

Distributed Predictive Control for Roundabout Crossing Modelled by Virtual Platooning

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Abstract: Roundabouts pose complex challenges for autonomous vehicles. Approaching and crossing them safely requires a significant amount of information, much of which is typically unavailable. With autonomous vehicles becoming increasingly prevalent on the roads, new approaches are necessary to address these upcoming issues. While platoons and distributed control have been extensively studied in the past decade, roundabouts have received less attention. This paper presents a distributed Nonlinear Model Predictive Control (NMPC) approach using the Alternating Direction Method of Multipliers (ADMM) to utilize virtual platooning and enhance the throughput of a roundabout without requiring approaching vehicles to come to a stop. Instead, it manages the velocity of each vehicle while maintaining a safe distance. The proposed approach is validated through two case studies.


1 INTRODUCTION


Platoons represent a fundamental step in autonomous driving to group vehicles by similar paths and increase road throughput.


Concerning autonomous driving, the additional degree of complexity in automating the displacement of multiple vehicles at once is given by the coordination that they need to provide overtime to avoid collision and ensure optimal behaviour for the other agents on the road. When dealing with highway scenarios, many works of literature provide techniques to handle one-dimensional displacement over time (Bozzi et al., 2021) comes up with a robust algorithm to optimally space and enhance safety on highways, while (Bengtsson et al., 2015) proposes protocols for highway platoon merge. On the other hand, one of the main bottlenecks in autonomous driving is represented by intersections. Huge efforts have to be put into designing safe and high-performing algorithms to handle the travelling order. Cooperative approaches may help in improving intersection throughput (Wei and He, 2022), even if when vehicles ap-


proach roundabout they usually are independent of the others. Thus, if they happen to share the same path after the intersection, a coordination mechanism is needed to merge them into a platoon for consequent road sections. Order criteria should consider both the distance from the insertion point and the long-term efficiency properties that a specific formation has, as stated in (Alam et al., 2014) (i.e. vehicles should be ordered by braking capacity). Control strategies can be applied to tackle intersection coordination with the ultimate goal of reducing the travel time of vehicles inside those areas (Wang et al., 2022). However, from the intersection perspective, the main goal surely remains the choice of platoons' order based on their arrival at the intersection. Afterwards, several works that analyse manoeuvres within a platoon may be considered to modify the initial formation (Lam and Katupitiya, 2013; Bozzi et al., 2022). In (Masi et al., 2022), virtual platooning is implemented to predict future situations of a roundabout by creating occupancy intervals and making decisions to avoid collisions.

The aforementioned works deal with the design of feasible trajectories for vehicles to pursue. As a matter of fact, to physically prompt input, the vehicle's dynamics should be taken into account. This is needed to ensure more reliable behaviour over time. With this aim, Model Predictive Control (MPC) is ex-

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ploited in different scenarios of autonomous driving. For instance, (Tang et al., 2020) couples the MPC with a kinematic model for path-tracking purposes, while (Graffione et al., 2022) uses a nonlinear MPC (NMPC) to handle both longitudinal and lateral displacement in the insertion and exit of a vehicle platoon.

In control engineering, the Alternating Direction Method of Multipliers (ADMM) is widely used to guarantee a robust, method for solving large problems by iteratively solving corresponding subproblems, which ensures a sort of coordination between each agent, as implemented in (Liu et al., 2022) for energy resources. There exists also a consensus-based approach in which agents communicate with each other until convergence. A recent example with multi-robot teams can be found in (Haksar et al., 2022).

The scalability and feasibility of the ADMM have been also tested in an urban traffic problem (Li and De Schutter, 2021). A distributed model-free adaptive predictive control has been proposed for multi-region urban traffic networks. The proposed method has been tested in a real case study in China in the traffic network of Linfen. The simulation results show that the distributed model proposed yields better performance than the fixed-time control and centralized MPC controller.

In the management and control of a fleet of fuel cell cars, a comparison of three different distributed control strategies based on dual decomposition has been shown (Alavi et al., 2019). The partial method has the most negligible loss of performance when the number of cars in the system is small.

