A New Approach to Addressing Uncertainty in Information Technology with Fuzzy Multi-Criteria Decision Analysis

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Abstract: The problem of reasoning under uncertainty is widely recognised as significant in information technology, and a wide range of methods has been proposed to address this problem. Uncertainty happens when imperfect information is the only available source to solve it using quantitative methods. Therefore, there is a need to implement a qualitative method when no numerical information is available. Linguistic uncertainties related to the qualitative part must be considered and managed wisely. Such uncertainty commonly involves in decision-making problem which depends on human perceptions. This study explores the relationship and difference between two variables, namely the level of uncertainty to the input and the output changes based on multi-criteria decision analysis. There is a positive relationship between these two variables. The novel generation interval type-2 fuzzy membership function technique is proposed based on this. It can accurately map the decision maker's perceptions to the fuzzy set model, reducing the potential for loss of information. In literature, the output ranking of the system is presented as a crisp number. However, this study proposed a new form of output in interval form based on multi-criteria decision analysis. Overall, this study provides new insight into how we should not ignore uncertainty when it affects the input. It provides an intelligent way to map human perceptions to the system using a fuzzy set.

1 INTRODUCTION

Life is always characterised by subjective judgements, which consist of different personal opinions that have been influenced by various factors such as personal views, experience, background or personal assessments of the different levels of variables of interest. They are made using a mixture of qualitative and quantitative information. Qualitative information cannot be directly measured—for example, human perceptions, feelings, emotions and words. However, quantitative information can be directly measured or computed from direct measurements such as the mean value of temperatures and standard deviations of days. Regardless of any information, either they are qualitative or quantitative, there always has uncertainty about it and the amount of uncertainty can exist from small to large.

Qualitative uncertainty can be distinguished from quantitative uncertainty; for example, words can be interpreted differently by different people. Therefore, their linguistic uncertainties need to be considered and managed wisely. Qualitative uncertainty commonly involves in decision-making problems as the problem is highly dependent on human perceptions. In this problem, for a specific context, it is highly dependent on words (i.e., perceptions and words), where words are utilised as the primary input to reach a desired decision. However, words are always characterised by uncertain and vague meanings, which result in increasing complexity of solving the decision-making problem. Fuzzy sets can be considered a successful traditional framework for dealing with uncertainty. The uncertainty is presented by the degree of membership within the range of [0,1] (i.e., certainty degree assigned to the elements belonging to the set or not). However, Mendel (Mendel, 2018) argued that the fuzzy set (Type-1 fuzzy set, T1FS) is unsuitable for modelling words. An extension of the T1FS

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set known as Type-2 fuzzy set (T2 FSs) was introduced by Zadeh in 1975 (Zadeh, 1975), where in this set, an additional dimension is associated with uncertainty about the degree of membership. For example, consider the room temperature; whether a temperature of 27 degrees Celsius belongs to this set or not may have a degree of membership of 0.9 with a certainty of 0.5 and a membership of 0.8 with a certainty of 0.6. Such a problem can be modelled using type-2 fuzzy sets (T2FSs). It is useful when existing uncertain information determines a set's precise and exact membership function. In most cases, however, providing crisp numbers, for example, using the Likert scale to assess something whether to determine the level of certainty or measure a degree of belonging to the set, is problematic (i.e. there could exist uncertainties about them), and thus it is more meaningful to provide intervals (Wu and Mendel, 2007). Therefore, the type-2 fuzzy set theory provides a valuable account of how uncertainties should be handled in the decision-making process when uncertainty about words is present.

Multi-criteria decision analysis problems are categorised as one of the decision-making issues which received considerable critical attention. It is a problem which concern finding the most desirable alternative(s) from a set of pre-determined alternatives, $A = A_1, A_2, \cdots, A_n$ concerning the decision information about criteria weights and criteria values provided by a group of decision-makers (DMs), $DM = DM_1, DM_2, \cdots, DM_m$. However, a significant problem with dealing with humans as decision-makers is that they exhibit variation in their decision (Garibaldi and Ozen, 2007). In order to design an intelligent decision-making method, such variations should be considered, especially in the initial of the system itself. In other words, the construction of a system should be aimed to better resemble human reasoning in conjunction with using approximate information and uncertainty to reach a decision.

