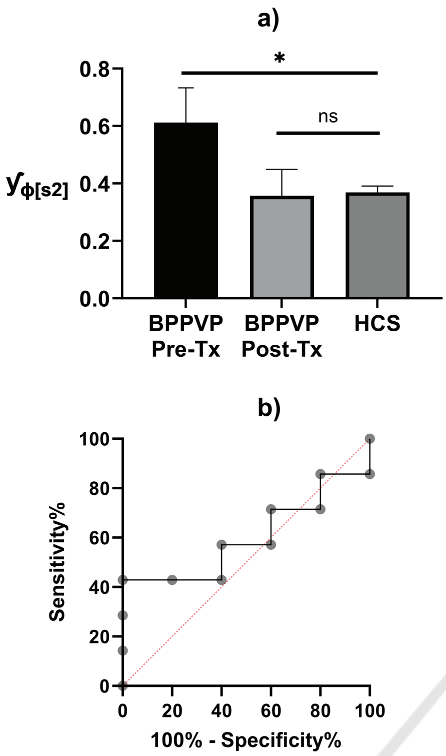

Figure 8: (a) mean ± SEM of the average $\mathcal{Y}_\phi[s2]$ values computed for BPPV patients (before and after the treatment) and healthy controls subjects (HCS): $n = 7$ (BPPV patients), $n = 5$ (HCS). p-values are calculated using the Mann-Whitney U test, $* : p \in (0.01, 0.05)$, ns: not significant. b) ROC curve related to $\mathcal{Y}_\phi[s2]$ for the evaluation of the index as a diagnostic test. AUC=0.5492; 95% confidence interval AUC=0.6000; 95% confidence interval AUC=[0.2686, 0.9314]; p-value AUC=0.5698; cut-off value ROC=0.5577; Sensitivity at cut-off value=42.86%; 95% CI=[15.82%,74.95%]; Specificity at cut-off value=80.00%; 95%CI=[37.55%,98.97%]; Likelihood Ratio (LR)=2.143.

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