This work proposes a dual-level controller to handle the crossing of roundabouts and consequent merging as a platoon. The high-level controller exploits the consensus-based ADMM to coordinate independent vehicles to form the platoon, while the decentralized low-level MPC tries to pursue the optimal trajectory for each element to respect the scenario provided by the other controller.

This paper is organised as follows: Section 2 presents the problem formulation and the proposed approach, while Section 3 introduces the two case studies analyzed and the results that validate the effectiveness of the work. Finally, the final remarks are in Section 4.

2 METHODS

This section will focus on the methods used to implement distributed control through virtual platooning.

2.1 Simulation Environment

Let's consider a roundabout with four approaching and leaving roads. Each road (roundabout included) has only one lane and is described by a sequence of points called "Joints" connected between each other with an "Arc". Each "Arc" is defined by a sequence of points, derivatives, lengths and directions (approaching lane, leaving lane or in a roundabout). Fig. 1 shows a roundabout used in this paper.

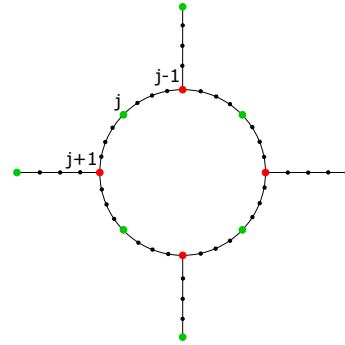


Figure 1: Roundabout example. Each big dot represents a Joint and each line connecting the two joints is an Arc. The smaller black dots are the points describing the Arc. The red Joints are the "Critical Joints" where a collision may occur.

Given an Arc length l and a path for each vehicle defined by a series of Arcs, the curvilinear distance $d_{i,J}$ between vehicle i and the "Critical Joint" J is

$$d_{i,J} = \sum_{k \in P_i} l_k \quad (1)$$

Given N homogeneous autonomous vehicles, the goal is to exploit virtual platooning to make a clean passthrough of the roundabout for all vehicles. It is also supposed to have a central unit placed in the roundabout with the sole objective of computing the platoon order and being a communication bridge between vehicles.

2.2 Vehicle Model

The vehicle in this work is described with a second-order differential equation that considers only the longitudinal displacement and velocity with acceleration as input. Thus, the discrete-time model for vehicle i is

$$\begin{cases} s_i(k+1) = s_i(k) + \Delta t V_i(k) + \frac{1}{2} \Delta t^2 a_i(k) \\ V_i(k+1) = V_i(k) + \Delta t a_i(k) \end{cases} \quad (2)$$

Where k is the time instant, Δt is the sample time, s is the longitudinal displacement, V is the longitudinal speed and a is the longitudinal acceleration.

The model (2) for vehicle i can be written in state space form $X_i(k+1) = AX_i(k) + BU_i(k)$ where $X_i(k)$ is the state vector at time k , $U_i(k)$ control vector at time k and A and B are state and control matrices. However, to have a general formulation of the approach, the following notation is chosen

$$\dot{X}_i = f_i(X_i, U_i) \quad (3)$$

Which is discretized through the Euler method, resulting in

$$X_i(k+1) = X_i(k) + \Delta t f_i(X_i(k), U_i(k)) \quad (4)$$

2.3 Virtual Platooning

Virtual platooning makes it possible to represent multiple vehicles inside a road infrastructure as a platoon moving in one direction. The goal is to find an order of vehicles such that it would be possible to enter a roundabout without stopping.

The controller's goal is to adjust the distance between platoon members ensuring safety during the roundabout crossing. Various studies discuss which technique and which information should be considered to make a platoon effective and stable. In this work, each member will consider the distance from the preceding vehicle and the leader as in (Graffione et al., 2022). This configuration reduces the oscillation caused by a string-like formation while giving great results in terms of stability and convergence time.