In the general framework of fuzzy multi-criteria decision analysis (MCDA), there exists a technique to assign a linguistic label (e.g., *Very Good*, *Very Poor*, *Fair*, etc.) with fuzzy membership functions (MFs) to represent the performance of each alternative concerning each criterion. For example, in one of the techniques, known as the Fuzzy Technique for Order Preference by Similarity to Ideal Solution (Fuzzy TOPSIS) (Chen, 2000), the performance of each alternative is evaluated against each criterion using the numerical scale, which then mapped into the fuzzy MFs with associated parameter (Table 1). The evaluation from the decision maker is mapped using a fuzzy set to enhance pre-screening evaluations, where the value of positive rating performance, for example, 'Good', can be approximated in a range of value, for example, $5-10$.

Table 1: Linguistic scale for rating of alternatives in Fuzzy TOPSIS method.

Poor (P)	Fair (F)	Good (G)
(0,0,5)	(0,5,10)	(5,10,10)

However, the conventional FTOPSIS used Type-1 fuzzy sets (T1FSs), characterised by precise membership functions in the range [0,1], resulting in the uncertainty disappearing once they have been chosen. In addition, humans as decision-makers exhibit dynamic behaviour, which causes dynamic variation in the decision-making process (Ozen and Garibaldi, 2004). Various frameworks based on fuzzy sets have recently been suggested to model uncertainty. The main challenge in constructing the model is the generation of the fuzzy MFs (C. Wagner, 2009; Mendel and Wu, 2007). In the MCDA framework, this will affect the overall ranking result at the end of the model. Additionally, a lack of investigations has been observed in the literature on how to construct the MFs and specify the parameter of MFs in MCDA paradigm. Thus, in this study, a series of experiments were carried out by introducing several different levels of small changes (i.e., uncertainty) in the MFs associated with the linguistic labels. The purpose of doing this is to explore any relationship between the amount of uncertainty (i.e., small changes level) introduced in MFs and to observe changes in overall decision support output. In addition, this experiment will lead towards a proposal of a novel and direct technique to generate Type-2 MFs for providing a better and more accurate model of uncertainty based on MCDA technique. Additionally, the output results remain in the same form of information which is in a range of values (i.e., interval form). This type of output result is the main difference as opposed to the standard MCDA technique, where in the classical one, it provides output results in a crisp rank. Thus, this novel technique is interesting when the input and output of the information are in the same form. Furthermore, it can minimise the potential for loss of information during the process by mapping all the information directly to fuzzy sets.

The paper is structured as follows: Section 2 briefly revises the fundamental concepts of fuzzy set theory and the MCDA method. Section 3 presents the experimental procedure implementing fuzzy TOPSIS method. Section 4 provides a discussion of experiment result in the comparison context. Finally, Section 5 gives conclusions with suggestions of future work.

2 BACKGROUND

This section reviews the background theory used in this paper based on one of MCDA methods, namely fuzzy TOPSIS as an application area.

2.1 Multi-Criteria Decision Analysis (MCDA)

Over the past four decades, many MCDA methods have been developed, and their number continues to grow. Based on the surveys conducted by Aruldoss et al. (Aruldoss et al., 2013), MCDA is a powerful tool for obtaining the best choice for complex decision-making situations. The MCDA methods have also been successfully applied in various domains.

