Fixed-Time Tracking Control for a Class of Nonlinear Systems via Command Filtered Backstepping

Wen-Nian Qi[1][a] and Rui-Qi Dong[2,*]

[1]School of Mechanical Engineering and Automation, Harbin Institute of Technology (Shenzhen), Shenzhen, P.R. China

[2]School of Mechanical Engineering, Tsinghua University, Beijing, P.R. China

Keywords: Nonlinear Systems, Fixed-Time Stability, Command Filtered Backstepping.

Abstract: The fixed-time tracking control problem is addressed for a class of nonlinear systems. A novel command filtered backstepping control law including virtual control signals, fixed-time command filters, and error compensation signals is constructed. By the introduced fixed-time command filters, the problem of "explosion of complexity" caused by backstepping approach is avoided. Simultaneously, the filtering errors produced by the introduced fixed-time command filters are eliminated by the designed error compensation signals. It is proven that the resulted closed-loop tracking system under the proposed command filtered backstepping control law is fixed-time stable.

1 INTRODUCTION

The control of nonlinear systems has been widely investigated, and many effective control methods, including backstepping control (Kanellakopoulos et al., 1991; Morawiec et al., 2020; Mazenc and Bliman, 2006), adaptive control (Tang et al., 2003), neural network control (Wang and Huang, 2005), etc, have been proposed. Among these control methods, the backstepping technique is widely utilized due to its superiority in dealing with mismatched uncertainties and disturbances (Kanellakopoulos et al., 1991). In recent works (Feng et al., 2020; Zhao et al., 2021; Tong et al., 2020), the backstepping approach was further incorporated with finite-time control, fault-tolerant control, and adaptive control. It should be noted that the backstepping control needs to construct virtual control laws step by step, and the derivative of virtual control signal in the last step is required to construct the virtual control law in the current step. Accordingly, repeatedly differentiating the virtual control signals causes the problem of "explosion of complexity" (Chen and Wang, 2021; Swaroop et al., 2000), and the complexity becomes severe especially for high-order dynamics. To address this problem, a dynamic surface control (DSC) approach was firstly proposed in (Swaroop et al., 2000) to avoid differentiating virtual control signals by introducing first-order filters in the backstepping design procedure. A drawback of DSC is that the filtering errors caused by the introduced first-order filters are ignored. To compensate the filtering errors, a command filtered backstepping was proposed in (Farrell et al., 2009), where the command filters were used to approximate the derivatives of virtual control signals, and the compensation mechanism were proposed to reduce the influence of filtering errors. Further, the command filtered backstepping was united with adaptive technique or neural network approximation method in (Dong et al., 2011; Shen and Shi, 2015), respectively, to eliminate the influence of uncertain nonlinearities.

The aforementioned command filtered backstepping control algorithms in (Farrell et al., 2009; Dong et al., 2011; Shen and Shi, 2015) merely achieve infinite-time convergence property of the closed-loop system. From the practical point of view, many engineering systems are often required to achieve fast tracking control in finite time. In view of this requirement, finite-time control methods combining with command filtered backstepping technique have been developed for many nonlinear systems. Typical works can be found in (Yu et al., 2018; Li, 2019; Fu et al., 2020; Wang et al., 2019; Cheng et al., 2023; Wang et al., 2021). In (Yu et al., 2018), the finite-time command filtered backstepping tracking controller was constructed for a class of nonlinear systems in strict-feedback form. In (Li, 2019), the parametric uncer-

[a] https://orcid.org/0000-0002-4627-1151

*Corresponding author.

Qi, W. and Dong, R.
Fixed-Time Tracking Control for a Class of Nonlinear Systems via Command Filtered Backstepping.
DOI: 10.5220/0012210200003543
In *Proceedings of the 20th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2023) - Volume 2*, pages 50-57
ISBN: 978-989-758-670-5; ISSN: 2184-2809

tainties and actuator faults were taken into consideration, and the finite-time tracking control law was developed by adaptive command filtered backstepping approach. In (Fu et al., 2020), the tracking control for uncertain switched nonlinear systems was considered, and a neural-network based command filtered backstepping controller was developed to achieve finite-time convergence of the closed-loop system. It should be pointed out that the initial conditions of the system limit the settling time ensured by finite-time control approaches (Guo et al., 2021). To solve the problem of initial condition dependence, the fixed-time control approach, which can guarantee that settling time is only depended on the parameters of the closed-loop system, have been developed for various control systems. In (Tian et al., 2017; Su and Zheng, 2019), the fixed-time control laws were constructed to stabilize second-order systems. In (Tian et al., 2018), the backstepping control approach was applied for high-order integrator systems, and the fixed-time tracking control was achieved. To our knowledge, the fixed-time command filtered backstepping tracking control is barely investigated for high-order nonlinear systems.

Motivated by the aforementioned discussions, the tracking control for a class of nonlinear systems is investigated in this paper. The control objective of this paper is to construct a control law such that the output of the nonlinear system can track a reference signal in fixed time. To achieve this control goal, a new fixed-time command filtered backstepping control method is proposed. Compared with most existing works, the main contributions are presented as follows.

(1) The proposed control law is constructed by combining fixed-time control approach and command filtered backstepping technique. Compared with the contribution in (Yu et al., 2018), in which the tracking error converges to a region around the origin in finite time, the proposed control law in this work makes sure that the tracking error converges to the origin in fixed time.