A platoon can be defined as a concatenation of each vehicle's system as follows

$$X(k+1) = (I \otimes A)X(k) + (I \otimes B)U(k) \quad (5)$$

Where X and U are the state and control vector of the platoon, N is the number of vehicles, I is an identity matrix of size N and the symbol \otimes represents the Kronecker product. The proposed algorithm (Alg. 1) to compute the virtual platoon is supposed to be executed by a central infrastructure in the roundabout and then propagated to the involved vehicles. Fig. 2 is an example of the application of the algorithm.

2.4 Platoon Distance Controller

The platoon obtained by the algorithm 1 needs to be controlled to keep its formation and safety distance while each member is approaching the roundabout.

Consider N vehicles indexed as $1, 2, \dots, N$ where 1 is the leader and the exchange of information is managed by a central unit placed in the roundabout. The information used by vehicle i is the distance between its preceding vehicle and the leader of the platoon. Thus, the proposed control model aims to minimize the square divergence of the longitudinal speed

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foreach Time instant do
    Search for common critical joints  $j$ 
    among vehicles based on their path;
    Check curvilinear distance  $d_{i,j}$  between
    each vehicle  $i$  and common critical joint
     $j$ ;
    Find the closer critical joint  $J$  and the
    corresponding vehicle  $I$ ;
    Sort the distance vector  $d_{i,J}$ , resulting
    order is the platoon formation;
end
    
