Survival Analysis as a Risk Stratification Tool for Threshold Exceedance Forecasting

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Keywords: Survival Analysis, Threshold Exceedance, Extreme Event Forecasting.

Abstract: This study presents a novel framework designed for predicting threshold exceedance in time series data through the use of survival analysis techniques. Contrasting the traditional binary classification methodologies typically applied to this problem, our approach offers a unique perspective, modeling and predicting not only the occurrence but also the time-to-event information. This significant differentiation furnishes an invaluable tool for understanding and anticipating extreme events, and more importantly, it enhances decision-makers' comprehension of temporal dynamics of such risks, enabling early intervention strategies. These facets are especially critical in various domains where timely action is essential. The effectiveness of our methodology has been empirically confirmed using both simulated and real-world datasets, showcasing our method's precision in forecasting threshold exceedance. An illustrative application within the food safety domain, leveraging real-world data related to food recalls over time, further demonstrates the practical utility of our approach, particularly in preventing and controlling high-risk hazards like salmonella. These findings underscore the wide-ranging implications of our method, particularly in applications where understanding the temporal dynamics of risks is paramount.

1 INTRODUCTION

The advent of extreme event forecasting holds tremendous significance across multiple domains, including finance, environmental sciences, engineering, public health, food safety and natural disaster management. The characteristically sporadic nature of extreme events, often marked by their rare occurrence yet substantial impact, presents formidable challenges to forecast accurately. Nevertheless, precise predictions of these events are integral to mitigating potential disruptions, minimizing losses, and informing decision-making processes.

Within the domain of finance, forecasting extreme market fluctuations facilitates informed investment decisions and risk management strategies. Environmental sciences greatly benefit from predictions of extreme weather events, aiding in disaster preparedness and infrastructure planning. The anticipation of extreme loads in engineering can guide the design of resilient structures, and in public health and food safety, accurate forecasting can bolster preventive measures and surveillance systems, notably in the context of infectious disease outbreaks or food-related incidents like salmonella.

However, the sparsity of extreme event data, coupled with their inherent uncertainty and potential non-stationarity and nonlinear behavior, can obstruct the path to robust forecasting models. The risk of overfitting becomes a stark reality, with models potentially capturing noise rather than true underlying patterns. Furthermore, the influence of external factors, such as climate change and socioeconomic variables, add to the complexity, necessitating a comprehensive understanding of underlying mechanisms.

In light of these challenges, this study introduces a novel framework leveraging survival analysis techniques to model and predict not only the occurrence of such events but, critically, the time until these events transpire. This shift from conventional binary classification methodologies provides a novel perspective on threshold exceedance prediction, offering a vital tool for understanding and anticipating extreme events. Furthermore, our method enhances the com-

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Marinos, G., Karvounis, M. and Athanasiadis, I.
Survival Analysis as a Risk Stratification Tool for Threshold Exceedance Forecasting.
DOI: 10.5220/0012234700003598
In *Proceedings of the 15th International Joint Conference on Knowledge Discovery, Knowledge Engineering and Knowledge Management (IC3K 2023) - Volume 3: KMIS*, pages 62-72
ISBN: 978-989-758-671-2; ISSN: 2184-3228

prehension of the temporal dynamics of extreme risks, crucial for facilitating timely interventions.

We demonstrate the efficacy of our approach using both simulated and real-world datasets, highlighting its precision in forecasting threshold exceedance. A practical application of our method within the context of food safety underscores its utility, especially in managing high-risk hazards. The ramifications of this approach are far-reaching, holding promise for a broad range of applications where understanding the temporal dynamics of risk is crucial. This universal framework, applicable to any univariate or multivariate time series dataset, presents a promising avenue to address the intricacies of extreme event forecasting.

2 RELATED WORK

Within this research field, we have identified two distinct sub-areas. The first sub-area focuses on extreme event forecasting, where researchers aim to develop predictive algorithms that account for the presence of extreme events. Neural networks are often utilized in these methods, as they exhibit sensitivity towards predicting extreme values. However, the primary objective in this sub-area is to forecast the exact value of the time series, leading to the evaluation of these methods using classic regression metrics.

On the other hand, the second sub-field, known as exceedance threshold forecasting, also involves predicting extreme values in a time series. However, a crucial difference lies in the approach taken. In this sub-field, a threshold is set, and the goal is to forecast whether or not there will be a threshold exceedance in the upcoming timestamps. Consequently, this can be framed as a binary classification problem, where the focus shifts to predicting the occurrence or non-occurrence of the threshold exceedance. In our work, we contribute to the second sub-field of exceedance threshold forecasting.

2.1 Extreme Event Forecasting

The authors in (Abilasha et al., 2022) address the challenge of accurately predicting extreme events in time series forecasting by proposing the Deep eXtreme Mixture Model (DXtreMM). This model combines Gaussian and Generalized Pareto distributions to better capture extreme points. It consists of a Variational Disentangled Auto-encoder (VD-AE) classifier and a Multi-Layer Perceptron (MLP) forecaster unit. The VD-AE predicts the possibility of extreme event occurrence, while the forecaster predicts the exact extreme value. Extensive experiments on real-world datasets demonstrate the model's effectiveness, comparable to existing baseline methods. The DXtreMM model, with its novel formulation and consideration of heavy-tailed data distributions, offers a promising approach for accurate extreme event prediction in time series forecasting.