(2) The fixed-time filter is introduced to replace the finite-time filter in (Yu et al., 2018; Li, 2019; Fu et al., 2020; Wang et al., 2019; Cheng et al., 2023; Wang et al., 2021). Therefore, the output of the introduced fixed-time filter can estimate the derivative of the virtual control law and the problem of "explosion of complexity" is obviated.

(3) Compared with (Yu et al., 2018; Li, 2019; Fu et al., 2020; Wang et al., 2019; Cheng et al., 2023; Wang et al., 2021), novel compensation mechanism is constructed to timely reduce the negative effect of the filtering error.

The rest of this paper is organized as follows. The system description and preliminaries are provided in Section 2. The control law design and stability analysis are presented in Section 3. An example is conducted in Section 4 to verify the effectiveness and advantages of the proposed control law. Finally, the conclusion of this paper is drawn in Section 5.

Notations. Throughout this paper, $\mathbb{R}$ denotes the real number, $\mathbb{R}^n$ denotes the n dimensional real vector, and $|\cdot|$ denotes the absolute value. For two integers $a \le b$, $\mathbb{I}[a,b]$ denotes the set $\{a,\ a+1,\ ...,\ b\}$. For any scalar $x \in \mathbb{R}$, define $\mathrm{sig}^{\gamma}(x) = \mathrm{sign}(x)\,|x|^{\gamma}$ where $\mathrm{sign}(\cdot)$ is the standard sign function. For any vector $x = [x_1\ x_2\ \cdots\ x_n]^{\mathrm{T}} \in \mathbb{R}^n$, $\|x\|$ is the 2-norm of the vector x. Furthermore, define

$$\mathrm{sign}\,(x) = \ [\mathrm{sign}(x_1)\ \mathrm{sign}(x_2)\ \cdots\ \mathrm{sign}(x_n)]^{\mathrm{T}},$$

and

$$\mathrm{sig}^{\gamma}(x) = [\mathrm{sig}^{\gamma}(x_1)\ \mathrm{sig}^{\gamma}(x_2)\ \cdots\ \mathrm{sig}^{\gamma}(x_n)]^{\mathrm{T}}.$$

2 SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following n-th order nonlinear systems

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)\,x_{i+1} \\ \dot{x}_n &= f_n(x) + g_n(x)\,u \\ y &= x_1 \end{aligned} \tag{1}$$

where $i \in \mathbb{I}[1,n-1]$, $x = [x_1\ x_2\ \cdots\ x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector, $\bar{x}_i = [x_1\ x_2\ \cdots\ x_i]^{\mathrm{T}} \in \mathbb{R}^n$; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the input and output of the system, respectively. The functions $f_i(\cdot)$ and $g_i(\cdot)$, $i \in \mathbb{I}[1,n]$, are assumed to be known. Denote the reference signal and its first-order derivatives by $y_0 \in \mathbb{R}$, $\dot{y}_0 \in \mathbb{R}$, respectively. Both y_0 and $\dot{y}_0$ are assumed to be bounded and known.

The control objective of this paper is to construct the control law u for system (1) such that the output y tracks the reference signal y_0 in a fixed time. The following assumption on the n-th order nonlinear systems (1) is presented.

Assumption 1. *For system* (1), *there exists an open set $\Omega_0 \subset R^n$ which includes the origin and the initial condition $x(0)$. (1) $f_i^{(m)}(\cdot)$ and $g_i^{(m)}(\cdot)$ are bounded in the closed set $\bar{\Omega}_0$ for $i \in \mathbb{I}[1,n-1]$, $m \in \mathbb{I}[1,n-i]$; (2) $f_n(\cdot)$ and $g_n(\cdot)$ and their first-order derivatives are bounded in the closed set $\bar{\Omega}_0$.*

In what follows, some useful definitions and lemmas are introduced.

Lemma 1. *(Hardy et al., 1952) For $x_i > 0$, $i \in \mathbb{I}[1,N]$*

there holds

$$\sum_{i=1}^{n} x_i^{\gamma} \geq \left(\sum_{i=1}^{n} x_i\right)^{\gamma}, \text{ if } 0 < \gamma \leq 1,$$

$$\sum_{i=1}^{n} x_i^{\gamma} \geq n^{1-\gamma} \left(\sum_{i=1}^{n} x_i\right)^{\gamma}, \text{ if } \gamma > 1.$$

Considered an autonomous system

$$\dot{x} = f(x, u), x(0), x \in U \subset \mathbb{R}^n, \tag{2}$$

where $f : U \times R^{+} \to \mathbb{R}^n$ is continuous on an open neighborhood U of the origin $x = 0$. Suppose for any initial condition $x(0) \in U$, there is a unique solution $x(t, x(0))$ of system (2).

Lemma 2. *(Polyakov, 2012) For the system* (2)*, if there is a Lyapunov function $V(x)$ with initial value $V(x(0))$, and some real numbers $a > 0$, $b > 0$, $0 < p < 1$, and $q > 1$, such that $\dot{V}(x) \leq -aV^p - bV^q$, then the origin of the system* (2) *is fixed-time stable, and the convergence time is bounded by*

$$T \leq \frac{1}{a(1-p)} + \frac{1}{b(q-1)}.$$

3 CONTROL LAW DESIGN AND STABILITY ANALYSIS

In this section, a command filtered backstepping control law is constructed for the system (1) to ensure that the output y tracks the reference signal y_0 in a fixed time, and the fixed time stability of the closed-loop system are analysed theoretically.

