A Novel Hybrid Approach Combining Beam Search and DeepWalk for Community Detection in Social Networks

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Abstract: In the era of rapidly expanding social networks, community detection within social graphs plays a pivotal role in various applications such as targeted marketing, content recommendations, and understanding social dynamics. Community detection problem consists of finding a strategy for detecting cohesive groups, based on shared interests, choices, and preferences, given a social network where nodes represent users and edges represent interactions between them. In this work, we propose a hybrid method for the community detection problem that encompasses both traditional tree search algorithms and deep learning techniques. We begin by introducing a beam-search algorithm with a modularity-based agglomeration function as a foundation. To enhance its performance, we further hybridize this approach by incorporating DeepWalk embeddings into the process and leveraging a novel similarity metric for community structure assessment. Experimentation on both synthetic and real-world networks demonstrates the effectiveness of our method, particularly excelling in small to medium-sized networks, outperforming widely adopted methods.

1 INTRODUCTION

The explosive growth in social media users, projected to reach nearly six billion by 2027 (Dixon, 2023), has highlighted the critical importance of community detection. This task, extending beyond traditional applications such as targeted advertising and trend analysis, content recommendation, and online security (Jain et al., 2020; Kumar et al., 2020; Jain et al., 2023), now finds utility in novel domains like epidemiology, political science, and social psychology, making it a central research focus in the contemporary digital landscape.

Community detection, a core problem in social network analysis, involves identifying distinct groups of nodes within a network characterized by robust interconnections. This is achieved by effectively partitioning network nodes into cohesive clusters with dense internal connections while maintaining sparse connections with nodes in other clusters.

Various approaches to addressing the community detection problem in social networks have been proposed, as highlighted in a recent survey of (Bara'a et al., 2021), in which researchers formulated the issue as a combinatorial optimization problem. This formulation allows for the use of various optimization methods, such as heuristics and meta-heuristics. Many functions to evaluate the quality of partitioning in network communities have been proposed, but Modularity emerged as one of the most well-known and widely used measures in this field (Newman and Girvan, 2004).

Tree-search (TS) algorithms, initially designed for exploring search spaces to identify optimal or near-optimal solutions, have demonstrated their efficacy in solving optimization problems. Such applications include Monte Carlo optimization algorithms for dense subgraph identification (Zhang and Chen, 2015), branch-and-bound methods for quasi-clique detection (Mahdavi Pajouh et al., 2014), and adaptations for diverse problems like puzzles (Cazenave, 2012) and the Container Pre-Marshalling Problem (CPMP) (Tanaka and Tierney, 2018). Furthermore, TS algorithms have found utility in community detection, with approaches such as efficient detection of disjoint communities (Palsetia et al., 2014) and

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BBmst for real-time community detection in evolving networks (Nath, 2022).

Lately, machine learning (ML) methods, particularly graph embeddings (Perozzi et al., 2014), have gained attention in community detection. Graph embeddings represent graph structures in low-dimensional vector spaces, capturing node and edge relationships and features. Graph Convolutional Networks (GCNs) (Kipf and Welling, 2017; Jia et al., 2019) have also seen extensive use in community detection.

Acknowledging the effectiveness of ML methods, their integration with tree-search (TS) algorithms has been explored to enhance performance. As noted by (Böther et al., 2022), these efforts have yielded mixed results, with some studies showing significant enhancements while others remain inconclusive. Such approaches include ML-guided heuristics for NP-hard combinatorial optimization problems (Khalil et al., 2017), employing an ML model to guide Beam Search in selecting promising nodes (Huber and Raidl, 2022), and Utilizing Graph Neural Networks for expansion within the search space (Li et al., 2018). However, to the best of the authors' knowledge, the hybridization of TS algorithms with ML have not yet been explored in the context of community detection.

In this work, our challenge not only aims to evaluate the effectiveness of TS algorithms in addressing the community detection problem but also delves into the impact of hybridizing them with ML techniques. Our contributions can be summarized as follows:

- We introduce a beam-search algorithm with a modularity-based agglomeration function as a baseline of our approach.
- To further boost the performance of our algorithm, we integrate DeepWalk embeddings into the process. We refer to the resulting hybrid algorithm as **BeamWalk**.
- We present the **Compound Similarity**, an new similarity metric designed for evaluating intermediate solutions discovered during the execution of BeamWalk.

