Interval Type-2 Fuzzy Control to Solve Containment Problem of Multiple USV with Leader's Formation Controller

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Abstract: An interval type-2 (IT2) fuzzy controller design method is proposed in this paper to simultaneously solve the formation and containment control problems of multi-unmanned surface vehicles (USVs) system. Via the construction of IT2 Takagi-Sugeno Fuzzy Model (IT2T-SFM), the control problem of nonlinear multi-USVs system can be transferred into the linear problem and the uncertain factors can be described more completely. Based on the IT2T-SFM, the IT2 fuzzy formation and containment controller is designed by the imperfect premise matching method to achieve the more flexible design process. When the IT2 fuzzy formation controller is designed for the leader USVs system, some problems are occurred in the containment analysis process. Therefore, the design concept for unknown leader's input is extended to solve the problem. And a technique is applied to obtain the less-conservative IT2 fuzzy controller design process for the containment purpose. Finally, the simulation results are presented to verify the proposed design method.

1 INTRODUCTION

By virtue of the unmanned feature, the developments of Unmanned Surface Vehicle (USV) and unmanned aerial vehicle have rapidly grown up (Yan et al., 2010 & Ucgun et al., 2022). Especially, USV has become an important role in the navy for every countries because it can efficiently put people out of critical danger in the extreme situations such as battlefields and nuclear regions. It is witnessed that USVs can efficiently substitute the human beings to achieve various required tasks. In addition, USV has also been extended to the control problem in daily-life (Manley, 2008). However, the dynamic of vessels whether the ships maneuvered by human or USV often consist of highly nonlinearities since the complex working environment. These nonlinearities will make USV difficult to perform well. In (Fossen, 1999), researcher has established the nonlinear system to represent dynamic behaviors of a navigating ship more completely. And some researchers have developed the control methods for USV with this kind of nonlinear system (Gonzalez-Garcia et al., 2021). Nevertheless, the complex environment and disturbances still make mathematical models not precise enough.

Over the past few decades, the control method based on the multi-agent systems has attracted a lot of attention. Benefiting from the rapid progress of wireless transmission technology, the multi-agent control system can be realized in various practical applications (Jiang et al., 2019). Moreover, the leader-following structure of multi-agent systems has been proposed to further distinguish the tasks of each agent (Jadbabaie et al., 2003). And the containment control problem has been proposed when the leader agents are more than one (Ji et al., 2008). Nowadays, the formation control and containment control issues are widely discussed for multi-USVs system (Zhou et al., 2020 & Wu and Tong, 2022). Nevertheless, the nonlinear systems are considered to develop the control methods in these researches. Due to the complex dynamic behaviors of a navigating USV, directly designing a nonlinear controller is a challenging task. Also, the controller is required to be designed with a nontrivial process.

Using the if-then fuzzy rule, Takagi-Sugeno Fuzzy Model (T-SFM) has been proposed to represent nonlinear systems by many linear fuzzy subsystems (Wang et al., 1996). Hence, the nonlinear controller design problem is efficiently transferred into linear problem. Over the past few decade, more

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and more research devoted their effort to solving the control problem of nonlinear system using the T-SFM (Precup et al., 2013). Moreover, some researchers have developed the fuzzy controller design method to improve control performances of a ship (Chang and Hsu, 2016 & Chang et al., 2019). However, these methods is based on the type-1 T-SFM, which isn't capable enough to deal with the uncertainties. It is obvious that the dynamic behavior of a USV is much more complex than the vehicles on the ground. In addition, a lot of biofouling will also adhere to the ship hull during the long-term transportation, which generates the resistance to ship dynamics. And these uncertain problems will become more serious in the multi-USVs system due to the error dynamic information continuously exchanged between USVs. Extending the type-1 mebership function and T-SFM, the Interval Type-2 T-SFM (IT2T-SFM) has been developed to better describe the nonlinear systems with uncertain factors (Bojan-Dragos et al., 2021 & Lam et al., 2013).

In this paper, an IT2 formation and containment fuzzy controller design method is proposed for multi-USVs systems. Firstly, the IT2T-SFM is constructed for nonlinear multi-USVs system based on leader-following structure with the effect of uncertainties. Referring to the results in (Lam et al., 2013), the IT2 fuzzy controller is designed for the IT2T-SFM using the imperfect premise matching method to obtain the less-conservative design process. Although the IT2 formation and containment control methods have already been developed in (Lin et al., 2022). Some imposed assumption is considered in the stability analysis to achieve the containment purposes. Extending the concept of (Li et al., 2021), the containment analysis method is proposed without the requirement of the assumption in this paper. However, the analysis process will also become too conservative by applying the method in (Li et al., 2021). Because of the reason, the analysis method of linear multi-agent system is extended to relax the stability analysis process (Xi et al., 2011). Finally, the simulation results of multi-USVs system is presented to verify the formation and containment control performances of the proposed design method.

2 SYSTEM DESCRIPTION AND PROBLEM STATEMENT

In this section, a nonlinear system and IT2T-SFM are established to describe the dynamic behaviours of multi-USVs by combining with the uncertain factors..