3.1 Control Law Design

Following the command filtered backstepping approach, the coordinate transformation is introduced as

$$z_1 = y - y_0, \tag{3}$$

$$z_i = x_i - \bar{\alpha}_i, \tag{4}$$

where $i \in \mathbb{I}[2, n]$, and $\bar{\alpha}_i$ is the output of a fixed-time command filter with the virtual control law α_i as the input. The fixed-time command filter is introduced as follows.

$$\begin{cases} \dot{\bar{\alpha}}_i = -\lambda_{1i}\mathrm{sig}^{\frac{1}{2}}(\bar{\alpha}_i - \alpha_i) - \lambda_{2i}\mathrm{sig}^{\gamma}(\bar{\alpha}_i - \alpha_i) + \phi_i, \\ \dot{\phi}_i = -\lambda_{3i}\mathrm{sign}(\phi_i - \dot{\alpha}_i) - \lambda_{4i}\mathrm{sig}^{2\gamma-1}(\bar{\alpha}_i - \alpha_i). \end{cases} \tag{5}$$

where $i \in \mathbb{I}[2, n]$, $\gamma > 1$, and λ_{1i}, λ_{2i}, λ_{3i}, and λ_{4i} are positive parameters. The fixed time convergence property of the signals $\bar{\alpha}_i$ and φ_i are shown in the following Lemma.

Lemma 3. *(Basin et al., 2017) For the system* (5) *with α_i as the input signal, and positive parameters λ_{1i}, λ_{2i}, λ_{3i}, λ_{4i}, and $\gamma > 1$, the outputs $\bar{\alpha}_i$ and φ_i converge to α_i and $\dot{\alpha}_i$ in a fixed time, respectively.*

In what follows, the details of the fixed-time command filtered backstepping control law design are presented in n steps.

Step 1: design of virtual control law α_2. From (1) and (3), the time derivative of z_1 is

$$\begin{aligned} \dot{z}_1 &= f_1 + g_1 x_2 - \dot{y}_0 \\ &= f_1 + g_1(x_2 - \bar{\alpha}_2 + \bar{\alpha}_2 - \alpha_2 + \alpha_2) - \dot{y}_0 \\ &= f_1 + g_1(z_2 + \bar{\alpha}_2 - \alpha_2 + \alpha_2) - \dot{y}_0. \end{aligned} \tag{6}$$

When we construct virtual control law α_2, a compensation mechanism is needed to reduce the influence of the filtering error $\bar{\alpha}_2 - \alpha_2$. The compensation signal ξ_1 is proposed as

$$\begin{aligned} \dot{\xi}_1 = &- l_{11}\mathrm{sig}^p(\xi_1) - l_{21}\mathrm{sig}^q(\xi_1) \\ &+ g_1(\bar{\alpha}_2 - \alpha_2) + g_1\xi_2, \end{aligned} \tag{7}$$

where $l_{11} > 0$, $l_{21} > 0$, $0 < p < 1$, and $q > 1$ are some designed parameters. ξ_2 is another compensation signal to reduce the influence of the error $\bar{\alpha}_3 - \alpha_3$, and will be designed in the next step.

Define the compensated tracking errors as

$$\chi_1 = z_1 - \xi_1, \tag{8}$$

$$\chi_2 = z_2 - \xi_2. \tag{9}$$

By using (6) and (7), the time derivative of χ_1 is

$$\begin{aligned} \dot{\chi}_1 =& \dot{z}_1 - \dot{\xi}_1 \\ =& g_1(\chi_2 + \alpha_2) + f_1 - \dot{y}_0 \\ &+ l_{11}\mathrm{sig}^p(\xi_1) + l_{21}\mathrm{sig}^q(\xi_1). \end{aligned} \tag{10}$$

For the first subsystem composed of (7) and (10), a Lyapunov candidate function can be chosen as

$$V_1 = \frac{1}{2}\chi_1^2 + \frac{1}{2}\xi_1^2. \tag{11}$$

Taking the time derivative of V_1 yields

$$\begin{aligned} \dot{V}_1 =& \chi_1\dot{\chi}_1 + \xi_1\dot{\xi}_1 \\ =& \chi_1(g_1\chi_2 + g_1\alpha_2 + f_1 - \dot{y}_0 + l_{11}\mathrm{sig}^p(\xi_1) \\ &+ l_{21}\mathrm{sig}^q(\xi_1)) + \xi_1(-l_{11}\mathrm{sig}^p(\xi_1) \\ &- l_{21}\mathrm{sig}^q(\xi_1) + g_1(\bar{\alpha}_2 - \alpha_2) + g_1\xi_2). \end{aligned} \tag{12}$$

Then for the first subsystem composed of (7) and (10), a virtual control law α_2 can be constructed as

$$\alpha_2 = \frac{1}{g_1}(-k_{11}\mathrm{sig}^p(\chi_1) - k_{21}\mathrm{sig}^q(\chi_1) + \dot{y}_0 - f_1), \tag{13}$$

where $k_{11} > 0$ and $k_{21} > 0$ are some designed parameters.

