# Tuning the Dynamic Response of a Redundant Robotic System Using Its Dominant Natural Frequencies

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Keywords: Redundant Robotic System, Dynamic Response, Dominant Natural Frequencies, Fast Fourier Transform, Robotic Manipulator.

Abstract: Robotic systems often encounter challenges in achieving desired dynamic responses, especially when they possess redundant degrees of freedom. This paper proposes a methodology to identify a redundant robotic system's dominant natural frequencies and tune its dynamic response through appropriate damping. The system's natural frequencies are accurately identified by analyzing displacement data and leveraging the power of fast Fourier transform tools. These frequencies serve as critical parameters for modifying the response behavior, enabling enhanced control and stability. To validate the effectiveness of the proposed methodology, simulations are conducted on a 7-degree-of-freedom redundant Panda robotic manipulator. The results demonstrate the methodology's potential to optimize the dynamic performance of complex robotic systems, opening avenues for improved efficiency, safety, and overall system performance.

## 1 INTRODUCTION

Robotic systems are pivotal in various industries, from manufacturing and automation to healthcare and space exploration (Cen and Melkote, 2017). Achieving precise and controlled dynamic responses is essential for ensuring these systems' optimal performance, safety, and efficiency. However, this task becomes more challenging in the presence of redundant degrees of freedom, which offer increased flexibility but also introduce complexities in controlling and tuning the system's response (Urrea and Pascal, 2017). In this paper, we propose a methodology that utilizes the fast Fourier transform (FFT) to identify the dominant natural frequencies of a redundant robotic system and subsequently tunes its dynamic response through appropriate damping.

Identifying modal parameters, such as natural frequencies, is crucial for understanding and characterizing the dynamic behaviour of a robotic system. By accurately identifying these frequencies, we can gain insights into the system's vibrational modes, allowing us to predict and manipulate its dynamic response (Gonul et al., 2019; Garnier and Subrin, 2022). Traditionally, identifying natural frequencies involved experimental modal analysis techniques, which required physical measurements on the robotic system. While these methods provide valuable information, they can be time-consuming, expensive, and may not always be practical for complex systems. The advent of computational tools and numerical simulations has opened up new possibilities for modal parameter identification, offering faster and more cost-effective alternatives (Chen et al., 2014).

The proposed methodology offers several advantages. Firstly, it eliminates the need for extensive experimental modal analysis, saving time and resources (Quqa et al., 2020). Secondly, computational tools provide a more flexible and versatile approach to modal parameter identification. Thirdly, tuning the system's response through damping modification enables enhanced control, stability, and performance of redundant robotic systems (İlman et al., 2022). It offers a more efficient and cost-effective approach to modal parameter identification by leveraging computational tools and numerical simulations. The fast Fourier transform algorithm allows us to convert displacement data from the time domain to the frequency domain, enabling the identification of dominant peaks

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Saldarriaga, C., Fajardo-Pruna, M., Helguero, C. and Leon-Torres, J.
Tuning the Dynamic Response of a Redundant Robotic System Using Its Dominant Natural Frequencies.
DOI: 10.5220/0012191000003543
In *Proceedings of the 20th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2023) - Volume 2*, pages 166-172
ISBN: 978-989-758-670-5; ISSN: 2184-2809

that correspond to the natural frequencies of the system. Finally, our methodology offers flexibility and adaptability, allowing it to be applied to various redundant robotic systems. The methodology presented in this paper is particularly useful for systems that have redundant degrees of freedom, but it is not a limitation. In case of non-redundant systems the process becomes somewhat simpler but the method still applies. In general mechanical systems (not necessarily a robotic manipulator) the redundancy property would be analogous to having an unconstrained system, or a system that is not attached to inertial frames, which cause the appearance of rigid body modes of motion with natural frequency $\omega_n = 0$.

The Panda robotic manipulator is a suitable case study for validating our methodology. With its 7 degrees of freedom and redundant kinematic structure, the Panda manipulator exhibits complex dynamic behaviour, making it an ideal candidate for testing the effectiveness of our approach. Through forward kinematics and dynamics simulations, we obtain displacement data that accurately represents the manipulator's response under specific operating conditions (Figure 1).

