Position/Velocity Aided Leveling Loop: Continuous-Discrete Time State Multiplicative-Noise Filter Case

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Abstract: The problem of leveling using a low cost Inertial Measurement Unit (IMU) is considered, where the IMU measurements are corrupted with white noise. In such a case the state equations are subject to state-multiplicative noise. To cope with this noise, a state-Multiplicative Kalman Filter (MKF) is applied. The state components for the Kalman filter implementation include the Body Position Vector (BPV), the Body Velocity Vector (BVV), which is just the Ground Velocities Vector (GVV), projected onto the body axes and the three direction cosines related to the roll and pitch angles. The BVB is assumed to be measured using a Doppler Velocity Log (DVL) device which consists of four antennas measuring the Doppler effect. Similarly, it is assumed that the corresponding BPV can be measured, for instance, using the received signal power at those four antennas. The paper includes numerical simulations and implementation aspects related to the sampled data nature of the estimation problem.

1 INTRODUCTION

Strap Down Inertial Navigation Systems (SDINS) require initialization of position, velocity and attitude. When the platform on which the SDINS is stationary, the roll and pitch of the SDINS may be measured directly from the accelerometers readings. When the platform moves and no transfer alignment is possible (e.g. no accurate reference INS is available), one may resort to the leveling loop approach providing so called coarse alignment, where the roll and pitch angles are estimated utilizing velocity measurements (see e.g. (Xu, 2017) and (Tal, 2017)) and accelerometers and rate sensors provided by an Inertial Measurement Unit (IMU). The present paper deals with the case where the IMU is a low cost one, providing measurements corrupted with white noise. In such a case (see also (Yaesh, 2013)) the state equations are subject to state-multiplicative noise, making related estimation problems readily tractable, using a state-multiplicative Kalman Filter (MKF), see (Stoica, 2009). The state vector components for the Kalman filter implementation include the Body Position Vector (BPV), the Body Velocity Vector (BVV), which is just the Ground Velocities Vector (GVV), projected onto the body axes and the three direction cosines related to the roll and pitch angles. In the present paper, the BVB is assumed to be measured using say a Doppler Velocity Log (DVL) device which consists of four antennas measuring the Doppler effect. Similarly, the BPV is measured from the four corresponding range (i.e. received power) measurements. We deal with the special case of a low range navigation mission, allowing a simple Cartesian formulation of equations of motion, where both Earth rotation and curvature are neglected. The resulting equations of motion are then linear equations of the states, simplifying both dealing with real time calculation of the transition matrix and exact modeling of the above mentioned multiplicative noise effect. It is well known in the inertial navigation community that navigation initialization with 'large' attitude-errors usually results in error divergence. In contrast, since in our case the states include the directions cosines and not just errors of the attitude angles (i.e. the rotation from true to calculated Local Level Local North), the leveling loop we consider, can deal with large initialization errors for both roll and pitch angles. In the present paper it is shown that when the IMU measurement noises are taken into account, a stochastic model with state-dependent noisy terms for the equations of motion is naturally obtained. Two specific Kalman fil-

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ters for systems corrupted with state-dependent (multiplicative) noises are presented. The first one is a discrete-time filter used in the case when measurements with high acquisition rate are available. The second one is a continuous-discrete Kalman filter having the representation of a system with finite jumps and it is used when the measurements have low acquisition rate.

2 PROBLEM STATEMENT

We consider the following equations of motion:

$$\begin{aligned} \frac{d\rho(t)}{dt} &= -\omega(t)\times\rho(t)+v(t)\\ \frac{dv(t)}{dt} &= -\omega(t)\times v(t)+a(t)+gc(t)\\ \frac{dc(t)}{dt} &= -\omega(t)\times c(t) \end{aligned} \tag{1}$$

where ρ and v are, respectively, the position and velocity vectors, g is the gravity constant, $a = [a_x, a_y, a_z]^T$ is the sensed accelerations vector, $\omega = [p, q, r]^T$ is the rates vector in body axes and $c = [c_1, c_2, c_3]^T$ is the vector of the third row in the direct cosine matrix (DCM) of Earth to body transformation, namely, $c_1 = -sin(\theta)$, $c_2 = sin(\phi)cos(\theta)$, $c_3 = cos(\phi)cos(\theta)$.