Substituting (13) into (12) yields

$$\begin{aligned}\dot{V}_1 =& -k_{11}|\chi_1|^{1+p} - k_{21}|\chi_1|^{1+q} + g_1\chi_1\chi_2 \\ &+ l_{11}\chi_1 \mathrm{sig}^p(\xi_1) + l_{21}\chi_1 \mathrm{sig}^q(\xi_1) - l_{11}|\xi_1|^{1+p} \\ &- l_{21}|\xi_1|^{1+q} + g_1(\bar{\alpha}_2 - \alpha_2)\xi_1 + g_1\xi_1\xi_2. \quad (14)\end{aligned}$$

Step i, $i \in \mathbb{I}[2, n-1]$: design of virtual control law α_{i+1}. From (1) and (4), the time derivative of z_i is

$$\begin{aligned}\dot{z}_i =& f_i + g_i x_{i+1} - \dot{\bar{\alpha}}_i \\ =& f_i + g_i(z_{i+1} + \bar{\alpha}_{i+1} - \alpha_{i+1} + \alpha_{i+1}) - \dot{\bar{\alpha}}_i. \quad (15)\end{aligned}$$

Proceeding similarly, to reduce the influence of the filtering error $\bar{\alpha}_{i+1} - \alpha_{i+1}$, the compensation signal ξ_i is designed as

$$\begin{aligned}\dot{\xi}_i =& -l_{1i}\mathrm{sig}^p(\xi_i) - l_{2i}\mathrm{sig}^q(\xi_i) + g_i(\bar{\alpha}_{i+1} - \alpha_{i+1}) \\ &+ g_i\xi_{i+1} - g_{i-1}\xi_{i-1}, \quad (16)\end{aligned}$$

where $l_{1i} > 0$ and $l_{2i} > 0$ are some designed parameters, and ξ_{i+1} is another compensation signal to reduce the influence of the error $\bar{\alpha}_{i+2} - \alpha_{i+2}$ in the next step.

Define the compensated tracking errors as

$$\chi_i = z_i - \xi_i, \quad (17)$$

$$\chi_{i+1} = z_{i+1} - \xi_{i+1}, \quad (18)$$

By using (15) and (16), the time derivative of χ_i is

$$\begin{aligned}\dot{\chi}_i =& g_i\alpha_{i+1} + f_i - \dot{\bar{\alpha}}_i + l_{1i}\mathrm{sig}^p(\xi_i) + l_{2i}\mathrm{sig}^q(\xi_i) \\ &+ g_i\chi_{i+1} + g_{i-1}\xi_{i-1}. \quad (19)\end{aligned}$$

For the i-th subsystem composed of (16) and (19), a Lyapunov candidate function can be chosen as

$$V_i = \frac{1}{2}\chi_i^2 + \frac{1}{2}\xi_i^2. \quad (20)$$

From (16) and (19), the time derivative of V_i is obtained as

$$\begin{aligned}\dot{V}_i =& \chi_i\left(g_i\alpha_{i+1} + f_i - \dot{\bar{\alpha}}_i + l_{1i}\mathrm{sig}^p(\xi_i) + l_{2i}\mathrm{sig}^q(\xi_i)\right. \\ &\left.+ g_i\chi_{i+1} + g_{i-1}\xi_{i-1}\right) + \xi_i\left(-l_{1i}\mathrm{sig}^p(\xi_i) - l_{2i}\mathrm{sig}^q(\xi_i)\right. \\ &\left.+ g_i(\bar{\alpha}_{i+1} - \alpha_{i+1}) + g_i\xi_{i+1} - g_{i-1}\xi_{i-1}\right). \quad (21)\end{aligned}$$

Then the virtual control law α_{i+1} can be constructed as

$$\begin{aligned}\alpha_{i+1} =& \frac{1}{g_i}\left(-k_{1i}\mathrm{sig}^p(\chi_i) - k_{2i}\mathrm{sig}^q(\chi_i)\right. \\ &\left.+ \dot{\bar{\alpha}}_i - f_i - g_{i-1}z_{i-1}\right), \quad (22)\end{aligned}$$

where $k_{1i} > 0$ and $k_{2i} > 0$ are some designed parameters.

Substituting (22) into (21) yields

$$\begin{aligned}\dot{V}_i =& -k_{1i}|\chi_i|^{1+p} - k_{2i}|\chi_i|^{1+q} - l_{1i}|\xi_i|^{1+p} - l_{2i}|\xi_i|^{1+q} \\ &+ l_{1i}\chi_i\mathrm{sig}^p(\xi_i) + l_{2i}\chi_i\mathrm{sig}^q(\xi_i) + g_i\chi_i\chi_{i+1} \\ &- g_{i-1}\chi_{i-1}\chi_i + g_i(\bar{\alpha}_{i+1} - \alpha_{i+1})\xi_i \\ &+ g_i\xi_i\xi_{i+1} - g_{i-1}\xi_{i-1}\xi_i. \quad (23)\end{aligned}$$