```

Algorithm 1: Virtual Platooning Algorithm.

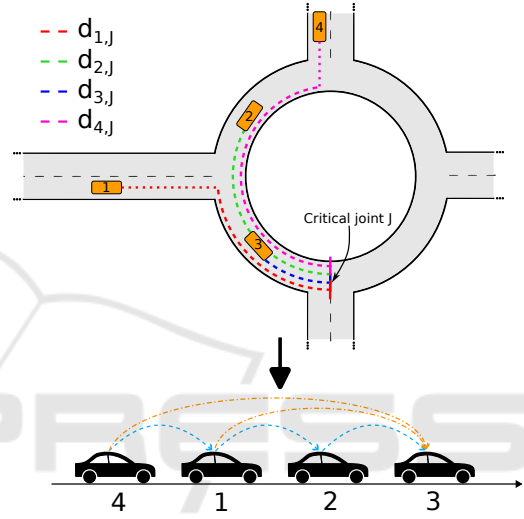


Figure 2: Example of virtual platooning. The distances of each vehicle from the critical joint J define the order of the virtual platoon and the initial condition for the controller. This process is repeated every time step by a centralized antenna that works also as an Access Point for all vehicles. Each member of the virtual platoon will consider its preceding vehicle and the leader as a reference for the inter-vehicular distance control.

of each vehicle to a reference value and the intra-vehicular distance between each member of the platoon i and its preceding vehicle and the leader. The proposed approach employs a non-linear predictive approach where the model (linear or non-linear) is used to predict the trajectory state along a time horizon to optimize the control action to the predicted state and the desired state.

The cost function at time k is defined as follows:

$$\begin{aligned}
 J(X(k), U(k)) = & \sum_{k=1}^{H_p-1} \left(\sum_{i=2}^N q_1 (s_1(k) - s_i(k) - (i-1)d_{des})^2 + \right. \\
 & + q_1 (s_i(k) - s_{i-1}(k) - d_{des})^2 + \\
 & + q_2 (V_i(k) - V_{ref})^2 + \\
 & \left. + r_1 a_i(k)^2 \right) \quad (6)
 \end{aligned}$$

Where H_p is the prediction horizon of the controller, q_1, q_2, q_3 and r_1 are weight parameters, d_{des} is the safety distance and V_{ref} is the reference speed.

Remark 1. *The reference speed of the leader is V_{ref} as well. The difference from the other vehicles lies in the fact that it does not consider the distance from other vehicles since it is in the front of the platoon.*

The minimization of the cost function defined in (6) is minimized every time step k , resulting in the following Non-Linear Model Predictive Control approach

$$\underset{X, U}{\text{minimize}} \quad J(X(k), U(k)) \quad (7a)$$

$$\text{subject to} \quad \text{equation(5)}, \quad (7b)$$

$$V_{min} \leq V_i(k) \leq V_{max}, \quad (7c)$$

$$a_{min} \leq a_i(k) \leq a_{max}, \quad (7d)$$

$$d_{min} \leq s_i(k) - s_{i-1}(k) \quad (7e)$$

Where for constraint (7b) $k = 1, \dots, H_c$ and for constraints (7c),(7d),(7e) $k = 1, \dots, H_p$ with $H_c \leq H_p$. Value H_c is the control horizon in which the control is optimized, which is usually between 10% and 20% of the prediction horizon. For $k > H_p$, the optimal control action is kept constant with the last computed value of U . To ensure safety and avoid collisions, it has been added a minimum distance between vehicles that consider the length of the vehicle. To improve the efficiency and accuracy of the controller, a Direct Multiple Shooting Method is used (Bock and Plitt, 1984). This approach approximates the time horizon in a set of n sub-intervals $[\tau_i, \tau_{i+1}]$. At each sub-interval, the initial condition of the state vector and control are parametrized as

$$\begin{aligned} U(k) &= V_i \quad \text{for } t \in [\tau_i, \tau_{i+1}] \\ X(k_j) &= h_i \\ & \quad i = 0, 1, \dots, n-1 \end{aligned} \quad (8)$$

Thus, to ensure the continuity of the solutions, two other constraints are added to the problem, and the cost function is slightly changed:

$$J(t) = \sum_{j=1}^{n-1} \int_{\tau_k}^{\tau_{k+1}} J(X_j(k), V_j(k)) d\tau \quad (9)$$