This (Dai et al., 2022) paper introduces a novel clustering algorithm, the Generalized Extreme Value Mixture Model (GEVMM), for accurate prediction of extreme values in scenarios with mixed distribution characteristics. The algorithm adaptively classifies block maximum data into clusters based on their weights in the population, creating a GEVMM that can forecast maximum values in a specified return period. The optimal number of clusters is determined using the elbow method, along with RMSE and R-squared, to prevent over- and under-fitting. The method is validated through theoretical examples and applied to traffic load effects on bridges using weight-in-motion data. Results demonstrate superior performance compared to existing methods, showcasing the potential of the proposed approach for accurate estimation of extreme values with mixed probability distributions across various fields.

The (Hill Galib et al., 2022) paper introduces DeepExtrema, a novel framework for accurate forecasting of extreme values in time series data. DeepExtrema combines a deep neural network (DNN) with the generalized extreme value (GEV) distribution to forecast the block maximum value. The authors address the challenge of preserving inter-dependent constraints among the GEV model parameters during DNN initialization. The framework enables conditional mean and quantile prediction of block maxima, surpassing other baseline methods in extensive experiments on real-world and synthetic data. DeepExtrema incorporates the GEV distribution into a deep learning framework, capturing nonlinear dependencies in time series data. The paper overcomes technical challenges such as positivity constraints and data scarcity through reparameterization of the GEV formulation and a model bias offset mechanism. Contributions include the novel framework, GEV constraint reformulation, model bias offset mechanism, and comprehensive experiments demonstrating DeepExtrema's superiority over baselines.

2.2 Threshold Exceedance Forecasting

In their study, (Taylor and Yu, 2016) propose an approach to manage financial risk using exceedance probability. They aim to predict if the returns of financial assets will surpass a certain threshold. To achieve this, they employ a method called CARL (conditional auto-regressive logit) that utilizes logistic regression. Building upon this (Taylor, 2017) extends CARL to handle multiple threshold problems. This extended model proves useful in predicting wind ramp events, where forecasting the extreme values on both ends of the distribution is crucial.

The (Krylova and Okhrin, 2022) study proposes a two-stage procedure for predicting exceedances of legal limits for PM10 and O3 concentrations in air pollution using machine learning. Hourly pollutant concentrations are forecasted using the Stochastic Gradient Boosting Model, demonstrating superior performance compared to similar studies. The forecasts are then utilized to predict exceedances of daily limits, achieving an average detection probability above 80% with low false alarm probability.

The (Gong and Ordieres-Meré, 2016) study focuses on forecasting daily maximum ozone threshold exceedances in the Hong Kong area using preprocessing techniques and ensemble artificial intelligence (AI) classifiers. The researchers address the imbalance data problem by employing methods such as over-sampling, under-sampling, and the synthetic minority over-sampling technique. Ensemble algorithms are proposed to enhance the accuracy of the classifiers. Additionally, a regional data set is generated to capture ozone transportation characteristics. The results demonstrate that the combination of preprocessing methods and ensemble AI techniques effectively predicts ozone threshold exceedances. The study provides insights into the relative importance of variables for ozone pollution prediction and highlights the significance of regional data. The findings can be utilized by Hong Kong authorities to enhance existing forecasting tools and guide future research in selecting appropriate techniques.

The (Zhou et al., 2016) study introduces a novel method for predicting traffic load effects on bridges. Traditionally, load effects were assumed to be identically and independently distributed, but this method recognizes that different loading events have different statistical distributions. The approach uses the peaks-over-threshold method with the generalized Pareto distribution to model the upper tail of load effects for each event type. The distributions are then combined based on their weights to determine the overall distribution. Numerical studies validate the method's effectiveness in predicting extreme values. The proposed method addresses the challenges of non-identically distributed load effects and provides a mathematical formulation for analysis.

The (Kazemi et al., 2023) paper proposes a machine learning approach to predict iron threshold exceedances in drinking water distribution networks. Using ten years of data, models were trained with Random Forests, Support Vector Machines, and RUSBoost Trees algorithms. The best model achieved prediction accuracies over 70% for the UK regulatory concentration. The predicted probabilities were used to rank relative risk and inform proactive management decisions. The aim is to provide predictive tools for proactive water quality management.