Step n: design of actual control law u. The time derivative of z_n is

$$\dot{z}_n = f_n + g_n u - \dot{\bar{\alpha}}_n. \quad (24)$$

The compensation signal ξ_n is designed as

$$\dot{\xi}_n = -l_{1n}\mathrm{sig}^p(\xi_n) - l_{2n}\mathrm{sig}^q(\xi_n) - g_{n-1}\xi_{n-1}. \quad (25)$$

where $l_{1n} > 0$ and $l_{2n} > 0$ are some designed parameters. The compensated tracking error χ_n is defined as

$$\chi_n = z_n - \xi_n. \quad (26)$$

By using (24) and (25), the time derivative of χ_n is

$$\begin{aligned}\dot{\chi}_n =& g_n u + f_n - \dot{\bar{\alpha}}_n + l_{1n}\mathrm{sig}^p(\xi_n) + l_{2n}\mathrm{sig}^q(\xi_n) \\ &+ g_{n-1}\xi_{n-1}. \quad (27)\end{aligned}$$

For the n-th subsystem composed of (25) and (27), a Lyapunov candidate function can be chosen as

$$V_n = \frac{1}{2}\chi_n^2 + \frac{1}{2}\xi_n^2. \quad (28)$$

Taking the time derivative of V_n yields

$$\begin{aligned}\dot{V}_n =& \chi_n\left(g_n u + f_n - \dot{\bar{\alpha}}_n + l_{1n}\mathrm{sig}^p(\xi_n) + l_{2n}\mathrm{sig}^q(\xi_n)\right. \\ &\left.+ g_{n-1}\xi_{n-1}\right) + \xi_n\left(-l_{1n}\mathrm{sig}^p(\xi_n) - l_{2n}\mathrm{sig}^q(\xi_n)\right. \\ &\left.- g_{n-1}\xi_{n-1}\right). \quad (29)\end{aligned}$$

Then the virtual control law α_{i+1} can be constructed as

$$\begin{aligned}u =& \frac{1}{g_n}\left(-k_{1n}\mathrm{sig}^p(\chi_n) - k_{2n}\mathrm{sig}^q(\chi_n) + \dot{\bar{\alpha}}_n\right. \\ &\left.- f_n - g_{n-1}z_{n-1}\right). \quad (30)\end{aligned}$$

where $k_{1n} > 0$ and $k_{2n} > 0$ are some designed parameters.

Substituting the control law u into (29), yields

$$\begin{aligned}\dot{V}_n =& -k_{1n}|\chi_n|^{1+p} - k_{2n}|\chi_n|^{1+q} - l_{1n}|\xi_n|^{1+p} - l_{2n}|\xi_n|^{1+q} \\ &+ l_{1n}\mathrm{sig}^p(\xi_n) + l_{2n}\mathrm{sig}^q(\xi_n) - g_{n-1}\chi_{n-1}\chi_n \\ &- g_{n-1}\xi_{n-1}\xi_n. \quad (31)\end{aligned}$$

In this subsection, the virtual control laws (13) and (22) are constructed step by step, and the fixed-time command filter (5) are provided to avoid repeatedly calculating the time derivative of the virtual control laws. Besides, the compensation mechanism provided by (7), (16) and (25) are provided to compensate the filtering errors caused by the fixed-time command filter (5). Finally, the actual control law u are constructed as in (30).

3.2 Stability Analysis

The fixed-time stability of the closed-loop system can be concluded in the following theorem.

Theorem 1. *For the system* (1) *satisfies Assumption 1, if the fixed-time command filter is chosen as in* (5)*, the virtual control laws are constructed as in* (13) *and* (22)*, and the compensation mechanism are design as in* (7)*,* (16) *and* (25)*, then the control law can be designed as in* (30) *such that the tracking error* z_1 *in* (3) *converges to origin in a fixed-time.*

Proof. For the closed-loop system, a Lyapunov function candidate can be selected as

$$V=\sum_{i=1}^{n}V_i,$$

Then by using (14), (23), and (31), the time derivative of V is

$$\begin{aligned}\dot V=&-\sum_{i=1}^{n}\left(k_{1i}|\chi_i|^{1+p}+k_{2i}|\chi_i|^{1+q}\right)+\sum_{i=1}^{n}(l_{1i}\chi_i\text{sig}^p(\xi_i)\\&+l_{2i}\chi_i\text{sig}^q(\xi_i))-\sum_{i=1}^{n}\left(l_{1i}|\xi_i|^{1+p}+l_{2i}|\xi_i|^{1+q}\right)\\&+\sum_{i=1}^{n-1}g_i(\bar\alpha_{i+1}-\alpha_{i+1})\xi_i.\end{aligned}\tag{32}$$

According to Young's inequality(Deng and Krstić, 1997), there holds

$$\begin{aligned}l_{1i}\chi_i\text{sig}^p(\xi_i)\le&\frac{l_{1i}}{1+p}|\chi_i|^{1+p}+\frac{pl_{1i}}{1+p}|\xi_i|^{1+p},\\l_{2i}\chi_2\text{sig}^q(\xi_i)\le&\frac{l_{2i}}{1+q}|\chi_i|^{1+q}+\frac{ql_{2i}}{1+q}|\xi_i|^{1+q}.\end{aligned}\tag{33}$$