According to the research for the analysis and control problems of ship's nonlinear dynamic behaviours (Fossen, 1999), the constructive process of nonlinear system for a moored tanker has been introduced based on the ship's parameters in (Fossen and Grovlen, 1998). Extending the nonlinear system, the nonlinear multi-USVs system can be presented as follows.

$$\dot{x}_1^{\vartheta}(t)=\left(cos\left(x_3^{\vartheta}(t)\right)+\Delta_{14}(t)\right)x_4^{\vartheta}(t)-\left(sin\left(x_3^{\vartheta}(t)\right)+\Delta_{15}(t)\right)x_5^{\vartheta}(t) \tag{1}$$

$$\dot{x}_2^{\vartheta}(t)=\left(sin\left(x_3^{\vartheta}(t)\right)+\Delta_{24}(t)\right)x_4^{\vartheta}(t)+\left(cos\left(x_3^{\vartheta}(t)\right)+\Delta_{25}(t)\right)x_5^{\vartheta}(t) \tag{2}$$

$$\dot{x}_3^{\vartheta}(t)=\left(1+\Delta_{36}(t)\right)x_6^{\vartheta}(t) \tag{3}$$

$$\dot{x}_4^{\vartheta}(t)=-0.0358x_1^{\vartheta}(t)-0.0797x_4^{\vartheta}(t)-0.9215u_1^{\vartheta}(t) \tag{4}$$

$$\begin{aligned}\dot{x}_5^{\vartheta}(t)=&-0.0208x_2^{\vartheta}(t)-0.0818x_5^{\vartheta}(t)-0.1224x_6^{\vartheta}(t)\\&+0.7802u_2^{\vartheta}(t)+1.4811u_3^{\vartheta}(t)\end{aligned} \tag{5}$$

$$\begin{aligned}\dot{x}_6^{\vartheta}(t)=&-0.0394x_2^{\vartheta}(t)-0.02254x_5^{\vartheta}(t)-0.2468x_6^{\vartheta}(t)\\&+1.4811u_2^{\vartheta}(t)+7.4562u_3^{\vartheta}(t)\end{aligned} \tag{6}$$

where $x_1^{\vartheta}(t)$ and $x_2^{\vartheta}(t)$ are north and east position, $x_3^{\vartheta}(t)$ is yaw angle, $x_4^{\vartheta}(t)$ and $x_5^{\vartheta}(t)$ are surge and sway motion, $x_6^{\vartheta}(t)$ is yaw angular velocity, and the uncertain factors are considered as $\Delta_{14}(t)\sim\Delta_{36}(t)$ in system (1)-(6). Note that the index $\vartheta=1,2,...,\Phi+\Xi$ denotes the agent number of USVs. The essential information for the interaction topology is given in the following definition according to graph theory.

Definition 1

For an undirected graph Λ, the structure of graph is represented by nodes and edges which denote the agents and the interaction between agents. The node set is defined as $N(\Lambda)=\{n_1,n_2,...,n_{\Phi+\Xi}\}$ and the edge set is defined as $E(\Lambda)\subseteq\{(n_\vartheta,n_\eta): n_\vartheta,n_\eta\in N(\Lambda)\}$. The set of neighbour agents from n_ϑ is $Z(\Lambda)=\{n_\vartheta\in N(\Lambda): (n_\vartheta,n_\eta)\in E(\Lambda)\}$. Then, the adjacency matrix is defined as $\mathbf{J}=\left[j_{\vartheta\eta}\right]\in \mathrm{R}^{(\Phi+\Xi)\times(\Phi+\Xi)}$. In matrix **J**, the element values $j_{\vartheta\eta}=1$ and $j_{\vartheta\eta}=0$ respectively denote there is and isn't an interaction between agent n_ϑ and n_η. The degree matrix is also constructed with the element of **J** as $\mathbf{D}=diag\left\{0_1,\ldots,0_\Phi,d_{(\Phi+1)},...,d_{(\Phi+\Xi)}\right\}$ where $d_\vartheta=\sum_{\eta=1}^{\Phi+\Xi}j_{\vartheta\eta}$ and $diag\{\bullet\}$ denotes the diagonal matrix with the item $\bullet$. Therefore, the Laplacian matrix is obtained as follows by $\mathbf{L}=\mathbf{D}-\mathbf{J}$ to represent the interaction relationship of all the agents.

$$\mathbf{L}=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{L}_2 & \mathbf{L}_1\end{bmatrix} \tag{7}$$

where $\mathbf{L}_1 \in \mathrm{R}^{\Xi\times\Xi}$ denotes the interaction between followers and $\mathbf{L}_2 \in \mathrm{R}^{\Phi\times\Xi}$ denotes the interaction from leaders to followers. To develop the IT2 fuzzy controller, the IT2T-SFM is constructed for nonlinear multi-USVs system (1)-(6) as follows

Model Rule α**:** If $x_3^{\vartheta}(t)$ is $\tilde{\mathrm{M}}_{\alpha}$, then

$$\dot{x}^{\vartheta}(t)=\mathbf{A}_{\alpha}x^{\vartheta}(t)+\mathbf{B}_{\alpha}u^{\vartheta}(t) \tag{8}$$

where $x^{\vartheta}(t)=\begin{bmatrix}x_1^{\vartheta}(t) & x_2^{\vartheta}(t) & x_3^{\vartheta}(t) & x_4^{\vartheta}(t) & x_5^{\vartheta}(t) & x_6^{\vartheta}(t)\end{bmatrix}^{\mathrm{T}}$, $u^{\vartheta}(t)=\begin{bmatrix}u_1^{\vartheta}(t) & u_2^{\vartheta}(t) & u_3^{\vartheta}(t)\end{bmatrix}^{\mathrm{T}}$ and $\alpha=1,2,3$. To demonstrate the effectiveness of IT2 membership function in the uncertain problem, the model matrices $\mathbf{A}_{\alpha}$ and $\mathbf{B}_{\alpha}$ are selected same as the type-1 T-SFM (Chang and Hsu, 2016 & Chang et al., 2019) according to the following operating points.

$$x_{op1}^{\vartheta}=\begin{bmatrix}0 & 0 & -90^{\circ} & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}},$$
$$x_{op2}^{\vartheta}=\begin{bmatrix}0 & 0 & 0^{\circ} & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}},$$
$$x_{op3}^{\vartheta}=\begin{bmatrix}0 & 0 & 90^{\circ} & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}.$$

Note that $\mathbf{A}_1$ and $\mathbf{B}_1$ of fuzzy rule 1 are related to x_{op1}^{ϑ} and so on. Considering three operating points, the IT2 membership function is designed in Figure 1 to more completely describe the uncertain factors.