In (Cerqueira and Torgo, 2022) authors explore the prediction of exceedance probability in the context of significant wave height forecasting. They propose a novel methodology that involves transforming the univariate time series data into a supervised learning format. This is achieved by using the lag values of the time series as input features in a standard regression algorithm. Once the regression model generates predictions for the future values of the time series, the authors proceed to estimate the probability of exceeding a predefined threshold (τ). To accomplish this, they rely on a selected distribution, which can be a well-known distribution such as the normal, Gumbel, generalized Pareto, or others. The authors calculate the (CDF) cumulative distribution function of the chosen distribution, utilizing the predicted values as the mean and the standard deviation computed from the training data. By applying the formula

$$p_i = 1 - \mathrm{CDF}_N(\hat{y}_i, \sigma_y^2)(\tau) \tag{1}$$

to the predicted values, the authors obtain an estimate of the exceedance probability for the specified threshold. This approach leverages the regression model's predictions ($\hat{y}_i$) and incorporates the statistical properties of the chosen distribution to provide probabilistic estimates of threshold (τ) exceedance. The proposed methodology offers a unique perspective on exceedance probability forecasting by combining regression modeling and distributional analysis. By utilizing well-established distributions, this methodology enables the estimation of exceedance probabilities based on the predicted values of the time series ($\hat{y}_i$) along with the standard deviation (σ_y^2) measured by the training set. Such an approach has the potential to improve risk assessment and decision-making in domains such as maritime operations, where accurate estimation of exceedance probabilities is crucial.

2.3 Contribution

This study unveils a novel framework for estimating the likelihood of threshold exceedance in time series data, pivoting on the utilization of survival analysis techniques (table 1). These techniques furnish the capacity to model and predict the time-to-event information, namely the duration until an anticipated event occurs. This is a departure from the conventional binary classification approach habitually used to tackle this issue, and it is precisely in this deviation that our method's novelty lies.

By employing survival analysis, our approach yields not merely an occurrence probability but the time until an event transpires, thereby transforming the data effectively to craft a dataset optimally tailored for training and forecasting. This provides an invaluable asset for comprehending and prognosticating extreme events, enhancing decision makers' understanding of the temporal dynamics of such risks, thereby allowing for timely action. This feature is particularly critical across various domains.

One of the primary merits of survival analysis is its utility in analyzing time-to-event data, enabling a robust examination of the temporal dynamics of outcomes. It is distinctly beneficial in scrutinizing the duration until an anticipated event materializes or a stipulated condition is fulfilled. This is particularly pertinent in the arena of threshold exceedance forecasting, where understanding the temporal dynamics holds paramount importance.

Survival analysis manifests its strength by adeptly managing censoring, an eventuality when certain subjects have not yet experienced the event of interest at the study's conclusion, or subjects are lost before the event occurs. By integrating duration variables and event indicators, survival analysis unravels the complex relationship between time and the probability of the event's occurrence.

Furthermore, survival analysis is proficient in handling censored observations, a common occurrence in threshold exceedance forecasting, where the threshold may remain uncrossed at the observation period's conclusion. Survival analysis capably leverages data from both observed events and censored instances, paving the way for superior estimations and more precise forecasts.

Survival analysis's competence extends to accommodating time-varying covariates, external factors, or changing conditions that may affect the event under consideration. By incorporating these covariates, survival analysis enables a more inclusive and dynamically adaptable modeling approach in threshold exceedance forecasting.

Significantly, survival analysis estimates the hazard function, reflecting the instantaneous risk of the event occurrence at any given time point. This capability is especially valuable in threshold exceedance forecasting as it sheds light on the intricate dynamics and patterns surrounding the event, permitting a more refined analysis that captures the fluctuating risk over time, thereby enhancing the precision of forecasts.

3 BACKGROUND

3.1 Survival Analysis Basics

Survival analysis is a statistical method used to analyze time-to-event data, where the event of interest is the occurrence of a particular event or outcome. This method is widely used in medical research, where the event of interest may be the occurrence of a disease, death, or relapse. However, survival analysis can also be applied to other fields, including finance, engineering, and environmental science.

In survival analysis, the key concept is the survival function, denoted by $S(t)$. The survival function (Lee and Wang, 2003) (Klein and Moeschberger, 2006) represents the probability that the time to the event of interest is not earlier than a specified time t. Often survival function is referred to as the survivor function or survivorship function in problems of biological survival and as the reliability function in mechanical survival problems. The survival function is represented as follows:

$$S(t) = P(T > t) \tag{2}$$

The function above denotes an individual that survives longer than t. Survival function decreases when the t increases. Its starting value is 1 for $t = 0$ which represents that at the beginning of the observation, all subjects survive.

Another important concept in survival analysis is the hazard function, denoted by $h(t)$. It is also called the force of mortality, the instantaneous death rate or the conditional failure rate (Dunn and Clark 2009). The hazard function $h(t)$ (Lee and Wang, 2003) and (Klein and Moeschberger, 2006) does not indicate the prospect or probability of the event of interest, but it is the rate of event at time t as long as no event occurred before time t. In this sense, the hazard is a measure of risk. The hazard function is defined as:

$$h(t) = \frac{f(t)}{S(t)} \tag{3}$$

In addition to the above relations, there is another important connection between h(t) (or H(t)) and S(t) given by

$$S(t) = exp(-\int_0^t h(x)dx) = exp(-H(t)) \tag{4}$$

Various parametric and non-parametric statistical techniques have been proposed over the years to estimate survival and hazard functions, providing valuable tools for analyzing time-to-event data and investigating the impact of covariates on survival time. Survival analysis also allows for the inclusion of covariates to investigate their impact on survival time.