The rest of the paper is organized as follows. In Section 2, we define the problem of community detection. In Section 3, we introduce Beam Search as a fundamental method and then present an enhanced ML-boosted variant. We outline the experiments conducted to evaluate the effectiveness of our approach, along with the obtained results and their interpretation in Section 4. Finally, Section 5 concludes the paper with our conclusions, and proposes potential avenues for future research.

2 PROBLEM STATEMENT

A social network can be modeled by a graph, $\mathcal{G} = (V,E)$ which includes $N = |V|$ nodes and $m = |E|$ edges. V is the set of nodes and E is the set of edges. Formally $V = \{v_1, \ldots, v_n\}$ and $E = e_{ij}{}_{i,j=1}^{n}$.

The problem of community detection involves finding a strategy for detecting communities given a network where the nodes represent users and the edges represent connections between two users. The similarity w_{ij} between users i and j is the weight of the edge connecting i and j (This value lies in the interval $[0,1]$ and indicates the degree of similarity in their interests). The result is a set of communities $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$. A node v_i classified in community C_k must satisfy the condition that its internal degree within the community is greater than its external degree. If $C_k \cap C_{k'} = \emptyset$ for all k, k', then $\mathcal{C}$ denotes disjoint communities; otherwise, overlapping communities.

3 PROPOSED APPROACH

Beam search is a TS algorithm that is widely used in computer science and natural language processing. The algorithm explores a graph by expanding the most promising node in a limited set, reducing the memory requirements of best-first search. Beam search is also similar to breadth-first search (BFS) (Sabuncuoglu and Bayiz, 1999), in that it progresses level by level without backtracking. However, unlike BFS, beam search only moves downward from the β most promising nodes (rather than all nodes) at each level, where β is referred to as the "beam width." The remaining nodes are simply ignored.

In this section, we present our proposed modeling of the problem of community detection into a TS problem, this will serve as the building block for the next section where we propose a hybridization with DeepWalk.

3.1 Proposed Beam Search Algorithm

To adapt beam search for community detection, we begin by representing the community partition as a list of sets, with each set denoting a distinct community of nodes. The subsequent step entails defining the foundational elements of the beam search algorithm. This encompasses the determination of the solution approach whether it be improvement-focused or constructive in nature. Within this framework, we must specify tree nodes, establish expansion rules,

formulate an evaluative function for guiding our exploration, devise a scoring mechanism to estimate solution quality, and ultimately establish the criteria governing the termination of the search.

In what follows, we will delve into a comprehensive exploration of the proposed algorithm, providing a detailed implementation of its step-by-step process in Algorithm 1.

- **Search Strategy.** We explore the solutions space using an improvement approach; starting with singletons as initial communities. Solutions are expanded by considering all potential community merges, thereby exploring every conceivable community partition that may arise from the initial partition.
- **Tree Node.** We define a tree node to be a correct community partitioning for that graph, more formally let $G = (V, E)$ be a network graph, a tree node is noted as C, where $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$ is denoting a set of disjoint communities, K being the number of partitions.
- **Expansion Rule.** We expand a node using all the possible community merges, the expansion is done by merging two communities at a time. For a node $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$, a child-node $\mathcal{C}'$ is defined as $\mathcal{C}' = \{C_1, C_2, C'_3 \ldots, C_{K-1}\}$ where $C'_3 = \{C_i, C_j\}$ $1 \leq i, j \leq K$, $i \neq j$ is the resulting community merge.
- **Evaluation and Score function.** Here we define both the evaluation and score function to be the modularity score (Newman and Girvan, 2004).

  Modularity would be employed to determine the best β communities, with their modularity values being evaluated at each tree level. The ultimate solution returned would be the community partitioning that exhibits the highest modularity score.
- **Stopping Criteria.** We define the stopping criteria as the measure of no improvement in terms of modularity score, for that the difference between the modularity score of the parent node and the child node is measured.

Figure 1 shows an example of our beam search modeling for the community detection problem.

3.2 Proposed BeamWalk Algorithm

The major problem with the proposed beam search approach is the time complexity in the modularity computation which is in the order of $O(mn)$. This makes the algorithm very slow and not suitable for large graphs hence the need for a more convenient and less complex metric.