MCDA can be classified into two main categories:1) Multi-Objective Decision Making (MODM); 2) Multiple-Attribute Decision Making (MADM)(Hwang and Yoon, 1981), (Kahraman, 2008). The MODM method is suitable for the design or planning model, whose main objective is to achieve an optimal solution by considering the various interactions among the given constraints. MADM is a method that makes selections among some elements in a set of actions with multiple, commonly conflicting attributes. We are particularly interested in one of MADM's methods, namely Fuzzy TOPSIS.

The selection of this method to be implemented in our experiments is motivated by a few findings provided by some studies. For example, Zanakis (Zanakis et al., 1998) concluded that the simulation experiment provided the result that TOPSIS has the fewest rank reversals among other MADM methods. Additionally, a survey conducted by (Behzadian et al., 2012) conclude that, among numerous MADM methods developed to solve real-world decision problems, the TOPSIS method works satisfactorily across different application areas. More recently, Yue (Yue, 2014) claimed that the TOPSIS method is suitable for cautious (risk avoider) decision maker(s) because the decision maker (s) may want to have a decision which not only makes as much profit as possible but also avoids as much risk as possible. Thus, in this study fuzzy TOPSIS method (Chen, 2000) is implemented in our experiment to observe changes in the overall decision support output when various uncertainty level is introduced in the membership functions. The reader is advised to refer to Figure 1 and (Madi et al., 2016) for further reference on the step-wise procedure in the fuzzy TOPSIS technique.

3 INTRODUCING UNCERTAINTY INTO MEMBERSHIP FUNCTIONS

3.1 Generation Type-1 Fuzzy Membership Functions

Fuzzy sets are commonly used to represent linguistic variables such as *height* or *goodness* every day. On real-world occasions, the decision maker commonly faced difficulty providing assessment in a conclusive and precise manner. Thus, using words instead of numerical values to provide assessments or evaluations is quite natural. In the standard fuzzy TOPSIS method, the scale is developed using TFNs. For example, as shown in Table 1. In this study, the evaluation given by a set of decision-makers to the fuzzy TOPSIS model should remain fixed, and the small changes (i.e., level of uncertainty present in MFs) to overall decision support output would be explored. The effect of introducing small changes to the MFs is investigated by using the type-2 fuzzy TOPSIS method. Next, the overall experimental procedure details are explained using the following case study.

3.1.1 Case Study

The case study is provided in which the three experts gave their opinions based on the criteria determined at the beginning of the study. The case study consists of a simulation of one mobile application that needs to perform an intensive task. The experts are presented with a list of criteria, and they need to give their opinion on the decision that should be made in the offloading task: whether to offload the task remotely (A_1) or remain local (A_2). In a specific scenario involving the use of video editing applications on a regular mobile phone (Samsung Galaxy S5), three experts in Cloud Computing utilized a video-editing application on their smartphones.

The experts, D1 and D2 are formed to conduct further evaluation to make decision based on three criteria, Battery Level (C1); Memory(C2), and Network Signal Level (C3). The three experts evaluate these two types of platform (i.e., local and remote) concerning the three criteria C1, C2 and C3, where the weighting vector is $w = (0.2, 0.4, 0.4)$. The experts use the linguistic variables scale (shown in Table 1) to evaluate these two types of platform. Assume that the evaluation given by experts D1 and D2 are summarized in Table 2.

Step 1: Construct normalized fuzzy decision matrix: $r_{ij} = \frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}$, for $j \in J$ (Benefit Criteria) OR $r_{ij} = \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{c_{ij}}$, for $j \in J'$ (Cost Criteria) ; where $c_j^* = \max_i c_{ij}$ if $j \in J$; and $a_j^- = \min_i a_{ij}$ if $j \in J'$.

Step 2: Construct the weighted normalized decision matrix: $v_{ij} = w_j r_{ij}$ where w_i is the weight for jth criterion.