Note that the above equations are written in the true variables. Since the components of ω, a provided by the IMU are corrupted with noise, we rewrite the state equation in terms of the Inertial Navigation states affected by these noise signals. To this end, we denote the state vector integrated by the INS scheme and the IMU measurements as

$$x = [\rho_m^T, v_m^T, c_m^T]^T \tag{2}$$

and

$$\omega_m = [p_m, q_m, r_m]^T, a_m = [a_{x,m}, a_{y,m}, a_{z,m}]^T. \tag{3}$$

The state equations are then given by the following Itô type stochastic differential equation (Jazwinski,1970)

$$dx(t) = (A_0x(t)+Bu(t))dt+\sum_{\ell=1}^{3} A_\ell x(t)d\xi_\ell(t)+B_w dw(t) \tag{4}$$

where $w(t)$ represents the accelerometers noise and $u(t)$ is the acceleration. Indeed, taking into account that $p_m(t) = p(t)+\varepsilon\xi_1(t)$, $q_m(t) = q(t)+\varepsilon\xi_2(t)$ and $r_m(t) = r(t)+\varepsilon\xi_3(t)$ where ε is the noise level, the system (1) may be written in the form (4) where

$$A_0(t) = \begin{pmatrix} -\Omega_0(t) & I_3 & 0 \\ 0 & -\Omega_0(t) & gI_3 \\ 0 & 0 & -\Omega_0(t) \end{pmatrix},$$

$$B = B_w = \begin{pmatrix} 0 \\ I_3 \\ 0 \end{pmatrix},$$

with the cross product matrix $\Omega_0(t)$ having the expression

$$\Omega_0(t) = \begin{pmatrix} 0 & -r_m(t) & q_m(t) \\ r_m(t) & 0 & -p_m(t) \\ -q_m(t) & p_m(t) & 0 \end{pmatrix}.$$

The coefficients of the state-dependent noise terms have the following expressions

$$A_\ell = \begin{pmatrix} -\Omega_\ell & 0 & 0 \\ 0 & -\Omega_\ell & 0 \\ 0 & 0 & -\Omega_\ell \end{pmatrix}, \ell = 1,2,3$$

where

$$\Omega_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\varepsilon \\ 0 & \varepsilon & 0 \end{pmatrix}, \Omega_2 = \begin{pmatrix} 0 & 0 & \varepsilon \\ 0 & 0 & 0 \\ -\varepsilon & 0 & 0 \end{pmatrix},$$

$$\Omega_3 = \begin{pmatrix} 0 & -\varepsilon & 0 \\ \varepsilon & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Our aim is to estimate the direction cosines vector c_m from noisy measurements of the body frame position and velocity vectors ρ_m and v_m respectively. We next formulate this problem as a state-multiplicative Kalman filtering problem.

3 DISCRETE MULTIPLICATIVE KALMAN FILTER

Discrete time model for the above state equation can be approximately written as

$$x(i+1) = (F_0 + \sum_{\ell=1}^{3} F_\ell \xi_\ell(i))x(i)+Gu(i)+G_w w(i), \tag{5}$$

$i = 0,1,\ldots$, where we added w to represent the measurement noise in the accelerations vector and where, for small enough sampling time $h > 0$, $F_0 = e^{A_0 h}$, $F_i = A_i\sqrt{h}, i = 1,2,3$, $G_w = B\sqrt{h}$ and $G = Bh$. The measurement equation is written as

$$y(i) = Hx(i)+v(i), i = 0,1,\ldots \tag{6}$$

with $H = [I_3\, 0_3\, 0_3]$. In the above equations $\xi(i)$ and $v(i)$ are white noise sequences of zero mean and respectively with Q, R covariances, independent of each other. The MKF for this system is (Stoica, 2009) :

$$\hat{x}(i+1) = F_0\hat{x}(i)+Gu(i)+L(i)(y(i)-H\hat{x}(i)), i = 0,1,\ldots \tag{7}$$

where

$$L(i) = F_0X(i)H^T(R+HX(i)H^T)^{-1}$$

and where the Kalman gain $L(i)$ is computed using the following coupled Riccati and Lyapunov type equations