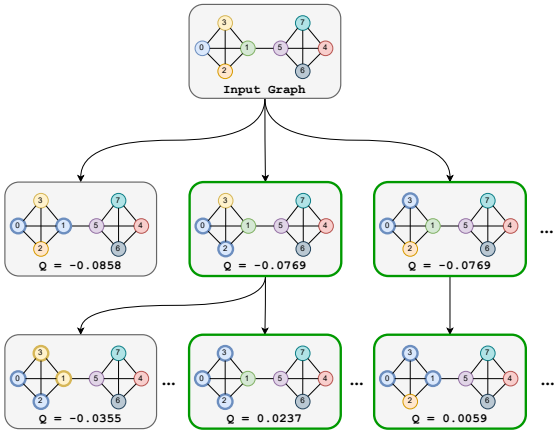


Figure 1: Beam search modeling for the problem of community detection with $\beta = 2$. Q refers to modularity.

Input: $G = (V, E)$, β, N_{min}
Output: S_{best}
beam_list = [$\{v_i\}$ for $v_i \in V$];
K : number of communities for the current level;
while $\Delta Q_i > 0$ ***and*** $K \geq N_{min}$ **do**
  foreach *node* ***in*** *beam_list* **do**
    Generate child nodes from the current node;
    Evaluate modularity for each child nodes ;
    Save ΔQ_i for $child_i$;
    Prune if $\Delta Q_i < 0$;
  end
  Select the top β nodes based on modularity; Re-initialize *beam_list* with the best β nodes for next iteration;
end
Return S_{best}, the best solution seen so far;

Algorithm 1: Modularity-Based Beam Search for Community Detection.

Therefore, in this paper, we propose an approach (Figure 2) that first maps the problem into a latent space representation in $\mathbb{R}^d$ using a graph embedding algorithm: *DeepWalk* (Perozzi et al., 2014). Then, we adapt our TS algorithm introduced before to the new modeling by introducing a new evaluation function based on the cosine similarity which is of the complexity of $O(d)$ where d is the learned vector dimension (usually ranges from 32-128).To further accelerate the search, we initialize the algorithm with Maximum Independent Cliques as initial communities, which helps accelerate the algorithm by restricting the search space, as cliques tend to be in the same community. To estimate the minimum number of communities N_{min}, a heuristic estimation is performed; in

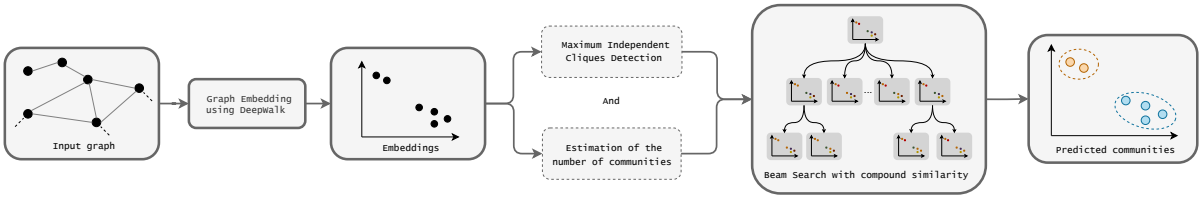


Figure 2: BeamWalk Process.

our experiments, we used the Elbow method. We also introduce a pruning strategy to speed up the search. Finally, a beam search is conducted on the initial set of communities, and the output is the best community S_{best} as measured by (2), meeting the minimum number of communities requirements.

The Beamwalk algorithm is recalled in Algorithm 2, and its main features are detailed in the subsections below.

3.2.1 DeepWalk Algorithm

DeepWalk learns a latent space representation of social interactions in $\mathbb{R}^d$. The learned representation encodes community structure so it can be easily exploited by standard classification methods. The algorithm consists of three main steps (Figure 3):

1. **Sampling Random Walks.** In this first step, random walks on the graph are generated. A random walk starts from a node and traverses the graph by randomly choosing one of its neighboring nodes as the next step. This process is repeated for a fixed number of steps or until a termination condition is met. By performing multiple random walks starting from different nodes, the algorithm aims to capture the structural information of the graph.
2. **Training Skip-Gram Model.** Once the random walks are generated, the next step is to use them to train a skip-gram model, which is a popular technique in natural language processing for learning word embeddings. In the context of DeepWalk, the skip-gram model is adapted to learn node embeddings. The skip-gram model takes a node as input and tries to predict the nodes that are likely to appear in its neighborhood based on the random walks. By optimizing the skip-gram model using the generated random walks, the algorithm learns meaningful representations for nodes in the graph.
3. **Computing Embeddings.** After training the skip-gram model, the final step is to obtain the embeddings for each node in the graph. The embeddings represent the learned low-dimensional vector representations that capture the structural properties of the graph. The embeddings can be computed by extracting the learned parameters of the skip-gram model, which contain the representations for each node.