$$\underset{X, U}{\text{minimize}} \quad J(X_j(k), V_j) \quad (10a)$$

$$\text{subject to} \quad h_0 - X(0) = 0, \quad (10b)$$

$$h_{j+1} - X_j(k_{j+1}, h_j, V_j) = 0, \quad (10c)$$

$$g(h_j, V_j) \geq 0 \quad (10d)$$

Where constraint (10b) is the initial condition constraint, (10c) is the continuity constraint that assures

that the computed solution is coherent among sub-intervals, (10d) contains all the constraints defined by (7c),(7d) and (7e) and other non-linear constraints if required.

2.5 Distributed Control

In a real-case scenario, a distributed control would be the best choice to handle such a computationally heavy problem. Thus, to design a Distributed NMPC, a distributed control approach based on the Consensus Alternating Direction of Multipliers is proposed.

The Consensus ADMM is employed in scenarios where multiple agents share a common objective but simultaneously face conflicting interests. By defining the common global variable \bar{z} and the local decision variables x_i , the problem is defined as follows

$$\begin{aligned} \underset{x_i}{\text{minimize}} \quad & \sum_{i=1}^N f_i(x_i) \\ \text{subject to} \quad & x_i - \bar{z} = 0 \quad i = 1, \dots, N \end{aligned} \quad (11)$$

The problem (11) is computed on each agent i and its result consists of an optimal control $u_i(k)$ and a state estimation of other agents (depending on the agents' interconnection in the system). All of these estimations are used to estimate an average state value for each agent resulting in a new value of \bar{z} to use in the next optimization step for each agent i . The platoon members will agree only with the vehicles they are interested in. Thus, the ADMM algorithm is as follows

$$x_i^{k+1} = \min_{x_i} \left(f_i(x_i) + y_i^{kT} x_i + \frac{\rho}{2} \|x_i - \bar{z}_i^k\|_2^2 \right) \quad (12a)$$

$$z^{k+1} = \min_z \left(\sum_{i=1}^m \left(-y_i^{kT} \bar{z}_i + \frac{\rho}{2} \|x_i^{k+1} - \bar{z}_i\|_2^2 \right) \right) \quad (12b)$$

$$y_i^{k+1} = y_i^k + \rho(x_i^{k+1} + \bar{z}_i^{k+1}) \quad (12c)$$

Where equation (12a) is the local variable optimization step, (12b) is the z-update step that results in the average of the agents according to the agents' interconnection and finally (12c) is the dual variable update. From an implementation point of view, steps (12a) and (12c) can be dragged out in parallel on each agent. Instead, step (12b), requires synchronization of the information exchange or some kind of prediction in case of missing data. In this paper, it is supposed that the vehicles are synchronized without data loss. Further work will include this event.

In order to use the ADMM formulation (12), it is required to define the functions $f_i(x_i)$ and the set of local variable x_i for each agent from the centralized cost function (9). Local variables for the platoon leader are related only to the leader itself since it is in

front of the platoon. The second vehicle has only the distance from the preceding vehicle which is also the distance to the leader. All the other vehicles will have both distances from the leader and the preceding vehicle. According to this reasoning, the local variables are as follows