$$\begin{array}{rcl} X(i+1) & = & \sum_{\ell=0}^{3} F_\ell Y(i) F_\ell^T - F_0 X(i) H^T \\ & & \times (R + HX(i)H^T)^{-1} HX(i) F_0^T \\ & & + G_w Q G_w^T \\ Y(i+1) & = & \sum_{\ell=0}^{3} F_\ell Y(i) F_\ell^T + G_w Q G_w^T \end{array} \tag{8}$$

where $X(i) := E(e(i)e^T(i)), e(i) := x(i) - \hat{x}(i)$ and $Y(i) := E(x(i)x^T(i))$, $i = 0, 1 \ldots$. Note that the signal $y - H\hat{x}$ is known as the innovation signal and is of zero mean and covariance of $HXH^T + R$. This approximate formulation is useful when the sampling rate of IMU and Doppler measurements are identical. In practice, the IMU measurements are available at a higher rate than the Doppler measurements. To this end we represent this situation as a continuous-discrete time problem (i.e. continuous with jumps) where the INS-like equation mechanization is approximated, due to its high rate, by a continuous-time equation.

4 CONTINUOUS-DISCRETE MULTIPLICATIVE KALMAN FILTER

In this case, the state equation of (5) is replaced by the continuous time Itô type stochastic differential equation of (4) whereas the measurement equation of (6) now reads

$$y(ih) = Hx(ih) + v(i), \, i = 0, 1, \ldots \tag{9}$$

with $H = [0_3 \, I_3 \, 0_3]$. The continuous-discrete MKF for this system is (Dragan, 2012):

$$\begin{array}{l} \dot{\hat{x}}(t) = A_0(t)\hat{x}(t) + Bu(t), \, t \neq ih \\ \hat{x}(ih^+) = \hat{x}(ih) + L(ih)(y(ih) - H\hat{x}(ih)), \, i = 0, 1, \ldots \end{array} \tag{10}$$

where the Kalman gain $L(ih) = X(ih)H^T(R + HX(ih)H^T)^{-1}$ is computed using the solution of the following system of coupled Lyapunov equations with jumps

$$\begin{array}{rcl} \dot{Y}(t) & = & A_0(t)Y(t) + Y(t)A_0^T(t) \\ & & + A_1 Y(t) A_1^T + A_2 Y(t) A_2^T \\ & & + A_3 Y(t) A_3^T + G_w Q G_w^T \\ \dot{X}(t) & = & A_0(t)X(t) + X(t)A_0^T(t) \\ & & + A_1 Y(t) A_1^T + A_2 Y(t) A_2^T \\ & & + A_3 Y(t) A_3^T + G_w Q G_w^T, t \neq ih \\ X(ih^+) & = & X(ih) - L(ih)HX(ih), i = 0, 1, \ldots \end{array} \tag{11}$$

Note that if $\hat{x}(0) = 0$ the first two equations of the system (11) have the same initial condition, $X(0) = Y(0) = E(x(0)x^T(0))$ and, therefore, their solutions would be identical if no measurement updates were applied, namely, if $L = 0$.

An outline of the proof of the above result is given as follows. Between the measurements, the estimation error evolves according

$$\begin{array}{rcl} de(t) & = & A_0(t)e(t)dt + \sum_{\ell=1}^{3} A_\ell x(t) d\xi_\ell(t) \\ & & + B_w dw(t), \, t \neq ih \end{array} \tag{12}$$

therefore between the measurements $X(t) = E(e(t)e^T(t))$ and $Y(t) = E(x(t)x^T(t))$ satisfy the first two equations (11), respectively. However, the measurement update is given by (10) and, therefore,

$$e(ih^+) = (I - L(ih)H)e(ih) - Lv(i), \, i = 0, 1, \ldots \tag{13}$$

obtaining

$$\begin{array}{rcl} X(ih^+) & = & (I - L(ih)H)X(ih)(I - L(ih)H)^T \\ & & + L(ih)RL^T(ih), \, i = 0, 1, \ldots, \end{array} \tag{14}$$

or equivalently,

$$\begin{array}{l} X(ih^+) = X(ih) + [L(ih) - X(ih)H^T(R + HXH^T)^{-1}] \\ \times (R + HX(ih)H^T) \\ \times [L(ih) - X(ih)H^T(R + HX(ih)H^T)^{-1}]^T \\ -X(ih)H^T(R + HX(ih)H^T)^{-1}HX(ih), \, i = 0, 1, \ldots. \end{array} \tag{15}$$

We readily see that $L(ih) = X(ih)H^T(R + HX(ih)H^T)^{-1}$ minimizes the right side of (15) and that $X(ih^+)$ is given by the third equation of the system with jumps (11).