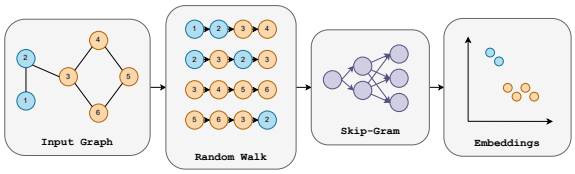


Figure 3: DeepWalk steps for learning a latent space representation of social interactions in $\mathbb{R}^d$.

3.2.2 Agglomeration Function

The agglomeration is performed using cosine similarity, as defined by Equation (1). Cosine similarity values range from -1 to 1. In our case, we only consider merges with positive similarity values. It's important to note that when measuring similarity between two communities, the centroid of each community is considered.

$$\text{Similarity}(A,B) = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2}\sqrt{\sum_{i=1}^{n} B_i^2}} \quad (1)$$

Where A and B are the two vectors representing node embedding.

3.2.3 Compound Similarity

The cosine similarity, as a scoring function, proves insufficient since it exclusively considers the most recent merge, thereby lacking the ability to provide prior insights into the quality of past merges or the overall performance of all branches within the partitioning process. Therefore, we define a new similarity measure that takes into account all the past merges and can serve for comparing solutions at the same level. This measure has proven to give very good results, better than the modularity measure. The compound similarity measure is computed according to equation 2:

$$\text{Compound-Similarity}\,(N_L) = \sum_{i=1}^{L} \log_2 \text{Similarity}\,(N_i) \quad (2)$$

where L is the length of the branch leading to the evaluated solution N_L, Similarity(N_i) is the cosine similarity calculated for each tree node N_i leading to N_L. We only consider positive merges, hence Similarity$(N_i) \in]0,1]$.

3.2.4 Similarity Threshold

In order to avoid exploring solutions we don't consider promising, we define a similarity threshold, where we prune any branch that would result in a below-threshold maximum merge similarity value as shown in Figure 4. The value of the threshold should be defined for each graph, next we define how we estimate a good working value for all graphs.

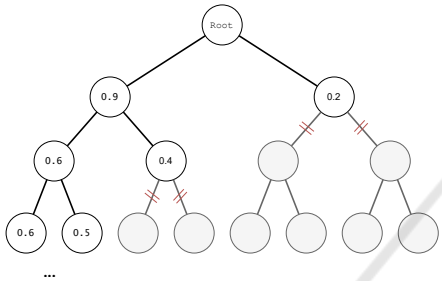


Figure 4: Pruning mechanism example, where solutions having a similarity below $threshold = 0.5$ are pruned.

We estimate the threshold for each graph $G = (V,E)$ as the mean of N random similarity measures sampled from the initial communities initialized as singletons. More formally, we define it as follows:

$$Threshold = \frac{1}{N} \sum_{(V_i,V_j) \in V}^{N} Sim(V_i,V_j) \qquad (3)$$

where N is the number of samples.

4 EXPERIMENTAL RESULTS AND DISCUSSION

To assess the efficiency and performance of our proposed algorithms, we conducted a comprehensive set of experiments on both synthetic and real-world networks using a personal computer running Windows 10 Enterprise, equipped with Intel(R) Xeon(R), 2.30GHz, 16 Cores, and 16 GB RAM.

For the synthetic networks, we used the well-known LFR benchmark (Lancichinetti et al., 2008). We generate networks using a mixing parameter ranging from 0.1 to 0.6 and nodes from 400 to $10k$. For real-world networks, experiments are conducted on six widely used real-world datasets namely Zachary's Karate (Zachary, 1977); Dolphins Football (Girvan and Newman, 2002); Polbooks (Lusseau et al., 2003);

Input: $G = (V,E)$, β
Output: S_{best}
X = Deepwalk(G);
$threshold$: as defined in (3);
$beam\_list$ =
 MaximumIndependentCliques(G);
N_{min} = Elbow(X)[1];
K : number of communities for the current level;
while ***not*** $beam\_list.empty()$ ***and*** $K > N_{min}$ **do**
 foreach *node* ***in*** $beam\_list$ **do**
 Generate child nodes from the current node;
 Evaluate the child nodes using (1) ;
 Prune if Similarity$(N_i) < threshold$;
 end
 Select the top β nodes based on evaluation (2);
 Re-initialize $beam\_list$ with the best β nodes for next iteration;
end
Return S_{best}, best solution for level N_{min} based on (2);

Algorithm 2: BeamWalk for community detection.