Step 3 Determine the positive ideal and negative ideal solutions: $A^* = \{v_1^*, \cdots, v_n^*\}$, [Positive Ideal Solution], where $v_i^* = (1,1,1)$; $A^- = \{v_1^-, \cdots, v_n^-\}$, [Negative Ideal Solution]; where $v_i^- = (0,0,0)$

Step 4 Calculate separation measures for each alternative using vertex method. The separation from positive ideal alternative: $s_i^* = \sum_{j=1}^{n} d(v_{ij}, v_{ij}^*)$, $i = 1, \cdots, m$.
The separation from the negative ideal alternative is: $s_i^- = \sum_{j=1}^{n} d(v_{ij}, v_{ij}^-)$, $i = 1, \cdots, m$; where $d(\cdot\,,\,\cdot)$ represent the distance between two fuzzy numbers by vertex method:
Let $\tilde{m} = (m_1, m_2, m_3)$ and $\tilde{n} = (n_1, n_2, n_3)$ be two triangular fuzzy numbers (TFN). Distance calculation of these to TFN is

$$d(\tilde{m}, \tilde{n}) = \sqrt{\frac{1}{3}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2]}$$

Step 5: Calculate the relative closeness to the ideal solution, CC_i : $CC_i = \frac{s_i^*}{(s_i^* + s_i^-)}$, $0 < CC_i < 1$
Select the alternative with CC_i closest to 1.

Figure 1: Stepwise procedure of Fuzzy TOPSIS(Chen, 2000),(Madi et al., 2016).

Table 2: Linguistic rating of each alternative and importance weight of each criterion.

		DM's rating	
Criteria	Alts.	$D1$	$D2$
C_1	A_1	G	F
	A_2	G	F
C_2	A_1	G	G
	A_2	F	P
C_3	A_1	P	P
	A_2	F	G

3.2 Novel Method in Generation of Interval Type-2 Fuzzy Membership Functions (IT2 MFs)

In this section, we explained how we generate IT2 MFs using T1 MFs by introducing a series of uncertainty in the MFs. Assume a T1 MFs is defined by: $\mu_l(x) = \frac{x-b}{a} + 1$, for left function which strictly increasing and $\mu_r(x) = \frac{b-x}{a} + 1$, for right function which strictly decreasing. The generation of IT2 MFs using T1 MFs is a symmetrical case where the value of a is the same for both the left and right functions. For non-symmetrical case, the properties of the function can be summarized as follow:

- a in left side, a_l, is not equal to a in right side,a_r,
- $a_l < a_r$,
- If $(b-a) < b$, then $b < (b+a)$

Then, assume we introduced uncertainty with level d to the same T1 MFs. The MF is now shifted to IT2 MFs where the area between standard T1 MFs with upper (UMF) and lower (LMF) bound of new MFs have now become footprint of uncertainty (FOU).

The left, μ_l and right, μ_r, MFs now have two functions each, where it is defined for UMF and LMF, respectively. The UMF and LMF for the left side are defined as in Eq. (1) and (2), respectively.

$$\mu_l^{UMF}(x) = \frac{x-b}{a-d} + 1 \tag{1}$$

$$\mu_l^{LMF}(x) = \frac{x-b}{a+d} + 1 \tag{2}$$

For the right side, the UMF and LMF are defined as in Eq. (3) and (4), respectively.

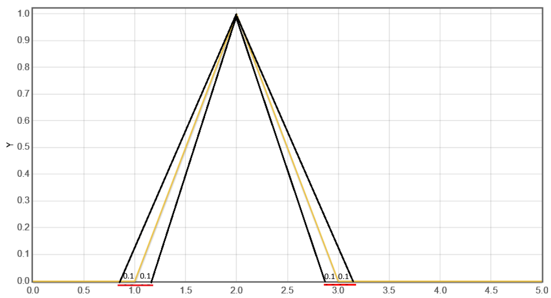

Figure 2: Triangular Fuzzy membership function.

$$\mu_r^{UMF}(x) = \frac{b-x}{a+d} + 1 \tag{3}$$

$$\mu_r^{LMF}(x) = \frac{b-x}{a-d} + 1 \tag{4}$$

From these definitions (i.e., Eqs. (1) to (4), the generation of any fuzzy MFs can be done directly by specifying the centre for triangular MFs and the approximate level of uncertainty (i.e., d). Next, we demonstrate the experiment procedure by using this novel fuzzy MF generation.