Table 1: Contribution of the proposed approach.

Contributions of Our Approach	Implications for Threshold Exceedance Forecasting
Outputs time-to-event information	Endorses survival analysis's suitability for capturing temporal aspects of threshold exceedance events, a unique and significant advantage over binary classification methods
Handles censored observations	Leverages data from both observed events and censored instances, leading to superior estimations and more precise forecasts
Accommodates time-varying covariates	Allows a dynamically adaptable modeling approach, considering external factors or changing conditions affecting the event
Estimates the hazard function	Provides insights on the instantaneous risk of the event at any given time point, permitting a refined analysis that captures fluctuating risk, thereby enhancing forecast accuracy

This can be done using regression models, such as the Cox proportional hazards model, which assumes specific parametric assumptions. Additionally, machine learning algorithms, including various regression and classification algorithms, can also be employed to explore the effects of covariates on the hazard function without the need for specific assumptions. These machine learning algorithms offer flexibility and versatility in handling complex relationships and can provide insights into the impact of covariates on survival time in a more data-driven manner.

In this paper, we introduce a new approach to extreme value forecasting using survival analysis. Specifically, we use survival analysis to model the time until the event of interest, which in our case is the appearance of threshold exceedance. By employing the proposed method, we can not only produce a binary outcome indicating the chance of a threshold exceedance but also retrieve valuable information about the survival distribution (or, conversely, the hazard distribution) of a data point. This capability enables us to delve deeper into the dynamics of extreme events and their probabilities, a critical aspect for enhancing the safety and quality of food production and supply chains. By gaining a comprehensive understanding of the hazard distribution of data points in the food production and supply chain, we can proactively identify situations with higher risk levels, allowing for targeted interventions and risk management strategies. This proactive approach not only helps prevent potential extreme events but also aids in minimizing the impact of adverse incidents, such as food recalls, which can have severe consequences on public health, consumer confidence, and industry reputation. The proposed methodology adopts a comprehensive two-phase approach to estimate the time until the next extreme event in a time series. Each phase consists of several well-defined steps, which are visually depicted in 2 and 4. Phase 1 involves the preprocessing of the time series data. This preparatory stage comprises multiple crucial steps, including data transformation and formatting, to ensure that the data are suitable for survival analysis. By carefully handling the data in this phase, we lay the foundation for accurate and meaningful insights into extreme event forecasting. Phase 2 focuses on applying survival analysis techniques to model the time-to-extreme-event data. Within this phase, various steps are performed, building upon the outcomes of Phase 1. These steps may include utilizing survival analysis models such as random survival forests, estimating the hazard function, and making predictions on the time until the next extreme event. By systematically conducting these analyses, we can extract valuable information about the survival distribution (or, conversely, the hazard distribution) of data points, enhancing our understanding of extreme event dynamics and their associated probabilities.

We demonstrate the effectiveness of our approach through a series of simulations and real-world case studies in the food sector, highlighting the potential of survival analysis in extreme value forecasting

4 PROBLEM FORMULATION

In our research work, we focus on the problem of utilizing lag values of a univariate time series dataset to predict whether the future timestamp's value will exceed a certain threshold. A univariate time series, denoted as Y, represents a temporal sequence of values $\{y_1, y_2, \ldots, y_n\}$, where each $y_i \in Y \subset \mathbb{R}$ represents the value at time i, and n is the length of the time series. The objective of our research is to develop a model that leverages the relationship between past observations and future outcomes. Specifically, we consider the most recent q known values of the time series as lagged predictors. Let $X_i = \{y_{i-1}, y_{i-2}, \ldots, y_{i-q}\}$ represent the lagged predictors for the i-th observation, where $X_i \in X \subset \mathbb{R}^q$ denotes the corresponding embedding vector. The difference with conventional timeseries forecasting is that we are not aim to predict $y_i \in Y \subset \mathbb{R}$, but instead the probability this value to be bigger than the predefined threshold denoted as follows:

- The probability of exceeding a predefined threshold τ in a given instant i, denoted as p_i, represents the likelihood that the value of a time series will surpass that threshold. This probability is captured by a binary target variable b_i, which can be defined as follows: b_i takes the value 1 if the value y_i of the time series is greater than or equal to the threshold τ, and 0 otherwise.

$$b_i = \begin{cases} 1 & \text{if } y_i \geq \tau, \\ 0 & \text{otherwise.} \end{cases}$$

Given the lagged predictors X_i, the objective is to build a forecasting model, denoted as f, that estimates whether or not a future value y_i will exceed a predefined threshold. Thus, the model can be represented as $y_i = f(X_i, Z_i)$, where Z_i represents any additional covariates or factors known at the i-th instance that may influence the outcome. The same applies also to our formulation so it can be used either for univariate or multivariate timeseries data.