Substituting (33) into (32) and properly choosing parameters $k_{1i}>\frac{l_{1i}}{1+p}$, $k_{2i}>\frac{l_{2i}}{1+q}$, $l_{1i}>\frac{pl_{2i}}{1+p}$, and $l_{2i}>\frac{ql_{2i}}{1+q}$, yield

$$\begin{aligned}\dot V\le&-\sum_{i=1}^{n}\left(\bar k_{1i}|\chi_i|^{1+p}+\bar k_{2i}|\chi_i|^{1+q}\right)-\sum_{i=1}^{n}\left(\bar l_{1i}|\xi_i|^{1+p}\right.\\&\left.+\bar l_{2i}|\xi_i|^{1+q}\right)+\sum_{i=1}^{n-1}g_i(\bar\alpha_{i+1}-\alpha_{i+1})\xi_i\end{aligned}\tag{34}$$

where $\bar k_{1i}=k_{1i}-\frac{l_{1i}}{1+p}$, $\bar k_{2i}=k_{2i}-\frac{l_{2i}}{1+q}$, $\bar l_{1i}=l_{1i}-\frac{pl_{2i}}{1+p}$, and $\bar l_{2i}=l_{2i}-\frac{ql_{2i}}{1+q}$. Besides, according to Lemma 3, the filtering error $\bar\alpha_{i+1}-\alpha_{i+1}=0$ can be achieved in a fixed time T_{1i} by properly choosing the parameters λ_{1i}, λ_{2i}, λ_{3i}, λ_{4i}, and γ. Then for $t\ge\max\{T_{1i}\}$, there holds

$$\begin{aligned}\dot V\le&-\sum_{i=1}^{n}\left(\bar k_{1i}|\chi_i|^{1+p}+\bar l_{1i}|\xi_i|^{1+p}\right)\\&-\sum_{i=1}^{n}\left(\bar k_{2i}|\chi_i|^{1+q}+\bar l_{2i}|\xi_i|^{1+q}\right).\end{aligned}\tag{35}$$

Denoting $\varpi_1=\min_i\{\bar k_{1i},\bar l_{1i}\}$, $\varpi_2=\min_i\{\bar k_{2i},\bar l_{2i}\}$, and applying Lemma 1, there holds

$$\begin{aligned}\dot V\le&-\varpi_1\left(\sum_{i=1}^{n}(\chi_i^2+\xi_i^2)\right)^{\frac{1+p}{2}}\\&-\varpi_2 n^{\frac{1-q}{2}}\left(\sum_{i=1}^{n}(\chi_i^2+\xi_i^2)\right)^{\frac{1+q}{2}}\\=&-\bar\varpi_1V^{\frac{1+p}{2}}-\bar\varpi_2V^{\frac{1+q}{2}}.\end{aligned}\tag{36}$$

where $\bar\varpi_1=2^{\frac{1+p}{2}}\varpi_1$, $\bar\varpi_2=2^{\frac{1+q}{2}}n^{\frac{1-q}{2}}\varpi_2$. According to Lemma 2, inequality (36) implies that $\chi_i=0$ and $\xi_i=0$, $i\in\mathbb{I}[1,N]$, are achieved in a fixed time T_2, which is bounded by

$$T_2\le\max_i\{T_{1i}\}+\frac{1}{\bar\varpi_1(1-p)}+\frac{1}{\bar\varpi_2(q-1)},$$

Since $z_1=\chi_1+\xi_1$, then it can be obtained that the tracking error $z_1=0$ is achieved in a fixed time T_2. Thus the proof is completed. □

In this section, the fixed-time tracking control law (30) is constructed based on the virtual control laws, fixed-time filters, and compensation mechanism. The problem of "explosion of complexity" is avoided by the introduced fixed-time filter (5). The influence of the filtering errors is reduced timely by the proposed compensation mechanism as in (7) and (25). Moreover, it is proved in Theorem 1 that the fixed-time tracking performance of the closed-loop system is ensured by the proposed control law.

4 SIMULATION RESULTS

In this section, the proposed fixed-time control law will be compared with the finite-time control law in (Yu et al., 2018) via an electromechanical system, composed of a single-link manipulator and motor. The dynamics of electromechanical system is described as follows.

$$\begin{cases}D\ddot q+B\dot q+N\sin(q)=\tau\\M\dot\tau+H\tau=D(u)-K_m\dot q\\y=q\end{cases}\tag{37}$$

where q, $\dot q$, and $\ddot q$ are the position, velocity, and acceleration of the link, respectively, τ is the motor shaft angle, and u represent the motor torque. The parameters are chosen as $D=1$, $B=1$, $M=0.05$, $H=0.5$, $N=10$, and $K_m=10$.

Define $x_1 = q$, $x_2 = \dot{q}$, and $x_3 = \tau/D$, the dynamics (37) can be written as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_3 - \dfrac{N}{D}\sin(x_1) + \dfrac{B}{D}x_2 \\ \dot{x}_3 = \dfrac{K_m}{MD}x_2 + \dfrac{H}{M}x_3 + u \end{cases} \tag{38}$$

The reference signal is

$$y_0 = 0.5\sin(t) + 0.5\sin(0.5t).$$

The designed parameters of virtual control law, actual control law, and compensation signals are taken as $k_{1i} = k_{2i} = 5$, $p = 0.6$, $q = 1.05$, and $l_{1i} = l_{2i} = 2$, $i \in \mathbb{I}[1,3]$.