$$\begin{aligned} x_1 &= \dot{X}_1, & x_2 &= \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} \\ x_i &= \begin{bmatrix} \dot{X}_1 \\ \dot{X}_{i-1} \\ \dot{X}_i \end{bmatrix} & \text{with } i &= 3, \dots, N \end{aligned} \quad (13)$$

and the functions $f_i(x_i)$ are

$$\begin{aligned} f_1(x_1) &= \sum_{k=1}^{H_p} q_2 (V_1 - V_{ref})^2 + r_1 a_1^2 \\ f_i(x_i) &= \sum_{k=1}^{H_p} (X_i - X_{r,i})^T Q_i (X_i - X_{r,i}) + U_i^T R_i U_i \end{aligned} \quad (14)$$

where $i = 2, \dots, N$. Thus

$$\begin{aligned} X_2 &= \begin{bmatrix} s_1 - s_2 \\ V_1 \\ V_2 \end{bmatrix} & X_{r,2} &= \begin{bmatrix} d_{des} \\ V_{ref} \\ V_{ref} \end{bmatrix} & U_2 &= \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \\ Q_2 &= \text{diag}(q_1, q_2, q_2) & R_2 &= \text{diag}(r_1, r_2) \\ X_i &= \begin{bmatrix} s_1 - s_{i-1} \\ s_1 - s_i \\ s_{i-1} - s_i \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} & X_{r,i} &= \begin{bmatrix} d_{des} \\ (i-1)d_{des} \\ d_{des} \\ V_{ref} \\ V_{ref} \\ V_{ref} \end{bmatrix} & U_i &= \begin{bmatrix} a_1 \\ a_{i-1} \\ a_i \end{bmatrix} \\ Q_i &= \text{diag}(q_1 I_{3 \times 3}, q_2 I_{3 \times 3}) & R_i &= \text{diag}(r_1, r_2, r_3) \end{aligned} \quad (15)$$

3 RESULTS

The proposed algorithm has been tested and evaluated in two different scenarios. The two case studies are reported to show the effectiveness of the proposed method. The case study does not consider real-size vehicles but Wheeled Mobile Robots (WMR) 25 cm long and 18 cm wide because further development includes a real-time implementation of physical steering WMR to validate the approach even in a real case scenario. However, this does not invalidate the results since vehicles and road sizes are simply scaled, ensuring consistency in the whole execution.

The first case study (Fig. 3) consists of three vehicles, two in the roundabout and one on an approaching lane, the second one (Fig. 4) consists of five vehicles, one inside the roundabout and four vehicles on three different approaching lanes.

Numerical simulations have been conducted on Matlab with a variable number of vehicles inside and outside the roundabout.

Figure 3a shows the initial condition of the roundabout where each vehicle has an initial speed of 0.1 m/s and an inter-vehicular distance that does not respect the bounds defined in Table 1.

Table 1: Main parameters used.

Param	Value	Unit	Description
q_1	1		Gain on platoon distances
q_2	10		Gain on state variable V
$r_{\{1,2,3\}}$	1		Gain on control variable a
T_s	0.1	s	Sampling time
H_p	10		Prediction Horizon
H_c	2		Control Horizon
d_{des}	0.55	m	Inter-vehicular distance
d_{min}	0.45	m	Minimum safety distance

In the first instant of the simulation, the algorithm 1 defines the order of vehicles as (1 – 3 – 2), and then the ADMM is initialized on each one (Fig. 3a). The estimation of other vehicles is initialized on each agent as a zeros state. At this point, the control makes vehicles 1 and 2 respectively accelerate and decelerate (as shown in the first seconds of Fig.5c) to create more space for vehicle 3 to enter. In the meanwhile, the velocity of all vehicles converges to the reference even if the distances have priority for safety reasons. Figure 3c shows the instant when vehicle 3 enters the roundabout, already at a safe distance from the other vehicles, finally able to continue on its path while maintaining the formation (Fig. 3d). The results shown in Fig. 5 prove the convergence of both distance and speed through the simulation.