A delta series is introduced in the standard T1 MFs. The delta, δ_i, $i = 1,2,..,m$, where $\forall \delta_i \in [0,1]$ are considered as uncertainty in the decision making process. In this experiment, we have chosen seven delta values; $\delta_7 = 0.10, 0.15, 0.17, 0.20, 0.30, 0.40, 0.50$, where these values are used to shift left and right of every MF according to Eqs. (1) to (4). We demonstrate for the first case, $\delta_1 = 0.10$ to be implemented in case study.

The introduction of $\delta_1 = 0.10$ is done by shifting left to 0.10 and shifting right to 0.10 of standard Type-1 MF (Figure 2).

Then, a fuzzy variable of 'Rating', with three linguistic labels, 'Poor', 'Fair' and 'Good', as shown in Figure 3, is now become an interval Type-2 fuzzy MFs (IT2 MFs), bounded with upper membership function (UMF) and lower membership function (LMF). For example, we defined UMF and LMF of the fuzzy label 'Poor' as in Eqs. (5) and (6), respectively.

$$\mu_{poor}^{UMF}(x) = 1 - \frac{x}{0.1} \tag{5}$$

$$\mu_{poor}^{LMF}(x) = 1 + \frac{x}{0.1} \tag{6}$$

Then, we can define the fuzzy label 'Poor' as an interval type-2 fuzzy number, $Poor = [(0,0,4.9),(0,0,5.1)]$, where the first element is indicated of lower value and the second element is indicated of upper value. The illustration of fuzzy MF 'Poor' is shown in Figure 3. Note that this generation of IT2 fuzzy MF is based on the original T1 fuzzy MF

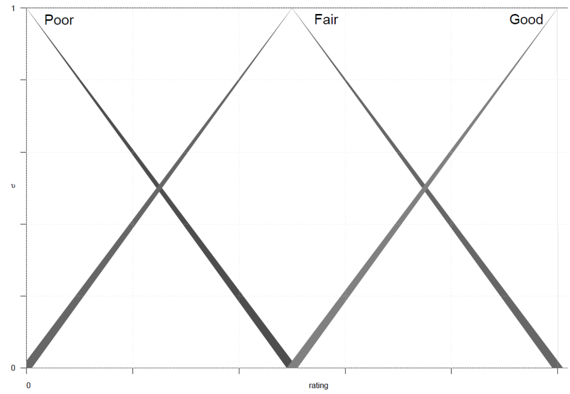

Figure 3: Interval Type-2 fuzzy variable 'Rating'.

from Table 1. The same procedure is applied to generate linguistic labels such as 'Fair' and 'Good'. The linguistic variable 'Rating' overall labels are now interval Type-2 fuzzy MFs shown in Figure 3. Thus, the labels in the linguistic variable 'Rating' can now be rewritten as IT2 fuzzy linguistic scale as in Table 3.

Next, the same fuzzy TOPSIS procedure as Case 1 (Section 3.1.1) is applied to get the rank of the alternative. However, since the IT2 fuzzy MFs bounded by LMF and UMF, we treated the value separately,i.e., instead of having one single ranking value for output results (CC_i), in this experiment, the result is an interval, which is a novel type of output. We present the experiment result in Table 4 and Figure 4. To recall, we used the following delta values in our experiment: $\delta_7 = (0.10, 0.15, 0.17, 0.20, 0.30, 0.40, 0.50)$.