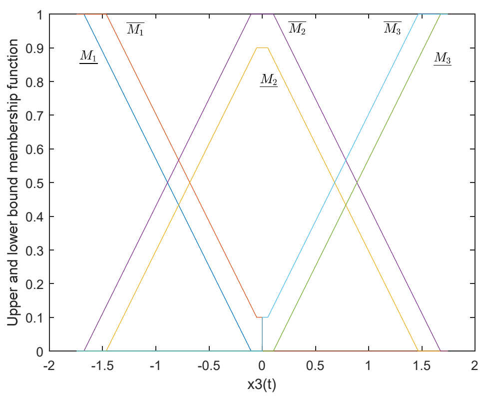

Figure 1: IT2 membership function of IT2T-SFM.

Based on the IT2 membership function the Fig. 1, the fired strength for IT2T-SFM (8) is obtained as

$$\tilde{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)=\left[\underline{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right),\overline{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\right] \tag{9}$$

where $\overline{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)$ and $\underline{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)$ denotes the upper and lower bound membership functions which satisfy $0\le\underline{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\le\overline{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\le 1$.

To achieve the formation purpose, the IT2T-SFM (8) is further presented into the following form.

Model Rule α**:** If $x_3^{\vartheta}(t)$ is $\tilde{\mathrm{M}}_{\alpha}$, then

$$\begin{cases}\dot{\varpi}^{\vartheta}(t)=\mathbf{A}_{\alpha}\varpi^{\vartheta}(t)+\mathbf{B}_{\alpha}u^{\vartheta}(t) & for \quad \vartheta=1,\dots,\Phi\\ \dot{x}^{\vartheta}(t)=\mathbf{A}_{\alpha}x^{\vartheta}(t)+\mathbf{B}_{\alpha}u^{\vartheta}(t) & for \quad \vartheta=\Phi+1,\dots,\Phi+\Xi\end{cases} \tag{10}$$

where $\varpi^{\vartheta}(t)=x^{\vartheta}(t)-\Re^{\vartheta}$ denotes the translated state vector with the desired value vector $\Re^{\vartheta}$ for the states of leader USVs. Then, the following overall IT2T-SFM is inferred from the model (10) with (9).

$$\dot{\varpi}^{\vartheta}(t)=\sum_{\alpha=1}^{3}\tilde{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\left\{\mathbf{A}_{\alpha}\varpi^{\vartheta}(t)+\mathbf{B}_{\alpha}u^{\vartheta}(t)\right\}$$
$$for \quad \vartheta=1,2,\dots,\Phi \tag{11}$$
$$\dot{x}^{\vartheta}(t)=\sum_{\alpha=1}^{3}\tilde{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\left\{\mathbf{A}_{\alpha}x^{\vartheta}(t)+\mathbf{B}_{\alpha}u^{\vartheta}(t)\right\}$$
$$for \quad \vartheta=\Phi+1,\Phi+2,\dots,\Phi+\Xi \tag{12}$$

where $\tilde{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)=\overline{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\overline{\Omega}_{\alpha}\left(x_3^{\vartheta}(t)\right)+\underline{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\underline{\Omega}_{\alpha}\left(x_3^{\vartheta}(t)\right)$, $\tilde{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\ge 0$ and $\sum_{\alpha=1}^{3}\tilde{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)=1$. Note that $\overline{\Omega}_{\alpha}\left(x_3^{\vartheta}(t)\right)$ and $\underline{\Omega}_{\alpha}\left(x_3^{\vartheta}(t)\right)$ denote the nonlinear functions which are unnecessary to be known. These functions satisfy $1\ge\overline{\Omega}_{\alpha}\left(x_3^{\vartheta}(t)\right)\ge\underline{\Omega}_{\alpha}\left(x_3^{\vartheta}(t)\right)\ge 0$ and $\overline{\Omega}_{\alpha}\left(x_3^{\vartheta}(t)\right)+\underline{\Omega}_{\alpha}\left(x_3^{\vartheta}(t)\right)=1$. According to (10), the IT2 fuzzy formation and containment controller can be designed as follows.

Controller Rule β**:** If $x_3^{\vartheta}(t)$ is $\tilde{\mathrm{N}}_{\beta}$, then

$$\begin{cases}u^{\vartheta}(t)=\mathbf{F}_{\beta}\varpi^{\vartheta}(t) & for \quad \vartheta=1,\dots,\Phi\\ u^{\vartheta}(t)=\mathbf{K}_{\beta}\sum\limits_{\eta\in N(\Lambda)} j_{\vartheta\eta}\left(x^{\vartheta}(t)-x^{\eta}(t)\right) & for \quad \vartheta=\Phi+1,\dots,\Phi+\Xi\end{cases} \tag{13}$$

where $\tilde{\mathrm{N}}_{\beta}$ denotes the IT2 fuzzy set and β denotes the rule number of IT2 fuzzy controller, $\mathbf{F}_{\beta}$ and $\mathbf{K}_{\beta}$ denote the feedback gains to be designed. And the IT2 membership function of (13) is designed in Figure 2.

Then, the IT2 fuzzy controller (13) is referred into overall fuzzy controller as follows.

$$u^{\vartheta}(t)=\sum_{\beta=1}^{2}\tilde{N}_{\beta}\left(x_3^{\vartheta}(t)\right)\left\{\mathbf{F}_{\beta}\varpi^{\vartheta}(t)\right\}$$
$$for \quad \vartheta=1,2,\ldots,\Phi \quad (14)$$

$$u^{\vartheta}(t)=\sum_{\beta=1}^{2}\tilde{N}_{\beta}\left(x_3^{\vartheta}(t)\right)\left\{\mathbf{K}_{\beta}\sum_{\eta\in N(\Lambda)} j_{\vartheta\eta}\left(x^{\vartheta}(t)-x^{\eta}(t)\right)\right\}$$
$$for \quad \vartheta=\Phi+1,\Phi+2,\ldots,\Phi+\Xi \quad (15)$$

where

$$\tilde{N}_{\beta}\left(x_3^{\vartheta}(t)\right)=\frac{\overline{N}_{\beta}\left(x_3^{\vartheta}(t)\right)\overline{\mho}_{\beta}\left(x_3^{\vartheta}(t)\right)+\underline{N}_{\beta}\left(x_3^{\vartheta}(t)\right)\underline{\mho}_{\beta}\left(x_3^{\vartheta}(t)\right)}{\sum_{\ell=1}^{2}\left\{\overline{N}_{\ell}\left(x_3^{\vartheta}(t)\right)\overline{\mho}_{\ell}\left(x_3^{\vartheta}(t)\right)+\underline{N}_{\ell}\left(x_3^{\vartheta}(t)\right)\underline{\mho}_{\ell}\left(x_3^{\vartheta}(t)\right)\right\}}\geq 0$$