The initial condition of the system states, command filters, and compensation signals are taken as

$$[x_1(0)\ x_2(0)\ x_3(0)] = [4\ 3\ 2],$$

$$[\bar{\alpha}_i(0)\ \varphi_i(0)] = [0\ 0], i = 2,3,$$

$$[\xi_1(0)\ \xi_2(0)\ \xi_3(0)] = [0\ 0\ 0].$$

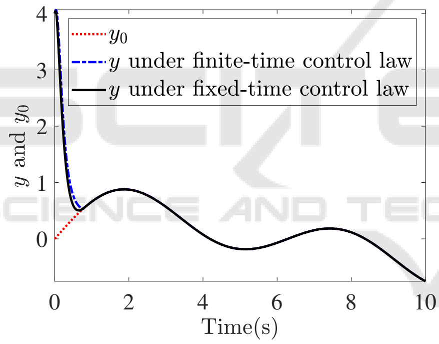


Figure 1: Trajectories of y and y_0.

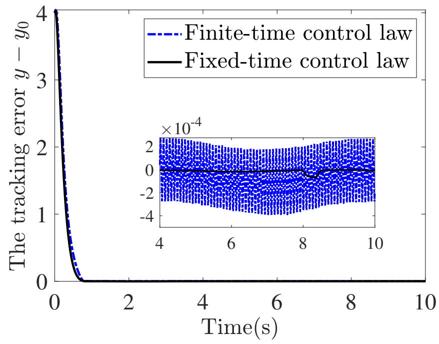
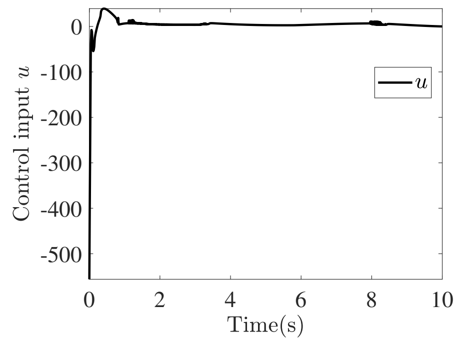
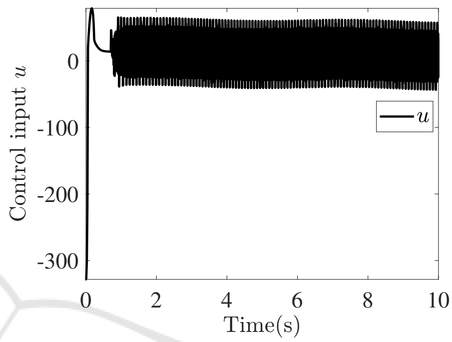


Figure 2: Tracking error $y - y_0$.

The simulation results are shown in Figures 1-3. In Figure 1, the tracking performance under the proposed fixed-time command filtered backstepping control law and finite-time command filtered backstepping control law in (Yu et al., 2018) are presented.



(a) u under the fixed-time control law in this paper



(b) u under the finite-time control law in (Yu et al., 2018)

Figure 3: Control input.

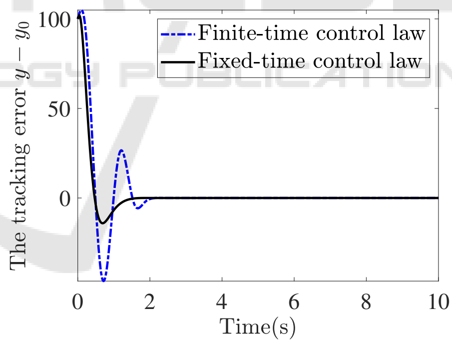


Figure 4: Tracking error $y - y_0$ for another condition.

Figure 2 depicts the tracking error. Figure 3 shows the time response of input signal u. From Figure 1 and 2, it can be found that the proposed fixed-time command filtered backstepping control law in this work achieves faster convergence rate and better tracking accuracy than the finite-time command filtered backstepping control law in (Yu et al., 2018). From Figure 3, it is observed that less chattering is suffered under the proposed fixed-time command filtered backstepping control law.

To better show the superiority of the proposed fixed-time control law, another initial condition

$$[x_1(0)\ x_2(0)\ x_3(0)] = [100\ 75\ 50]$$

is chosen. The tracking errors under such condition are depicted in Figure 4, from which we can find that the proposed fixed-time tracking control law possesses faster time response compared with the finite-time tracking control law in (Yu et al., 2018) when the initial condition of the system is far away from the target value.

5 CONCLUSION

In this paper, a novel fixed-time adaptive command filtered backstepping control approach is proposed to solve the tracking control problem for a class of nonlinear systems. According to this approach, a group of novel virtual control laws and the actual control law are constructed to achieve the fixed-time convergence of the closed-loop system. The fixed-time differentiator is introduced to approximate the time derivative of virtual control laws in a fixed time. The new compensation mechanism is developed to reduce the negative effect of the filtering error. By using the fixed-time stability criterion, the fixed-time tracking performance of the closed-loop system under the proposed command filtered backstepping control law is analysed, and a rigorous theoretical proof is presented.

ACKNOWLEDGEMENTS

This work was supported in part by the National Natural Science Foundation of China under Grant No. 62203255.