4 DISCUSSION

There is a significant difference in overall decision output for the case uncertainty present. For comparison purposes, we present the result of Type-1 fuzzy TOPSIS, where in this synthetic example, the closeness coefficient values for both cars are 0.1971524 and 0.2005322, respectively. However, when a series of delta (i.e., uncertainty) is introduced to the uncertain spread MFs, the overall output result has a slight difference on the output (i.e., Closeness Coefficient value) (Table 4). One reason for having slightly different values on output is because only the 'Fair' MF shifted to the left and the right direction, while two other MFs, 'Poor' and 'Good', shifted to the right and left, respectively.

Based on this, the difference among various output values should be considered when implementing any decision-making process. As this generation of fuzzy MFs is entirely straight away, there could mini-

Table 3: Linguistic scale for rating of alternatives in Interval Type-2 fuzzy TOPSIS method.

Poor (P)	Fair (F)	Good (G)
[(0,0,4.9),(0,0,5.1)]	[(0.1,5,9.9),(-0.1,5,10.1)]	[(5.1,10,10),(4.9,10,10)]

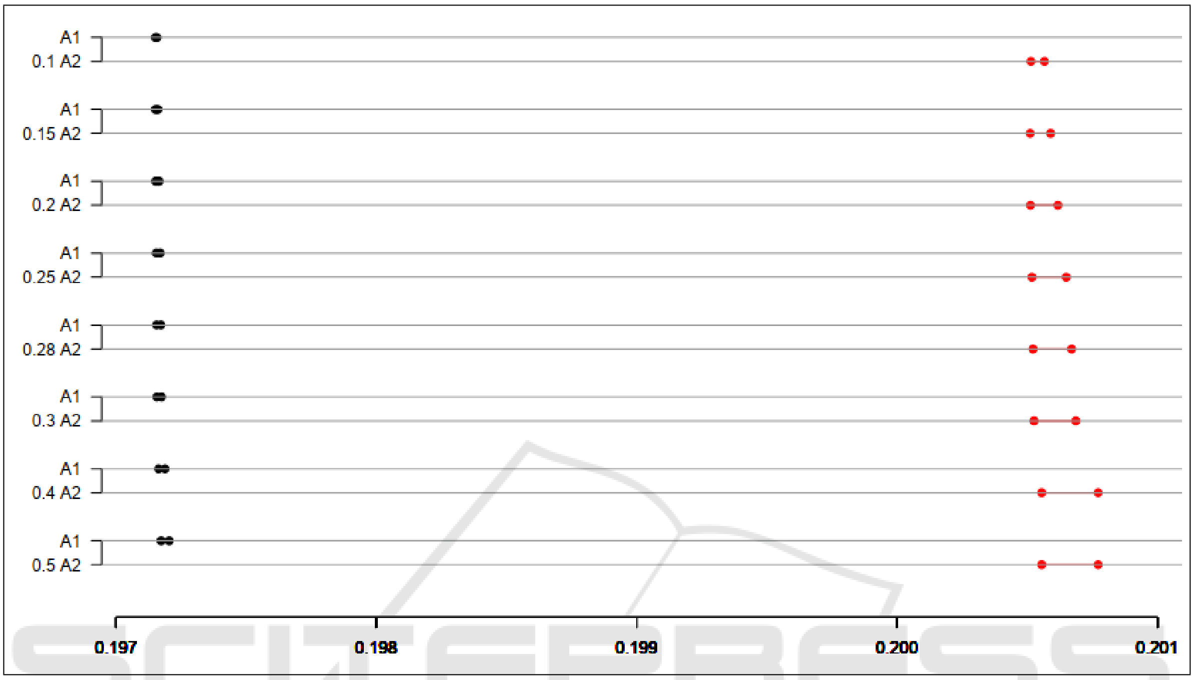

Figure 4: Case 2 result: Interval closeness coefficient, where A1 = Car 1, A2 = Car 2.

Table 4: Result: Interval Closeness Coefficient for Uncertain spread MFs.