, $\sum_{\beta=1}^{2}\tilde{N}_{\beta}\left(x_3^{\vartheta}(t)\right)=1$, $\overline{N}_{\beta}\left(x_3^{\vartheta}(t)\right)$ and $\underline{N}_{\beta}\left(x_3^{\vartheta}(t)\right)$ denote the upper and lower bound membership functions presented in Figure 2, $\overline{\mho}_{\beta}\left(x_3^{\vartheta}(t)\right)$ and $\underline{\mho}_{\beta}\left(x_3^{\vartheta}(t)\right)$ denote the predefined functions.

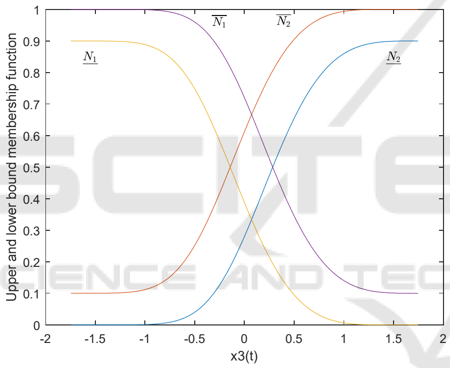

Figure 2: IT2 membership function of fuzzy controller.

Respectively substituting the IT2 fuzzy formation and containment controller of (14)-(15) into the IT2T-SFM (11)-(12), the following closed-loop IT2 fuzzy model can be obtained.

$$\dot{\varpi}^{L}(t)=\sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\tilde{M}_{\alpha}\left(x_3^{L}(t)\right)\tilde{N}_{\beta}\left(x_3^{L}(t)\right)\left\{\left(\mathbf{I}_{L}\otimes\left(\mathbf{A}_{\alpha}+\mathbf{B}_{\alpha}\mathbf{F}_{\beta}\right)\right)\varpi^{L}(t)\right\} \quad (16)$$

$$\dot{x}^{F}(t)=\sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\tilde{M}_{\alpha}\left(x_3^{F}(t)\right)\tilde{N}_{\beta}\left(x_3^{F}(t)\right)$$
$$\times\left\{\left(\mathbf{I}_{F}\otimes\mathbf{A}_{\alpha}+\mathbf{L}_1\otimes\mathbf{B}_{\alpha}\mathbf{K}_{\beta}\right)x^{F}(t)+\left(\mathbf{L}_2\otimes\mathbf{B}_{\alpha}\mathbf{K}_{\beta}\right)x^{L}(t)\right\} \quad (17)$$

where $\varpi^{L}(t)=\left[\varpi^{1}(t) \quad \cdots \quad \varpi^{\Phi}(t)\right]^{\mathrm{T}}$, $x^{L}(t)=\left[x^{1}(t) \quad \cdots \quad x^{\Phi}(t)\right]^{\mathrm{T}}$, $x^{F}(t)=\left[x^{\Phi+1}(t) \quad \cdots \quad x^{\Phi+\Xi}(t)\right]^{\mathrm{T}}$, $\mathbf{I}_L$ and $\mathbf{I}_F$ denote the identity matrix with the proper dimension, $\otimes$ is Kronecker product. For the containment purpose, the containment error system is constructed as follows

$$e^{F}(t)=x^{F}(t)+\left(\mathbf{L}_1^{-1}\mathbf{L}_2\otimes\mathbf{I}_6\right)x^{L}(t) \quad (18)$$

where $e^{F}(t)=\left[e^{\Phi+1}(t) \quad e^{\Phi+2}(t) \quad \cdots \quad e^{\Phi+\Xi}(t)\right]$ and $e^{\vartheta}(t)=\sum_{\eta\in N(\Lambda)} j_{\vartheta\eta}\left(x^{\vartheta}(t)-x^{\eta}(t)\right)$ for $\vartheta=\Phi+1,\ldots,\Phi+\Xi$.

Then, the error dynamic system can be obtained as follows via the closed-loop system (16)-(17) and (18).

$$\dot{e}^{F}(t)=\sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\tilde{M}_{\alpha}\left(x_3^{F}(t)\right)\tilde{N}_{\beta}\left(x_3^{F}(t)\right)$$
$$\times\left\{\left(\mathbf{I}_{F}\otimes\mathbf{A}_{\alpha}+\mathbf{L}_1\otimes\mathbf{B}_{\alpha}\mathbf{K}_{\beta}\right)e^{F}(t)-\left(\mathbf{L}_1^{-1}\mathbf{L}_2\otimes\mathbf{A}_{\alpha}\right)\Re^{L}\right\} \quad (19)$$

Referring to the design method in (Li et al., 2021), the containment error dynamic system (19) can be represented with the following form.