By developing an accurate forecasting model using lag values, we aim to provide insights into the occurrence of threshold exceedance in future timestamps, enabling proactive decision-making and risk management strategies.

5 PROPOSED METHODOLOGY

The proposed methodology aims to estimate the time until an extreme value occurrence in a time series using survival analysis techniques. To achieve this goal, we first transform the univariate time series to a supervised dataset with the binary event indicator and the duration variables which are needed for the survival dataset, where the event of interest is the occurrence of a threshold exceedance, and the survival time is the time counted until the threshold exceedance. Finally, we apply a range of survival analysis methods to model the survival dataset and make predictions.

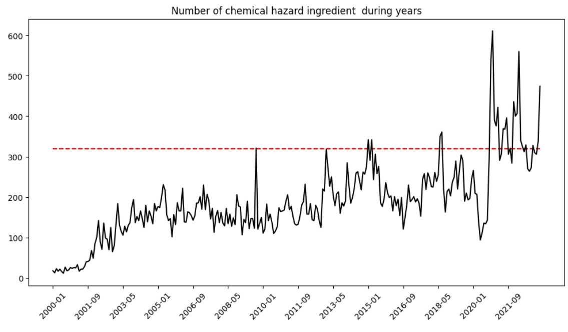


Figure 1: Real-world data showcase.

The proposed methodology can be applied to various time series datasets to identify the time until the next extreme event and provide insights for decision-making in various domains.

The rest of this section is divided into two main parts. The first one aims to describe the pre-processing steps that we follow and the second one is the post-processing pipeline until we reach the prediction outcome.

5.1 Pre-Processing Steps

In order to perform survival analysis on our time series for threshold exceedance forecasting, it is imperative to undertake a data transformation process to align the data with the requisite format. The format for survival analysis comprises three essential components. Firstly, the data predictors which, in our case, correspond to the lagged values of the univariate time series confined within the predetermined time window. Secondly, the binary event indicator, and lastly, the duration variable. This entails the generation of a binary event indicator as well as a duration variable for each observation within the series.

Having as a starting point the time-series dataset which can be either univariate or multivariate the first action in the pre-processing pipeline in our methodology is the re-framing (step 2 in figure 2) of the time-series dataset into a supervised form using the X vector of lagged values $X_i = \{y_{i-1}, y_{i-2}, \ldots, y_{i-q}\}$ as predictor variables as defined in the problem formulation section and the b_i as the variable we want to predict. The number of lag values is adjustable and is denoted by the user.

Right after, as is demonstrated in the second step of 2 is the identification of extraordinary occurrences within each individual time series. As previously indicated, owing to the adaptable nature of our approach, the threshold can be established either through manual determination by the user or by heeding the guidance of a domain expert. Nevertheless, in our empirical investigations, we present out-

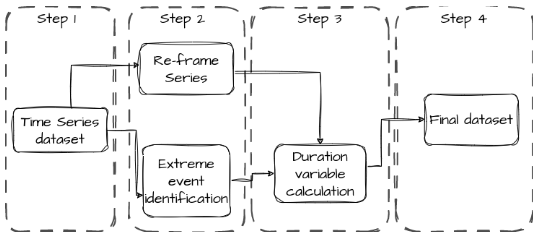


Figure 2: Phase 1: Pre-processing Steps.

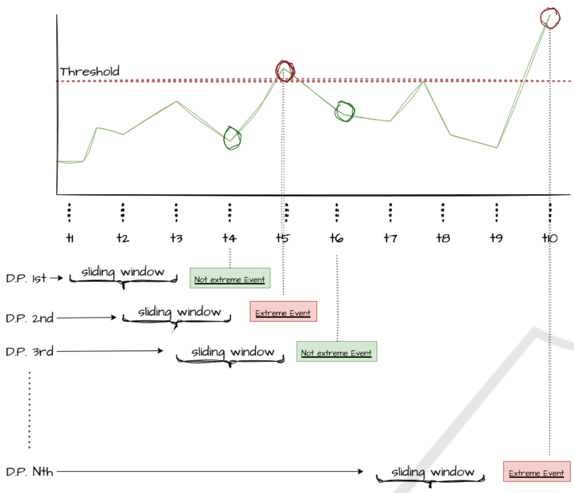


Figure 3: Schematic representation of the proposed method showcasing the sequential transformation of time-series dataset to survival analysis-ready dataset which consist of N data points (D.P).

comes employing a uniform procedure introduced in the scholarly work of (Cerqueira and Torgo, 2022) which is predicated upon a data-centric technique relying on the n-th percentile, specifically the 95th percentile, of the training dataset that is accessible.