	$\delta = 0.10$	$\delta = 0.15$	$\delta = 0.20$	$\delta = 0.25$
Local	[0.1971508, 0.1971569]	[0.1971511,0.1971602]	[0.1971521,0.1971643]	[0.197154,0.197169]
Remote	[0.2005143, 0.2005666]	[0.2005114,0.2005901]	[0.2005125,0.2006179]	[0.200518,0.200650]

	$\delta = 0.28$	$\delta = 0.30$	$\delta = 0.40$	$\delta = 0.50$
Local	[0.197155,0.197172]	[0.1971562,0.1971746]	[0.1971632,0.1971878]	[0.1971730,0.1972040]
Remote	[0.200522,0.200671]	[0.2005264,0.2006865]	[0.2005556,0.2007729]	[0.2005556,0.2007729]

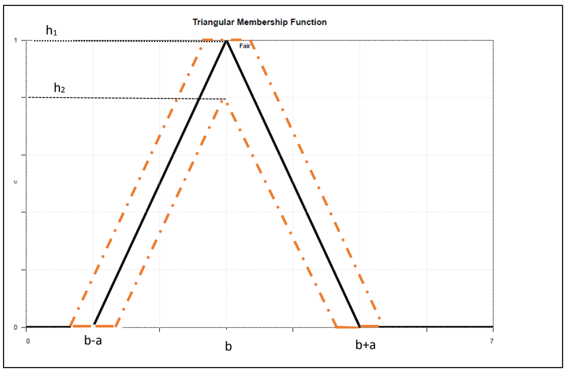

Figure 5: Uncertain mean fuzzy membership function.

mize any potential of loss of information when transferring the evaluation made by decision-makers to any decision support system. The width of the interval output denotes the decision makers' certainty in their evaluation; a narrow interval is used when they are sure where on the scale the answer lies, and a wider one is where they are less specific. Thus, whenever the uncertainty effect the input, it should be considered that every step in the process has that uncertainty. Each value in the interval has a specific meaning that we should not ignore, mainly when applied in a medical context as this context commonly deals with the life and death of humans.

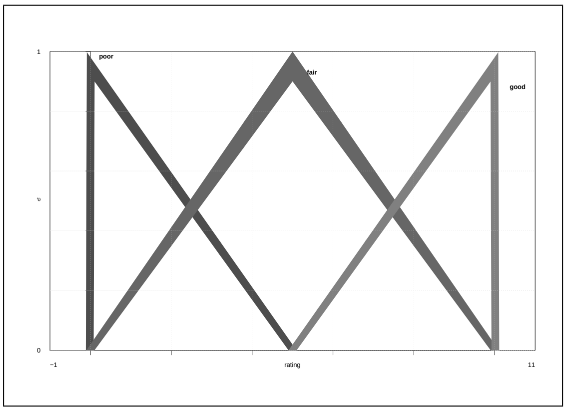

Figure 6: Uncertain mean fuzzy membership function.

5 CONCLUSION

In this paper, an experiment is conducted by introducing several different levels of uncertainty in the Type-1 MFs associated with the linguistic variables. The purpose of doing this is to explore any relationship between the uncertainty introduced in MFs and to observe changes in overall decision support output. The presence of uncertainty causes output values to fluctuate. Further, this successful experiment led towards proposing a novel and direct technique to generate Type-2 MFs for providing a better and more accurate uncertainty model based on the MCDA method. This method has a few advantages as it is a direct way of generating the MFs. Thus, it can minimise the potential loss of information decision-makers give. Additionally, the output results remain in the same information form in interval-based numbers. This type of output result is the main difference as opposed to the standard MCDA method, where in the classical approach, it provides output results in a rank of crisp number. Thus, this novel technique is interesting when the input and output of the information are in the same form. Each value in the interval is considered and can support the decision maker's decision. Accurately modelling preference information to fuzzy MFs can reduce the potential of making any misleading decision. In future, we will explore different techniques and methods of ranking intervals. We will develop our ranking algorithm based on this, specifically focusing on various interval values.

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