REFERENCES

Basin, M., Yu, P., and Shtessel, Y. (2017). Finite-and fixed-time differentiators utilising hosm techniques. *IET Control Theory & Applications*, 11(8):1144–1152.

Chen, L. and Wang, Q. (2021). Finite-time adaptive neural dynamic surface control for non-linear systems with unknown dead zone. *IET Control Theory & Applications*, 15(1):40–50.

Cheng, F., Wang, H., Zong, G., Niu, B., and Zhao, X. (2023). Adaptive finite-time command-filtered control for switched nonlinear systems with input quantization and output constraints. *Circuits, Systems, and Signal Processing*, 42(1):147–172.

Deng, H. and Krstić, M. (1997). Stochastic nonlinear stabilization–i: A backstepping design. *Systems & Control Letters*, 32(3):143–150.

Dong, W., Farrell, J. A., Polycarpou, M. M., Djapic, V., and Sharma, M. (2011). Command filtered adaptive backstepping. *IEEE Transactions on Control Systems Technology*, 20(3):566–580.

Farrell, J. A., Polycarpou, M., Sharma, M., and Dong, W. (2009). Command filtered backstepping. *IEEE Transactions on Automatic Control*, 54(6):1391–1395.

Feng, C., Wang, Q., Hu, C., and Zhang, S. (2020). Finite-time backstepping control with command filter for a class of nonlinear systems with parametric uncertainties. *Transactions of the Institute of Measurement and Control*, 42(12):2297–2307.

Fu, C., Wang, Q., Yu, J., and Lin, C. (2020). Neural network-based finite-time command filtering control for switched nonlinear systems with backlash-like hysteresis. *IEEE Transactions on Neural Networks and Learning Systems*.

Guo, X., Ma, H., Liang, H., and Zhang, H. (2021). Command-filter-based fixed-time bipartite containment control for a class of stochastic multiagent systems. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*.

Hardy, G. H., Littlewood, J. E., Pólya, G., Pólya, G., et al. (1952). *Inequalities*. Cambridge university press.

Kanellakopoulos, I., Kokotovic, P. V., and Morse, A. S. (1991). Systematic design of adaptive controllers for feedback linearizable systems. In *1991 American control conference*, pages 649–654. IEEE.

Li, Y. (2019). Finite time command filtered adaptive fault tolerant control for a class of uncertain nonlinear systems. *Automatica*, 106:117–123.

Mazenc, F. and Bliman, P.-A. (2006). Backstepping design for time-delay nonlinear systems. *IEEE Transactions on Automatic Control*, 51(1):149–154.

Morawiec, M., Strankowski, P., Lewicki, A., Guziński, J., and Wilczyński, F. (2020). Feedback control of multiphase induction machines with backstepping technique. *IEEE Transactions on Industrial Electronics*, 67(6):4305–4314.

Polyakov, A. (2012). Nonlinear feedback design for fixed-time stabilization of linear control systems. *IEEE Transactions on Automatic Control*, 57(8):2106–2110.

Shen, Q. and Shi, P. (2015). Distributed command filtered backstepping consensus tracking control of nonlinear multiple-agent systems in strict-feedback form. *Automatica*, 53:120–124.

Su, Y. and Zheng, C. (2019). Global fixed-time output feedback stabilization for a class of double integrator systems. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 67(10):1954–1958.

Swaroop, D., Hedrick, J. K., Yip, P. P., and Gerdes, J. C. (2000). Dynamic surface control for a class of nonlinear systems. *IEEE transactions on automatic control*, 45(10):1893–1899.

Tang, X., Tao, G., and Joshi, S. M. (2003). Adaptive actuator failure compensation for parametric strict feedback systems and an aircraft application. *Automatica*, 39(11):1975–1982.

Tian, B., Lu, H., Zuo, Z., and Wang, H. (2018). Fixed-time stabilization of high-order integrator systems with mismatched disturbances. *Nonlinear Dynamics*, 94(4):2889–2899.

Tian, B., Zuo, Z., Yan, X., and Wang, H. (2017). A fixed-time output feedback control scheme for double integrator systems. *Automatica*, 80:17–24.

Tong, S., Li, K., and Li, Y. (2020). Robust fuzzy adaptive finite-time control for high-order nonlinear systems with unmodeled dynamics. *IEEE Transactions on Fuzzy Systems*, 29(6):1576–1589.

Wang, D. and Huang, J. (2005). Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form. *IEEE transactions on neural networks*, 16(1):195–202.

Wang, H., Kang, S., and Feng, Z. (2019). Finite-time adaptive fuzzy command filtered backstepping control for a class of nonlinear systems. *International Journal of Fuzzy Systems*, 21:2575–2587.

Wang, K., Liu, X., and Jing, Y. (2021). Adaptive finite-time command filtered controller design for nonlinear systems with output constraints and input nonlinearities. *IEEE Transactions on Neural Networks and Learning Systems*, 33(11):6893–6904.

Yu, J., Shi, P., and Zhao, L. (2018). Finite-time command filtered backstepping control for a class of nonlinear systems. *Automatica*, 92:173–180.

Zhao, D., Jiang, B., and Yang, H. (2021). Backstepping-based decentralized fault-tolerant control of hypersonic vehicles in pde-ode form. *IEEE Transactions on Automatic Control*, 67(3):1210–1225.