$$\dot{e}^{F}(t)=\sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\tilde{M}_{\alpha}\left(x_3^{F}(t)\right)\tilde{N}_{\beta}\left(x_3^{F}(t)\right)$$
$$\times\left\{\left(\mathbf{I}_{F}\otimes\mathbf{A}_{\alpha}+\mathbf{L}_1\otimes\mathbf{B}_{\alpha}\mathbf{K}_{\beta}\right)e^{F}(t)+\left(\mathbf{I}_{F}\otimes\mathbf{A}_{\alpha}\right)\Re_{Lc}\right\} \quad (20)$$

where $\Re_{Lc}^{\vartheta}=-\sum_{\wp=1}^{\Phi}h_{\vartheta\wp}\Re^{\wp}$ for $\vartheta=\Phi+1,\ldots,\Phi+\Xi$, $h_{\vartheta\wp}$ is the ($\vartheta,\wp$)-*th* element in the matrix $-\mathbf{L}_1^{-1}\mathbf{L}_2$ which satisfies $-h_{\vartheta\wp}\geq 0$ and $\sum_{\wp=1}^{\Phi}h_{\vartheta\wp}=1$. Then, the more relaxed IT2 fuzzy controller design process can be obtained by referring to (Xi et al., 2011) as follows. For a Laplacian matrix $\mathbf{L}_1$, the Jordan canonical form is defined as $\mathbf{\Theta}=\mathbf{Z}^{-1}\mathbf{L}_1\mathbf{Z}$ with the non-singular matrix $\mathbf{Z}$. And λ^{ϑ} is defined as eigenvalue of matrix $\mathbf{L}_1$ whose number is related to the follower USVs. Then, the eigenvalues are rearranged with the relationship $Re\left\{\lambda^{\Phi+1}\right\}<Re\left\{\lambda^{\Phi+2}\right\}<\cdots<Re\left\{\lambda^{\Phi+\Xi}\right\}$ where $Re\{\bullet\}$ and $Im\{\bullet\}$ denotes the real and imaginary part of $\bullet$.

Lemma 1 (Xi et al., 2011)

Considering the eigenvalue λ^{ϑ} of Laplacian matrix, the following relation can be obtained.

If $\Pi_1+Re\left\{\tilde{\lambda}^{\ell}\right\}\Pi_2+Im\left\{\tilde{\lambda}^{\ell}\right\}\Pi_3<0$ for $\ell=1,2,3,4$ is satisfied, then

$$\Pi_1+Re\left\{\lambda^{\vartheta}\right\}\Pi_2+Im\left\{\lambda^{\vartheta}\right\}\Pi_3<0$$
$$\text{for } \vartheta=\Phi+1,\ldots,\Phi+\Xi$$

is also satisfied, where the eigenvalue $\tilde{\lambda}^{\ell}$ is defined with $\tilde{\lambda}^{1,2} = Re\left\{\lambda^{\Phi+1}\right\} \pm j\bar{\kappa}$ and $\tilde{\lambda}^{3,4} = Re\left\{\lambda^{\Phi+\Xi}\right\} \pm j\bar{\kappa}$, $\bar{\kappa}$ is defined as $\bar{\kappa} = max\left\{\lambda^{\vartheta}\right\}$ for $\vartheta = \Phi+1,\ldots,\Phi+\Xi$.

Applying the matrix $\mathbf{Z}$, the error dynamic system (20) can be further transferred into the following form.

$$\dot{\tilde{\tilde{e}}}^{\vartheta}(t) = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\tilde{\mathrm{M}}_{\alpha}\left(x_3^{\vartheta}(t)\right)\tilde{\mathrm{N}}_{\beta}\left(x_3^{\vartheta}(t)\right)\left\{\left(\tilde{\mathbf{A}}_{\alpha} + \aleph^{\vartheta}\tilde{\mathbf{B}}_{\alpha}\tilde{\mathbf{K}}_{\beta}\right)\tilde{\tilde{e}}^{\vartheta}(t) + \tilde{\mathbf{A}}_{\alpha}\tilde{\tilde{\mathfrak{R}}}_{Lc}\right\} \tag{21}$$

where $\tilde{\mathbf{A}}_{\alpha} = diag\{\mathbf{A}_{\alpha},\mathbf{A}_{\alpha}\}$, $\tilde{\mathbf{B}}_{\alpha} = diag\{\mathbf{B}_{\alpha},\mathbf{B}_{\alpha}\}$,

$\tilde{\tilde{e}}^{\vartheta}(t) = \begin{bmatrix} Re\left(\tilde{e}^{\vartheta}(t)\right) \\ Im\left(\tilde{e}^{\vartheta}(t)\right) \end{bmatrix}$, $\tilde{\tilde{\mathfrak{R}}}_{Lc} = \begin{bmatrix} Re\left(\tilde{\mathfrak{R}}_{Lc}\right) \\ Im\left(\tilde{\mathfrak{R}}_{Lc}\right) \end{bmatrix}$,

$\aleph^{\vartheta} = \begin{bmatrix} Re\left\{\lambda^{\vartheta}\right\}\mathbf{I}_6 & -Im\left\{\lambda^{\vartheta}\right\}\mathbf{I}_6 \\ Im\left\{\lambda^{\vartheta}\right\}\mathbf{I}_6 & Re\left\{\lambda^{\vartheta}\right\}\mathbf{I}_6 \end{bmatrix}$. Note that the error signal $\tilde{e}^{\vartheta}(t)$ and desired value $\tilde{\mathfrak{R}}_{Lc}$ are obtained from $e(t) = \left(\mathbf{Z}^{-1} \otimes \mathbf{I}_d\right)\tilde{e}(t)$ and $\mathfrak{R}_{Lc} = \left(\mathbf{Z}^{-1} \otimes \mathbf{I}_d\right)\tilde{\mathfrak{R}}_{Lc}$.

Therefore, an IT2 fuzzy formation and containment controller design method is proposed in next section with the closed-loop system (16)-(17).

3 IT2 FUZZY FORMATION AND CONTAINMENT CONTROLLER DESIGN

Via the IT2 fuzzy controller design with (14)-(15), the formation and containment purposes can be achieved with the following theorem.