5 SIMULATION RESULTS

The continuous discrete MKF was simulated with sampling time of 0.001 sec for the velocity updates and of 0.1 sec for the position updates. The accelerometers noise was taken to be a zero mean white noise of $3m/\sec/\sqrt{hour}$ whereas the gyros noise is $50deg/\sqrt{hour}$. Since the direction cosines square sum to unity, normalization of $[\hat{x}_4, \hat{x}_5, \hat{x}_6]$ has been performed, following (Zanetti, 2006) after each measurement update. Figure 1 depicts the results of 10 Monte Carlo runs using the discrete-time model and filter. The comparison is in terms of the normalized estimation errors, using the 3σ predictions of the errors using the diagonal terms in X, of the velocities and of the direction cosines $c_i, i = 1, 2, 3$. The errors are well within the corresponding predicted standard deviations. The roll and pitch angle estimation errors of the Monte-Carlo runs depicted in Figure 2, whereas the first Monte-Carlo run, roll and pitch angles are respectively depicted in Figure 3 and in Figure 4. The results are satisfactory and encourage the implementation of the presented algorithm.

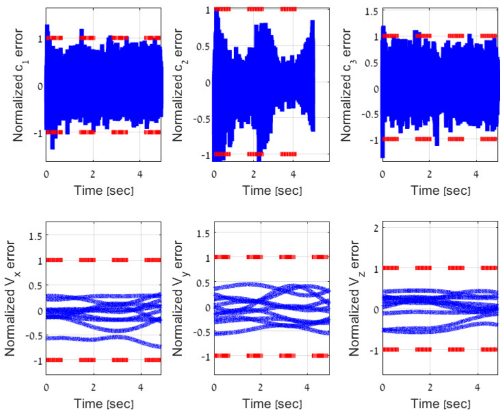

Figure 1: Continuous Discrete MKF results after 10 MC runs.

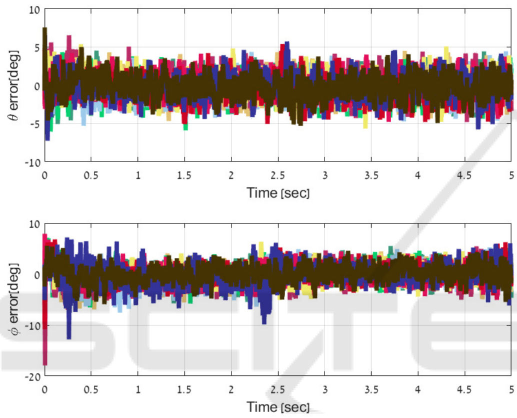

Figure 2: Normalized Errors.

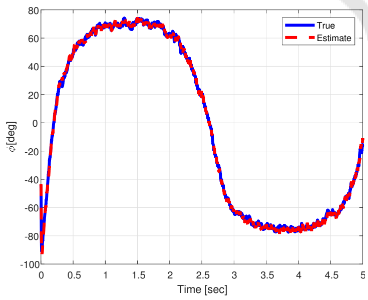

Figure 3: Roll angle.

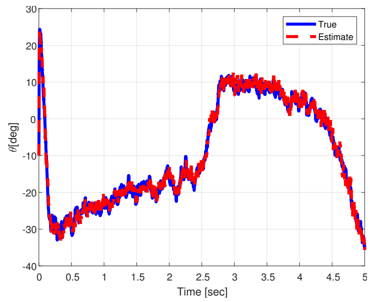

Figure 4: Pitch angle.

6 CONCLUSIONS

We considered the problem of estimating the roll and pitch angles in order to initiate the SDINS mechanization on a moving platform. We assumed using both velocity and position measurements in body axes with different sampling rates allowed by the continuous-discrete time formulation of the state-multiplicative noise Kalman filter we used. More rigorous normalization procedure for the directions cosines within the proposed filter, is left as topic for further research.

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