(Rossi and Ahmed, 2015); Email ((Yin et al., 2017)); and Actor ((Tang et al., 2009)).

We adopted two widely used criteria to evaluate the accuracy of community detection algorithms: the normalized mutual information (NMI) (Danon et al., 2005) to measure the similarity between the ground truth community structure and the predicted one; and the modularity (Newman and Girvan, 2004) to measure the quality of the uncovered community structure.

4.1 Parameters Settings

The BeamWalk approach consists of two stages: obtaining embeddings using Deepwalk and then applying beam search for community partitioning. The parameters being investigated are those of DeepWalk: the length of walk sequences denoted as t, the number of walks conducted per vertex denoted as γ, the dimensionality of the vector representation space denoted as d, and the window size denoted as w.

In order to assess the impact of these parameters on the performance of the model, we carried out a

[01] Note that once in a latent space, the minimum number of communities could be estimated using different techniques, the one used in our implementation is the Elbow method(Thorndike, 1953).

rigorous process of systematic parameter adjustment. During this process, three parameters were kept constant while the remaining parameter was systematically varied. The resulting vertex vector representations were then documented, and community detection was performed to evaluate the efficacy of the model using the NMI score. This analysis was conducted on the synthetic networks, the outcomes of this investigation are presented in Figure 5.

In Figure 5a, 5c, 5e, and 5g, we kept the walking length t, dimension d, and window size w constant while altering the number of walks γ to observe the corresponding NMI score. By varying the number of walks, we obtained an NMI curve. The results indicate that the learning algorithm achieves optimal community detection performance and stabilizes when the number of walks γ reaches 60. Furthermore, when the window size w is set to 5, the community detection results demonstrate a higher NMI score compared to other window sizes. Therefore, for more accurate community partitioning, we used a window size of 5 and a number of walks of 60.

In graphs 5b,5d,5f and 5h, we conducted an in-depth exploration of the influence of different vector representation dimensions and walk lengths on the NMI score. We observed that for complex networks like LFR4($\mu = 0.4$), the NMI score stabilizes after reaching a vector representation dimension of 128. However, for simpler networks, the optimal dimension size may be lower, indicating that the community detection performance benefits from higher-dimensional vector representations up to a certain limit, which is dependent on the complexity of the network. Similarly, we noticed a similar trend for walk lengths, where longer walk lengths tend to result in higher NMI scores. Furthermore, the mixing parameter for the graphs exerts a noticeable influence on the observed fluctuations. As the mixing parameter increases, we observe a corresponding rise in fluctuations, indicating a greater impact of randomness on the community detection outcomes. Nevertheless, our method maintains stability and delivers satisfactory performance despite these fluctuations. Based on our findings, we used a dimension size of 128 for the vector representation, as it consistently yields favorable results. For achieving more accurate community detection, we suggest using a high window size value of 60 when the mixing parameter value is lower than 0.4. However, if the mixing parameter value exceeds 0.4, we advise utilizing a higher window size value of 80 to mitigate the effects of increased randomness.

4.2 BeamWalk Performance Analysis

Figure 6 depicts the NMI score and modularity values for real datasets. The results show that our hybrid method gives the best NMI scores in comparison to other approaches like SSLPA (Cordasco and Gargano, 2010) and Louvain (Blondel et al., 2008). BeamWalk achieves a maximum NMI score of 1 in the Karate graph and outperforms other methods in other graphs. When it comes to more complex graphs like Actor, our method outperforms the others. The modularity score isn't what our method is trying to maximize thus we could expect other modularity-based algorithms like Louvain to score better. In general, our method gives modularity values relative to the quality of the solutions found.

During the analysis of the resulting partitioning quality for varying beam widths in Figure 7, a consistent improvement in quality is observed as the beam width increases. However, it is worth noting that this improvement tends to plateau after reaching a beam width of $\beta = 10$. It is important to acknowledge that in certain cases, the quality may even decrease. This behavior can be attributed to the imperfect positive relationship between optimizing the NMI score and the defined compound similarity metric used in our approach.