Theorem 1

If there exist the positive matrices $\mathbf{Q}_L$, $\mathbf{Q}_F$, $\mathbf{W}_{L\alpha\beta}$, $\mathbf{W}_{F\alpha\beta}$, common symmetric matrices $\mathbf{M}_L$, $\mathbf{M}_F$, the matrices $\mathbf{G}_{L\beta}$, $\mathbf{T}_{F\beta}$ such that the following sufficient conditions are satisfied with the given positive scalars $\bar{\delta}_{\alpha\beta i_1 q}$, $\underline{\delta}_{\alpha\beta i_1 q}$, ε, ϕ, then the leader USVs can achieve the stability and complete the formation. Additionally, the containment is achieved for all follower USVs.

$$\mathbf{Q}_L, \tilde{\mathbf{Q}}_F, \mathbf{W}_{L\alpha\beta}, \mathbf{W}_{F\alpha\beta} > 0 \tag{22}$$

$$\boldsymbol{\Gamma}_{\alpha\beta} + \mathbf{W}_{L\alpha\beta} + \mathbf{M}_L > 0 \tag{23}$$

$$\sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\left(\bar{\delta}_{\alpha\beta i_1 q}\boldsymbol{\Gamma}_{\alpha\beta} - \left(\underline{\delta}_{\alpha\beta i_1 q} - \bar{\delta}_{\alpha\beta i_1 q}\right)\mathbf{W}_{L\alpha\beta} + \bar{\delta}_{\alpha\beta i_1 q}\mathbf{M}_L\right) - \mathbf{M}_L < 0 \tag{24}$$

$$\boldsymbol{\Psi}_{\alpha\beta} + \mathbf{W}_{F\alpha\beta} + \mathbf{M}_F > 0 \tag{25}$$

$$\sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\left(\bar{\delta}_{\alpha\beta i_1 q}\boldsymbol{\Psi}_{\alpha\beta} - \left(\underline{\delta}_{\alpha\beta i_1 q} - \bar{\delta}_{\alpha\beta i_1 q}\right)\mathbf{W}_{F\alpha\beta} + \bar{\delta}_{\alpha\beta i_1 q}\mathbf{M}_F\right) - \mathbf{M}_F < 0 \tag{26}$$

$$\begin{bmatrix} \tilde{\mathbf{Q}}_F & \tilde{\mathbf{Q}}_F \\ * & \phi^2\mathbf{I}_{12} \end{bmatrix} > 0 \tag{27}$$

$$min\left\{\phi^2\right\} \tag{28}$$

where $\boldsymbol{\Gamma}_{\alpha\beta} = \mathbf{A}_{\alpha}\mathbf{Q}_L + \mathbf{B}_{\alpha}\mathbf{G}_{L\beta} + \mathbf{Q}_L\mathbf{A}_{\alpha}^{\mathrm{T}} + \mathbf{G}_{L\beta}^{\mathrm{T}}\mathbf{B}_{\alpha}^{\mathrm{T}}$,

$$\boldsymbol{\Psi}_{\alpha\beta} = \begin{bmatrix} \tilde{\mathbf{A}}_{\alpha}\tilde{\mathbf{Q}}_F + \aleph^{\vartheta}\tilde{\mathbf{B}}_{\alpha}\tilde{\mathbf{T}}_{F\beta} + \tilde{\mathbf{Q}}_F\tilde{\mathbf{A}}_{\alpha}^{\mathrm{T}} + \aleph^{\vartheta}\tilde{\mathbf{T}}_{F\beta}^{\mathrm{T}}\tilde{\mathbf{B}}_{\alpha}^{\mathrm{T}} + \varepsilon\tilde{\mathbf{Q}}_F & \tilde{\mathbf{A}}_{\alpha} \\ * & -\varepsilon\mathbf{I}_{12} \end{bmatrix}$$

,

$\mathbf{G}_{L\beta} = \mathbf{F}_{\beta}\mathbf{Q}_L$, $\tilde{\mathbf{T}}_{F\beta} = \tilde{\mathbf{K}}_{\beta}\tilde{\mathbf{Q}}_F$, $\mathbf{Q}_L = \mathbf{P}_L^{-1}$, $\tilde{\mathbf{Q}}_F = \tilde{\mathbf{P}}_F^{-1}$.

proof

Because the limitation of this paper, the main derivation related to the contribution of this paper is provided instead of detailed proof. The stability analysis process of closed-loop IT2 fuzzy model (16) is similar to the general control method based on T-SFM. Moreover, the imperfect premise matching and related stability analysis have been completely introduced for IT2T-SFM and fuzzy controller design method in (Lam, Deters et al., 2013). Referring to (Lam et al., 2013) and defining the Lyapunov function of $V_L(t) = \left(\varpi^1(t)\right)^{\mathrm{T}}\mathbf{P}_L\varpi^1(t)$, the stability conditions (22)-(24) can be derived. In Theorem 1, the parameter $\bar{\delta}_{\alpha\beta i_1 q}$ and $\underline{\delta}_{\alpha\beta i_1 q}$ are obtained from IT2 membership function such that the more relaxed stability analysis process than the type-1 fuzzy control method can be proposed. It is worth notice that the stability analysis process is only required to be developed for leader USV 1. And the stability of all other leader USVs is also ensured because of the homogenous property. Therefore, the leader USVs can be controlled to the desired position by properly setting the desired value of states $\mathfrak{R}^{\vartheta}$.

However, the redundant item related to $\mathfrak{R}^{\vartheta}$ can be seen in error dynamic system (19) which causes the analysis problem in IT2 fuzzy containment controller design method. To solve the problem, the stability analysis method can be developed as follows by referring to (Li et al., 2021). According to the properties of $-\mathbf{L}_1^{-1}\mathbf{L}_2$, whose element is nonnegative and individual row sum is equal to 1, the error dynamic system (19) is transferred into (20). The detailed information can be referred to (4)-(9) in (Li, Jabbari et al., 2021). And the system (20) is further transferred into the Jordan canonical form (21). Then, defining Lyapunov function $V_F(t) = \left(\tilde{\tilde{e}}^{\vartheta}(t)\right)^{\mathrm{T}} \tilde{\mathbf{P}}_F \tilde{\tilde{e}}^{\vartheta}(t)$ where $\tilde{\mathbf{P}}_F = diag\{\mathbf{P}_F, \mathbf{P}_F\}$ and ellipsoid $\sigma : \left\{\tilde{\tilde{e}}^{\vartheta}(t) \mid \left(\tilde{\tilde{e}}^{\vartheta}(t)\right)^{\mathrm{T}} \tilde{\mathbf{P}}_F \tilde{\tilde{e}}^{\vartheta}(t) \le \Im \mu^2\right\}$, the stability conditions (25-26) for error dynamic system (21) can be obtained as follows

$$\dot{V}_F(t) + \varepsilon\left(V_F(t) - \tilde{\tilde{\mathfrak{R}}}_{Lc}^{\mathrm{T}} \tilde{\tilde{\mathfrak{R}}}_{Lc}\right) < 0 \tag{29}$$