The third step as it is shown in figure 2 as well as in the algorithm 1 is the calculation of the duration variable which plays a critical role in our methodology. The computation of the duration variable is executed through the utilization of a sliding window technique as depicted in 3, where the parameters defining the window, including the number of lag values, are determined by the user. For each individual data point within the series, there exists a corresponding start date and end date. Notably, the start date remains constant until an observed data point exhibits an extreme event in its most recent value. In such cases, the start date transitions to the date associated with that specific data point, while the end date remains the concluding date of the respective sliding window. By incorporating these three fundamental components, our methodology facilitates the analysis of survival patterns, enabling a comprehensive exploration of the impact of extreme events on the duration of the time series.

Following the transformation, we apply survival analysis algorithms to the dataset, akin to conventional machine learning. The data is split into training and test sets, and various algorithms are evaluated. Survival analysis excels in modeling time-to-event data, especially when events are rare or censored. In our proposed methodology, we explore non-parametric methods like Decision Survival Trees, RandomSurvival Forest, Survival SVM, and neural network-based models such as DeepSurv.

Our methodology is configurable, allowing users to experiment with different methods and tailor their analysis. For instance, the selection of extreme events can be customized based on domain knowledge. The user has the flexibility to experiment with different numbers of the lagged values, different percentiles to determine the threshold at which values are considered extreme, various survival analysis algorithms, and even different train and test sizes. This allows for customization in defining the criteria for extreme values within the dataset.

5.2 Post-Processing Steps

The objective of this study is to predict whether or not a threshold exceedance will occur in the next timestamp, necessitating the conversion of survival analysis results into binary output. Our approach encompasses two alternatives, each described in detail below.

In the first alternative, we construct a survival analysis model and we predict the survival distribution for each data point. Subsequently, we extract the mean probability from the distribution, signifying the sharpness of the probability distribution. The intuition behind this is the idea that the greater the survival probability of the data point the most probable that we will not have a threshold exceedance in the future timestamp. To transform this output into a binary outcome, analogous to a probability prediction of a binary classifier, we employ a threshold-based approach. By comparing the mean probability against a pre-defined threshold which is regularly at 0.5 like in the conventional classification tasks, we assign a binary label indicating exceedance or non-exceedance of the threshold.

The second alternative in the proposed methodology involves utilizing the survival analysis model to predict the risk associated with each data point. Inspired by the concept presented in Cerqueira et al. (Cerqueira and Torgo, 2022), we employ the trained survival model to estimate the survival risk for each data point. Subsequently, in order to obtain a binary outcome indicating the occurrence or absence of a

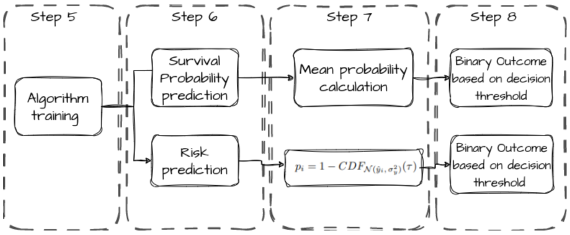


Figure 4: Phase 2: Post-processing Steps.

threshold exceedance in the future, we apply the formula used in Cerqueira et al. (Cerqueira and Torgo, 2022), which is given by:

$$p_i = 1 - \mathrm{CDF}_N(\hat{y}_i, \sigma_y^2)(\tau) \tag{5}$$

The above function is utilized to transform the survival risk into a binary outcome. Leveraging a well-known distribution, such as the Normal distribution, where the predicted survival risk ($\hat{y}_i$) is set as the mean of the distribution for each data point and the standard deviation (σ_y^2) is computed from the training set, we can predict whether the survival risk will exceed a specific threshold (τ) for each data point. This transformation enhances the practicality and interpretability of the survival analysis results, facilitating informed decision-making in the context of threshold exceedance predictions.

As previously elucidated, within the context of time series reframing, a specific time window is designated, allowing for systematic traversal along the temporal axis. Subsequently, the binary event indicator is assigned a value of 1 when an extreme value manifests itself at the conclusion of the aforementioned time window.

Conversely, if an extreme value fails to materialize at the conclusion of the time window, the binary event indicator assumes a value of 0. Pertinent to note is that data points exhibiting an extreme value at the termination of the time window are deemed uncensored, while those lacking an extreme value are classified as censored.

6 EXPERIMENTS

In order to assess the performance of our framework, we rigorously evaluate it using multiple real-world and synthetic datasets. This evaluation process serves to validate the effectiveness and robustness of our proposed approach.

By conducting extensive evaluations on both real and synthetic datasets, we ensure the thorough examination of our framework's capabilities and provide evidence of its efficacy in exceeding threshold forecasting.

Algorithm 1: Duration variable calculation.