The increased effectiveness achieved through the hybridization of the initial beam search can be observed in Figure 8, wherein the extent of improvement is positively correlated with both the size and complexity of the network. Our experimental investigation, carried out on the LFR dataset ($\mu = 0.6$) in Figure 9, demonstrates the time scalability of our approach as the network size increases. Notably, the time complexity associated with the hybridization is relatively low due to the avoidance of intricate modularity calculations and the utilization of quality pruning. As a result, our method is particularly suitable for small to medium-sized networks.

Upon comparison with the vanilla beam search and other methods, Figure 10 illustrates the robustness of the BeamWalk method, particularly complex networks. Significantly improved results are achieved when the value of the μ parameter surpasses 0.5. As demonstrated in Figure 11, our method exhibits robustness when faced with an increase in the number of nodes. Particularly, it yields highly satisfactory results for small to medium-sized networks.

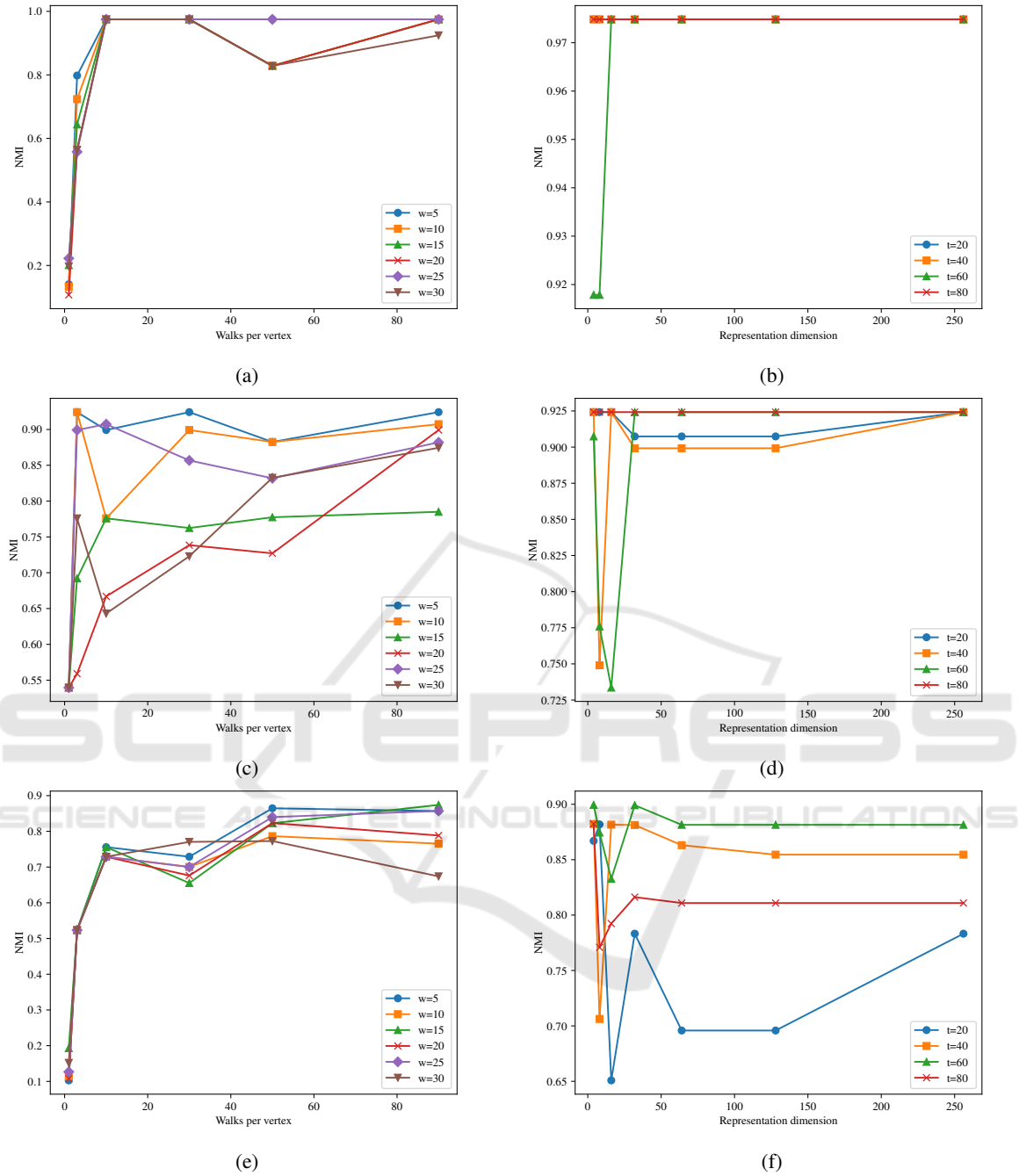
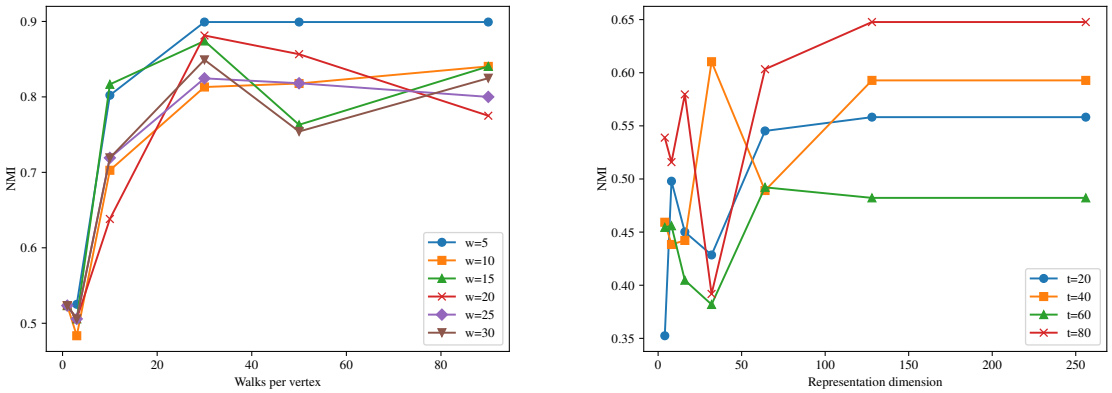
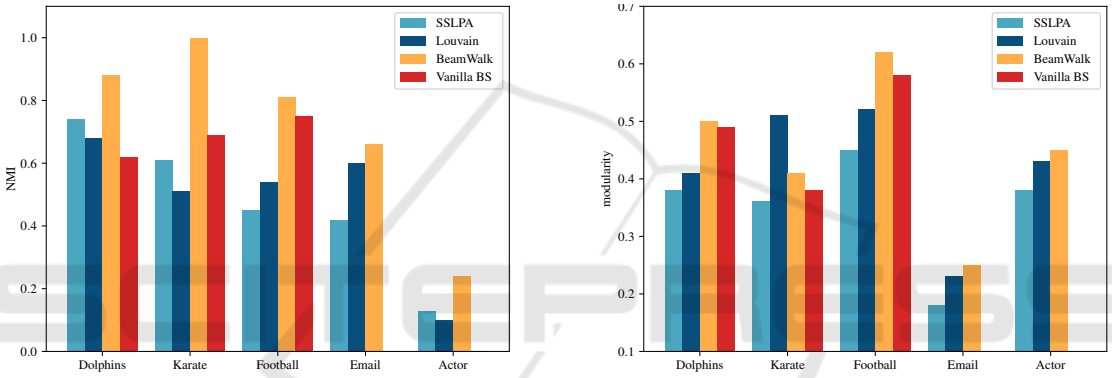