Referring to (Li et al., 2021), one can know that if the condition (29) is satisfied, σ is an attractive invariant set for (21). And the Problem 1 in (Li et al., 2021) for the containment purpose is achieved with the effect of the item related to $\tilde{\tilde{\mathfrak{R}}}_{Lc}$. In ellipsoid, the symbol $\Im$ is defined for the upper bound for the leader's unknown input in (Li et al., 2021). It is worth notice that no matter the designed desired value $\tilde{\tilde{\mathfrak{R}}}_{Lc}$ or the leader control input in this paper is definitely finite due to the convergence of states. Obviously, if the sufficient conditions (25)-(26) are satisfied by Theorem 1, then the stability condition (29) is satisfied. And the relationship $\left\|\tilde{\tilde{e}}^{\vartheta}(t)\right\| \le \phi\sqrt{\Xi}\Im^2$ can be obtained according to the definition of ellipsoid and condition (27) where Ξ denote the follower numbers. Via the minimization with condition (28), the upper bound from $\tilde{\tilde{\mathfrak{R}}}_{Lc}$ to $\tilde{\tilde{e}}^{\vartheta}(t)$ is minimized.

However, the conservative stability conditions (25)-(28) is also caused due to the minimization of (28) on a common positive definite condition. To solve the problem, Lemma 1 is applied to obtain the more relaxed stability analysis process. Regardless of the USV's number, the sufficient conditions (25)-(28) is only required to be satisfied for four kinds of eigenvalue.

4 SIMULATION OF FORMATION AND CONTAINMENT FOR MULTI-USVS SYSTEM

In the simulation of this section, the IT2 fuzzy controller design method in Theorem 1 is applied to simultaneously solve the formation and containment control problems of multi-USVs system (1)-(6). Thus, the control gains are obtained as follows by solving the conditions (22)-(28) with MATLAB.

$$\mathbf{F}_1 = \begin{bmatrix} -0.6703 & -0.3381 & -0.0034 & -2.7782 & -0.7549 & 0.0070 \\ 0.6315 & -1.3098 & 0.5664 & 1.3769 & -5.4683 & 0.6326 \\ -0.1243 & 0.2727 & -0.3430 & -0.2710 & 1.1520 & -0.2953 \end{bmatrix}$$
$$\mathbf{F}_2 = \begin{bmatrix} -0.6719 & 0.3372 & 0.0034 & -2.7848 & 0.7529 & -0.0070 \\ -0.6297 & -1.3124 & 0.5667 & -1.3730 & -5.4795 & 0.6335 \\ 0.1239 & 0.2732 & -0.3431 & 0.2702 & 1.1540 & -0.2955 \end{bmatrix} \tag{30}$$

$$\mathbf{K}_1 = 10^4 \times \begin{bmatrix} -1.5121 & -0.0185 & 0.0013 & -1.7365 & -0.0213 & 0.0020 \\ -0.0006 & -0.8910 & 0.0271 & -0.0008 & -1.0260 & 0.0310 \\ -0.0188 & 0.1737 & -0.0130 & -0.0216 & 0.2001 & -0.0140 \end{bmatrix}$$
$$\mathbf{K}_2 = 10^4 \times \begin{bmatrix} -1.4753 & -0.0124 & 0.0017 & -1.6942 & -0.0144 & 0.0024 \\ -0.0597 & -0.8703 & 0.0265 & -0.0684 & -1.0021 & 0.0303 \\ 0.0302 & 0.1702 & -0.0128 & 0.0346 & 0.1960 & -0.0138 \end{bmatrix} \tag{31}$$

Then, the state responses of nonlinear multi-USVs system (1)-(6) are obtained in Figures 3-8 by applying the IT2 fuzzy controller (14)-(15) with the gains (30)-(31).

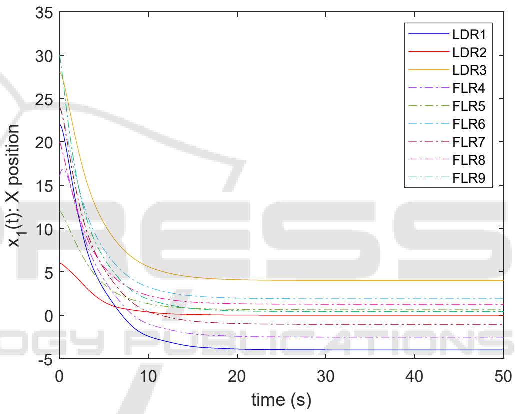

Figure 3: State $x_1^{\vartheta}(t)$ responses of multi-USVs system.

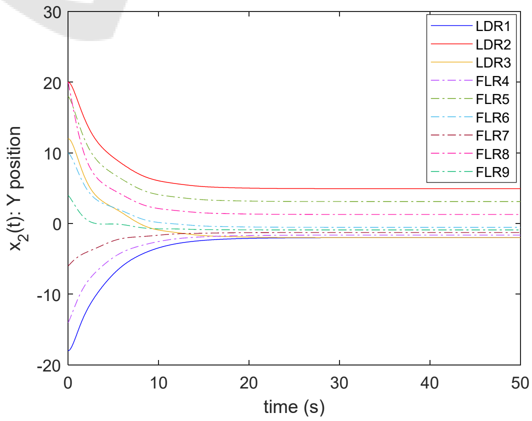

Figure 4: State $x_2^{\vartheta}(t)$ responses of multi-USVs system.