Input : DataFrame df with time series column, date column, and binary column denoting extreme events, $window\_size$

Output: DataFrame df_new with lag columns, real value column, extreme event column, start date column, stop date column, duration column

start_date $\leftarrow df[0]$; `// set the start date to the first date in the time series`

lags $\leftarrow$ df[i:i+window_size].shift(n) ; `// create lag variables from time series within the window for` n `in range(window_size)`

target_value $\leftarrow df[i + window\_size + output\_steps - 1][time\_series]$; `// get the real value`

threshold $\leftarrow$ [threshold value] ; `// set the threshold value`

for $i \leftarrow 0$ **to** $[len(df) - window\_size - output\_steps]$ **do**

  stop_date $\leftarrow df[i + window\_size + output\_steps - 1][date]$; `// set the stop date to the end of the time window`

  if ***target_value*** $>$ ***threshold*** **then**

    threshold_exceedance $\leftarrow 1$; `// set threshold exceedance to 1`

    duration $\leftarrow (stop\_date - start\_date).days$; `// calculate the duration`

    start_date $\leftarrow df[i + window\_size + output\_steps - 1][date]$; `// set the start date to the date where the extreme event occurred`

  else

    threshold_exceedance $\leftarrow 0$; `// set threshold exceedance to 0`

    duration $\leftarrow (stop\_date - start\_date).days$; `// calculate the duration`

  df_new $\leftarrow [lags, target\_value, threshold\_exceedance, start, stop, duration]$; `// add the new row to the output DataFrame`

return df_new ; `// return the new DataFrame`

6.1 Simulated Data

The data used in this research paper were simulated using random seeds for reproducibility. The data were generated using a covariance matrix and multivariate normal distribution, with mean values set to zero for all series. Some random extreme values were added to each series. Additionally, trends and seasonality were added. The data were decomposed into the trend, seasonality, and residual components. The trend, seasonality, and residual components were added to the data, and negative values were set to zero.

6.2 Real Data

In this study, data on food incidents caused by systemic factors led to massive food recalls across the globe. A web crawler was used to systematically search for official announcements from authorities in different countries. The collected data were then stored in a database. Named entity extraction was performed using a deep learning algorithm to extract relevant information from the texts, such as hazard types and food product types. A team of food experts curated the data, removing any erroneous data points and ensuring accurate extraction of hazards and products. The curated data were analyzed to identify patterns and trends in food incidents across regions and food categories. Time stamps of the incidents were also recorded to project the absolute number of incidents over time. The use of a web crawler and data curation process ensured comprehensive and reliable data for analysis.

6.3 Results

In this section, we present the results of our experiments, which are visually depicted in table 2 and in figure 5. Our objective is to compare the performance of our proposed method with a baseline approach encompassing the common core approach which is mentioned in the related work which is the binary classification. This comparison is based on several evaluation metrics, including accuracy, F1-score, and ROC curve. The experiments were conducted on diverse datasets, encompassing both real-world time series which consist of food incidents from public announcements as well as synthetic datasets. Through the analysis of boxplots, it becomes evident that our approach yields improved results compared to the baseline approach across the aforementioned metrics. The observed performance enhancements underscore the effectiveness of our method in addressing various real-world and synthetic data scenarios.

Table 2: Classification Results.

Dataset	Approach	Mean Performance Measures				
		Accuracy	Precision	Recall	F1-score	Auc-score
Real	Binary clas.	59%	67%	48%	46%	58%
	S.A. Alter A	66%	50%	91%	61%	41%
	S.A. Alter B	67%	54%	88%	63%	61%
Synthetic	Binary clas.	77%	51%	20%	25%	62%
	S.A. Alter A	57%	25%	74%	35%	43%
	S.A. Alter B	77%	54%	49%	44%	67%

6.4 Discussion

In this study, we have embarked on a journey towards redefining the way we approach the challenging task of threshold exceedance forecasting in time series data. By departing from the conventional binary classification methods, our novel framework leverages the power of survival analysis techniques, thus ushering in a fresh perspective in this domain.

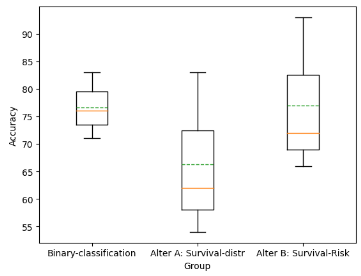
The essence of our approach lies in its ability not only to predict the occurrence of events but also to unravel the temporal dimension by estimating the time until an event takes place. This fundamental departure from the binary classification paradigm provides us with a unique advantage. By shifting our focus to understanding the duration until an anticipated event materializes, we transform our data into a more informative and tailored dataset, ripe for training and forecasting.

The implications of this paradigm shift are far-reaching and of immense value across diverse domains. For decision-makers, our framework offers a deeper comprehension of the temporal dynamics of extreme events. Armed with insights into when these events are likely to occur, timely actions can be taken to mitigate risks and enhance preparedness.

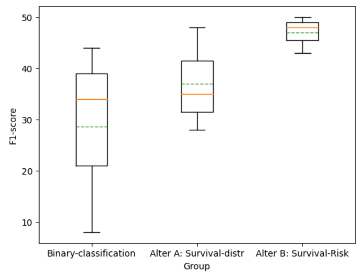
Survival analysis, as a central component of our framework, brings several advantages to the table. First and foremost, it excels in the analysis of time-to-event data, enabling a robust examination of the temporal dynamics of outcomes. This strength is particularly pertinent in the context of threshold exceedance forecasting, where understanding when an event is likely to occur is of paramount importance.