Figure 5: NMI for different LFR benchmark networks. (a) and (b) correspond to LFR1 ($\mu = 0.1$), (c) and (d) LFR2 ($\mu = 0.2$); (e) and (f) LFR3 ($\mu = 0.3$); (g) and (h) LFR4 ($\mu = 0.4$).



(g) (h)

Figure 5: NMI for different LFR benchmark networks. (a) and (b) correspond to LFR1 ($\mu = 0.1$), (c) and (d) LFR2 ($\mu = 0.2$); (e) and (f) LFR3 ($\mu = 0.3$); (g) and (h) LFR4 ($\mu = 0.4$) (cont.).



(a) (b)

Figure 6: NMI and modularity scores.

5 CONCLUSION

In this study, we introduced a novel hybrid approach for community detection in social networks, which combines a deep learning component: DeepWalk and Beam search. To the best of our knowledge, this is the first method that proposes to address this problem using a tree search approach. Our proposed method has exhibited several desirable characteristics:

- **Effectiveness.** The NMI scores achieved by our method are comparable to, and in some cases surpass, those of other existing approaches. This demonstrates the effectiveness of our hybrid approach in accurately detecting community structures.
- **Stability.** Our method has demonstrated stability and robustness when faced with increasing network complexity. This suggests that our approach can reliably identify communities even in highly complex networks.
- **Relatively Fast.** Despite being a TS algorithm, our method exhibits reasonable computational time. By leveraging a good initialization, a hybrid approach, and an effective pruning strategy, we have achieved reasonable timing performance.