The second case study considers vehicles at an initial speed of 0.1 m/s. The computed platoon's order is (4 – 3 – 5 – 1 – 2), indicating critical inter-vehicular distances between vehicles 1 and 5, as well as between 3 and 5. Conversely, vehicles 3 and 4 have substantial distances to reduce to mitigate congestion. Fig. 4a shows the initial position of vehicles and 6b shows in detail the initial distances. As a consequence, the required control action of each agent is stronger resulting in a slower convergence time (about 15 seconds against 10 seconds in the first case study).

Fig. 4b and Fig. 6b show that vehicles 1 and 2 slow down to increase space between 1 and 3 and let 5 enter the roundabout. Vehicle 2 slows down as a consequence of vehicle 1 deceleration. Fig. 4c shows moments before vehicle 1 enters the roundabout, followed by vehicle 5 which is placed right before in the platoon order. Finally, Fig. 4d shows all vehicles in formation and continuing their programmed path (vehicle 4 is leaving the roundabout).

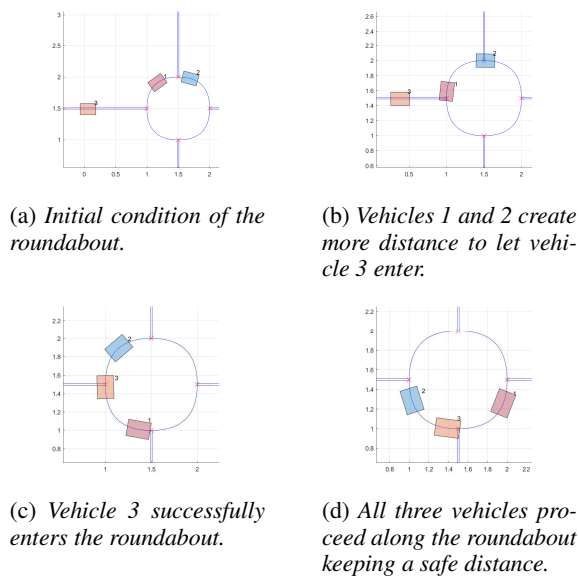


Figure 3: First case study. Two vehicles were inside the roundabout and one on the approaching lane.

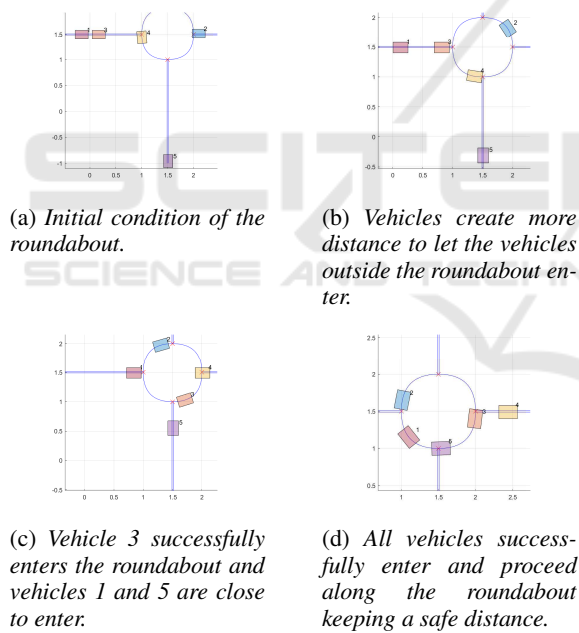


Figure 4: Second case study. Five vehicles are involved in this scenario. Four vehicles are approaching the roundabout and the last one is already in it. Fig. 6b shows the distances during the simulation. Fig. 6c shows the speeds during simulation.

4 CONCLUSIONS

The proposed approach consists of a distributed NMPC through ADMM exploiting virtual platooning to let vehicles enter a roundabout without stopping.

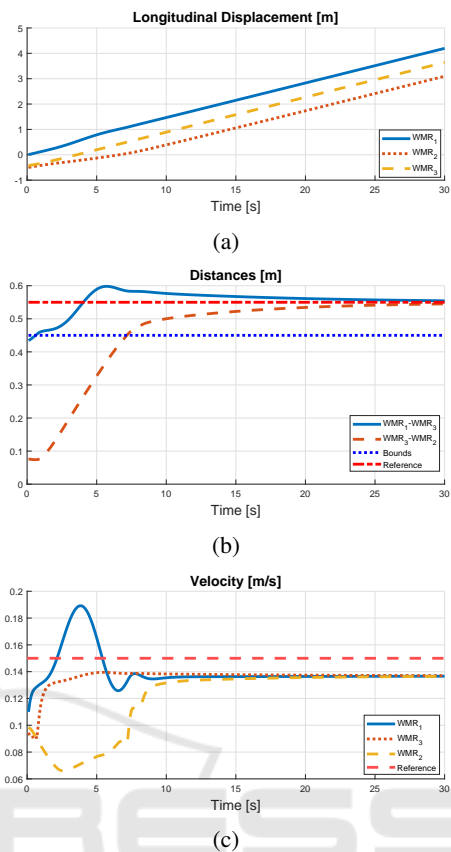


Figure 5: Results of the first case study. Fig. 5a shows the longitudinal displacement of vehicles and the platoon's order. Fig. 5b and Fig. 5c show the distances and speeds.

Two case studies with three and five vehicles are provided to prove the effectiveness and broadly assess the scalability of the algorithm. The results demonstrate satisfactory performance, despite the simplicity of the algorithm used to construct the virtual platoon. Improvements can be made by considering the initial inter-vehicular distance to ensure a smoother response from the controller. Additionally, incorporating "time-to-arrival" prediction would be essential for assessing collision risks caused by the controller's slow convergence.

Although the results consider a case study with a limited number of vehicles, it is crucial to anticipate unexpected behaviors when scaling up to accommodate a larger number of member. Further studies will be conducted to evaluate the effectiveness of this implementation depending on the number of vehicles inside the roundabout.

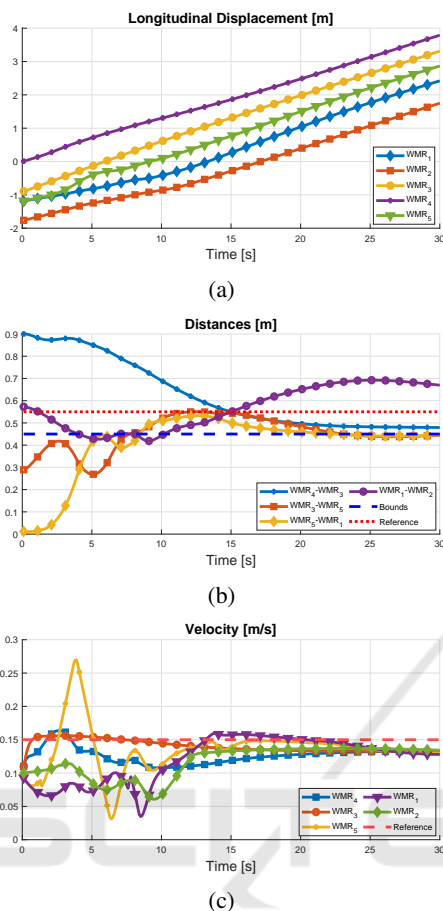


Figure 6: Results of the second case study. Fig. 6a shows the longitudinal displacement of vehicles and the platoon's order. Fig. 6b and Fig. 6c show the distances and speeds.

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