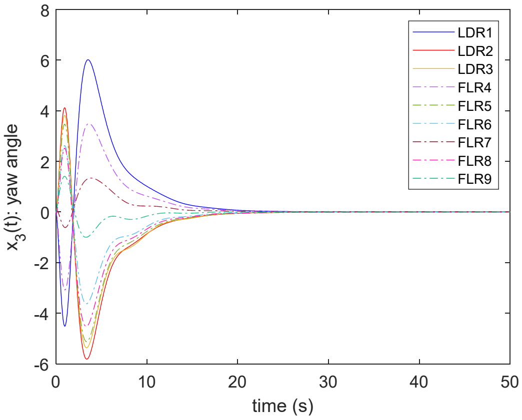

Figure 5: State $x_3^{\vartheta}(t)$ responses of multi-USVs system.

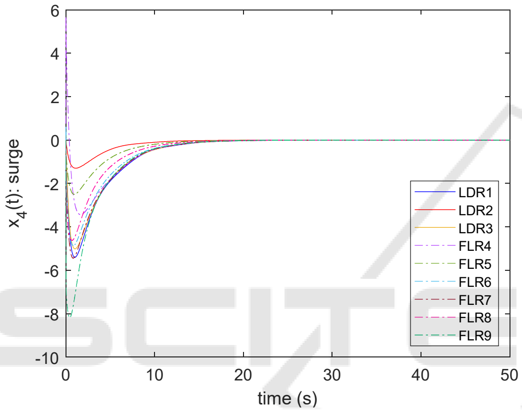

Figure 6: State $x_4^{\vartheta}(t)$ responses of multi-USVs system.

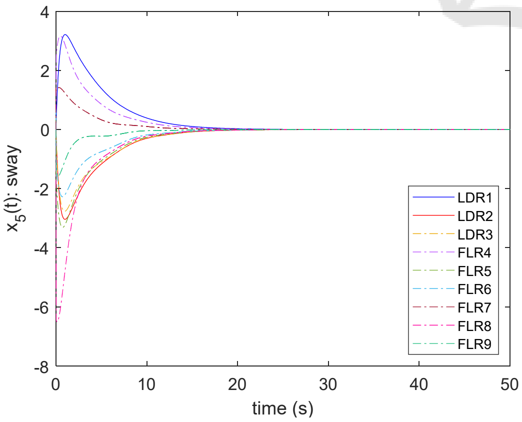

Figure 7: State $x_5^{\vartheta}(t)$ responses of multi-USVs system.

According to the simulation results in Figures 3-4, it is seen that the X position and Y position of all leader USVs can achieve the stability and converge to the desired value. Following the states of leader USVs, the states of follower USVs simultaneously complete

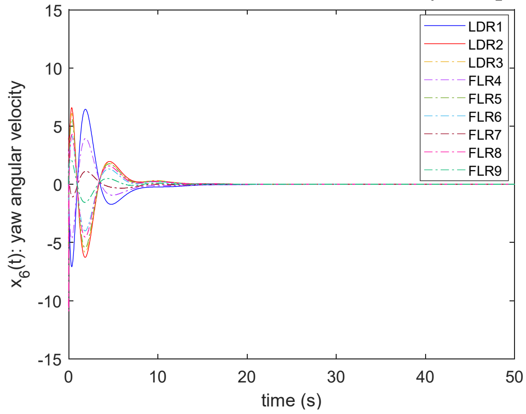

Figure 8: State $x_6^{\vartheta}(t)$ responses of multi-USVs system.

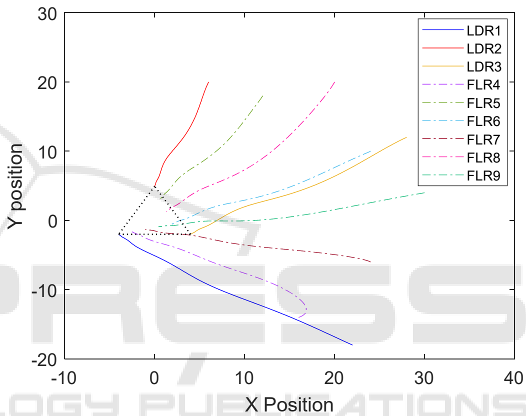

Figure 9: Trajectories of multi-USVs system.

the containment task such that all the states are forced into the interval formed by leader USVs. Moreover, the states of follower USVs can also be forced to zero if the states of all leader USVs are set to zero value in Figures 5-8. Based on the results of Figures 3-4, the trajectories of all the USVs in the nonlinear multi-USVs system (1)-(6) are also presented in Figure 9. It is obvious that the triangular region is successfully formed by three leader USVs via the IT2 fuzzy formation controller design method in this paper without the communication between USVs. And all follower USVs are controlled into the triangular region. In this simulation, the effect of uncertainties is considered as $\Delta_{14}(t) \sim \Delta_{36}(t) = 0.1sin(t)$. It is worth notice that the good formation and containment control performances can be obtained in Figures 3-8. And the smooth trajectories of all USVs can be obtained in Figure 9. Thus, it is said that the IT2 fuzzy controller design method of Theorem 1 in this paper is a good choice to simultaneously achieve the formation and containment purposes for a nonlinear multi-USVs system with uncertain problem.

5 CONCLUSIONS

In this paper, an IT2 fuzzy formation and containment controller design method is developed for the multi-USVs system based on the IT2T-SFM. Using the imperfect premise matching method, the IT2 fuzzy formation and containment controller can be designed with the different IT2 membership function from the model. The design concept for leader's unknown input is successfully extended to solve the analysis problem. And the analysis method according to the Jordan canonical form of Laplacian matrix is applied to obtain a more relax IT2 fuzzy controller design process. From the simulation results, the smooth responses to achieve the formation and containment purposes can be obtained even under the effect of uncertainties.

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