While our study has presented promising results, there are several unresolved issues that warrant further investigation. One such issue is the adaptation of our method to social networks with additional node information. Additionally, accelerating these algorithms remains a challenge. Exploring efficient initialization algorithms that provide a promising starting point for the tree search, where valuable neighborhoods can still be explored, could be a potential avenue for improvement. Furthermore, designing a dynamic pruning strategy that adapts the threshold during the search based on solution acceptance and local similarity could enhance the overall performance of the method.

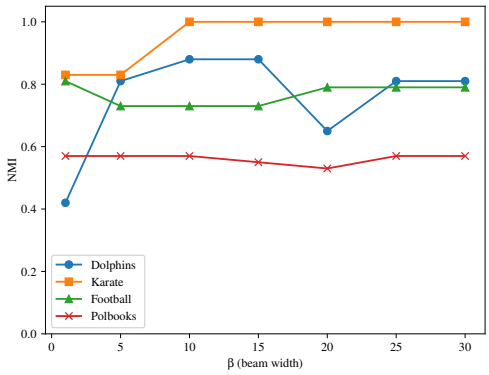


Figure 7: Quality comparison on real-world datasets when augmenting the beam width.

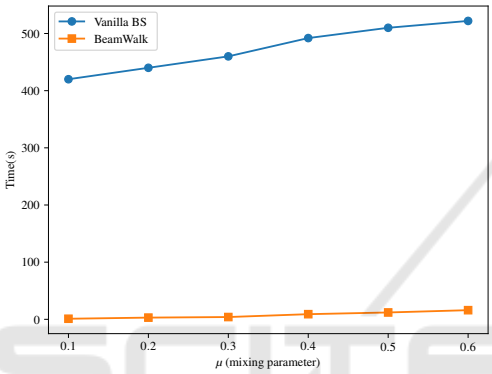


Figure 8: Time scaling for large-sized networks.

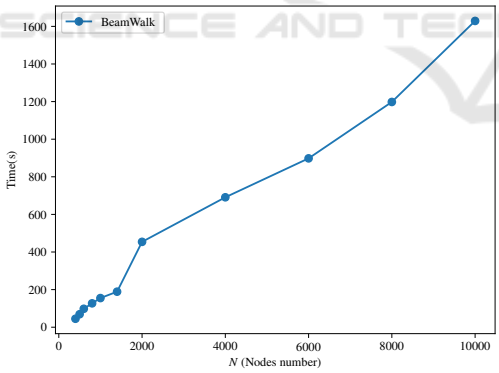


Figure 9: Time scaling when augmenting the number of nodes, the test was conducted on LFR Networks of $\mu = 0.6$.

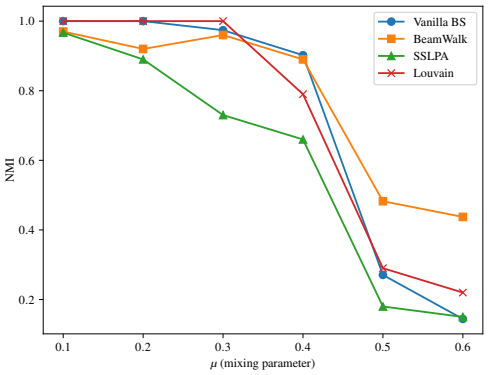


Figure 10: Quality comparison between other methods and ours when augmenting the complexity of the network, the test was conducted on LFR Networks of $N = 128$.

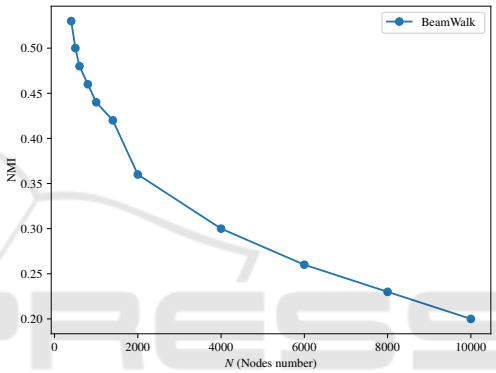


Figure 11: Quality scaling when augmenting the number of nodes, the test was conducted on LFR Networks of $\mu = 0.6$.

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