One of the key strengths of survival analysis is its adept handling of censoring, a common occurrence in our domain. By seamlessly integrating duration variables and event indicators, survival analysis unravels the intricate relationship between time and the probability of event occurrence. This capability is pivotal in scenarios where certain subjects have not yet experienced the event of interest at the conclusion of the study, or when subjects are lost before the event unfolds.

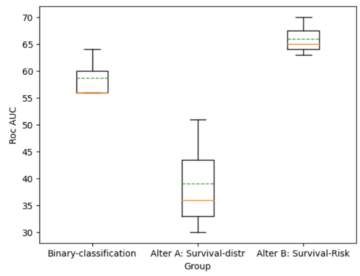
Moreover, survival analysis excels in accommodating censored observations, a frequent scenario in threshold exceedance forecasting. Here, the threshold



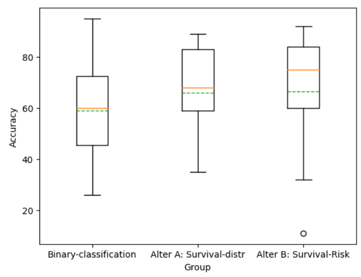
(a) Accuracy metrics boxplot for the synthetic datasets.



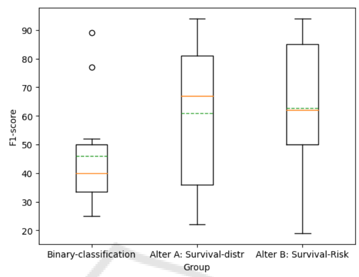
(b) F1-score metrics boxplot for the synthetic datasets.



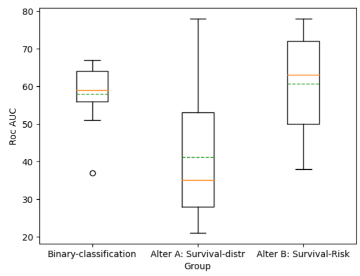
(c) Roc-auc metrics boxplot for the synthetic datasets.



(d) Accuracy metrics boxplot for the real-world datasets.



(e) F1-score metrics boxplot for the real-world datasets.



(f) Roc-auc metrics boxplot for the real-world datasets.

Figure 5: Metrics observed from the application of our approach to real-world and synthetic datasets.

may remain uncrossed at the end of the observation period. Survival analysis ingeniously leverages data from both observed events and censored instances, enabling more accurate estimations and precise forecasts.

While our novel framework brings fresh perspectives to threshold exceedance forecasting, it's important to acknowledge certain limitations that are inherent to this domain. One of the primary challenges we face is the issue of imbalanced data. In scenarios where extreme events, our positive cases, are relatively rare compared to non-extreme events, our dataset becomes inherently skewed. This imbalance can pose a significant challenge, as the framework's performance may be affected. The scarcity of positive cases can lead to decreased performance metrics, making it challenging to achieve the same level of accuracy and precision as in more balanced datasets. Addressing this limitation is a critical area of future research, and we recognize the need for innovative techniques to handle imbalanced data effectively. Strategies such as oversampling, undersampling, or the use of specialized algorithms tailored for imbalanced datasets are avenues worth exploring to mitigate this limitation and further enhance the framework's robustness.

In addition to our innovative framework, it is essential to highlight potential directions for future research that can build upon the foundation laid in this study. A primary challenge that warrants further exploration is the issue of imbalanced data, particularly when dealing with rare extreme events. As mentioned, data imbalance can affect the performance of the framework. To address this limitation, future work could delve deeper into advanced techniques for handling imbalanced datasets. Strategies such as cost-sensitive learning, synthetic data generation, or ensemble methods tailored for imbalanced scenarios can be investigated to improve the framework's resilience in such situations. Furthermore, expanding the framework to accommodate additional sources of temporal information, such as external factors or dynamic covariates, represents another promising avenue for future research. By incorporating these elements, we can enhance the adaptability and predictive power of the framework, making it even more valuable for various applications. In summary, while this study marks a significant milestone, there is ample room for further innovation and refinement to unlock the full potential of threshold exceedance forecasting in time series data.

7 CONCLUSION

In conclusion, our innovative framework, anchored by survival analysis, brings a profound transformation to threshold exceedance forecasting. By delving into the temporal intricacies and unveiling the instantaneous risk through hazard function estimation, our approach enhances precision and timeliness in forecasting extreme events. This paradigm shift is not only a pioneering step but also a potential game-changer in how we comprehend and act upon extreme events. It offers resilience in the face of censoring, adaptability to changing conditions, and the promise of more effective risk mitigation. The ability to understand when events are likely to occur, regardless of data imbalance, empowers decision-makers across diverse domains, making our framework an invaluable asset in the realm of temporal risk assessment.

ACKNOWLEDGEMENTS

The research presented in this paper has been supported with funding from the European Union's Horizon Europe programme under the Extreme Food Risk Analytics (EFRA) project with grant agreement No 101093026.

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