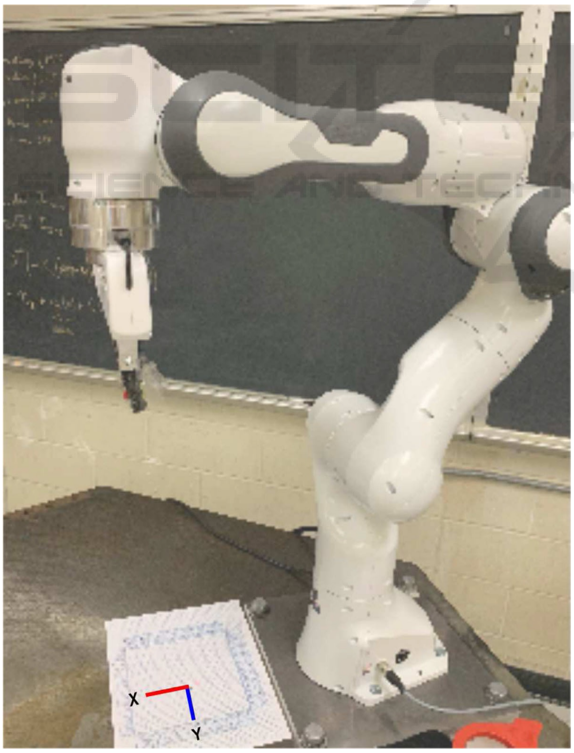



Figure 1: Franka Emika Panda robot (Saldarriaga et al., 2022).

Applying the fast Fourier transform to the obtained displacement data. We analyze the resulting frequency spectrum to identify the dominant natural frequencies of the Panda robotic manipulator. These frequencies represent the system's inherent vibrational modes and provide valuable insights into its dynamic behavior. Determining the natural frequencies lays the foundation for optimizing the system's response through damping modification.

Once the dominant natural frequencies are identified, we proceed to tune the dynamic response of the redundant robotic system through appropriate damping. Damping is crucial in controlling oscillations and attenuating unwanted vibrations within a system. By strategically introducing damping at specific frequencies, we can effectively modify the system's response behaviour and enhance its stability and performance (Kao and Saldarriaga, 2021).

The damping modification technique proposed in our methodology considers the relative magnitudes of the peaks in the frequency spectrum. By analyzing the spectral content of the system, we identify the frequencies that require additional damping to achieve the desired response. The damping modification can be implemented in an actual physical system using various techniques, such as introducing passive dampers or adjusting the control parameters of the robotic system, largely limited by the sampling of the system and the accuracy of the dynamic model.

Through simulations on a 7-degree-of-freedom redundant Panda robotic manipulator, we validate the effectiveness of our methodology. The results demonstrate our approach's potential to improve the dynamic performance of complex robotic systems.

## 2 METHODOLOGY

In the proposed methodology, the FFT algorithm converts the time-domain displacement data obtained through simulations or physical measurements into the frequency domain, clearly representing the system's spectral content. We can identify the dominant peaks that correspond to the system's natural frequencies by analyzing the resulting frequency spectrum.

To illustrate the efficacy of our methodology, we conduct simulations on a 7-degree-of-freedom redundant Franka Emika Panda robotic manipulator.

With the obtained joint displacement data, we perform the FFT analysis to extract the natural frequencies of the Panda manipulator. The FFT algorithm decomposes the displacement data into its constituent frequency components, providing a spectrum that highlights the dominant frequencies present in the system. By identifying the peaks in the frequency spectrum, we can precisely determine which natural frequencies characterize the dynamic behaviour of the Panda manipulator the most.

Our methodology proposes a damping modification technique based on the identified natural frequencies that contribute the most to the response.

Once the dominant natural frequencies are identified, we modulate the system's dynamic response through appropriate damping selection. Damping is a critical parameter that affects the decay rate of oscillations in a system. By strategically updating certain damping elements that affect specific frequencies, we can effectively control and tune the response behavior of the robot.

The dynamic equation of motion of a robotic system is governed by

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}(t)+\mathbf{G}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}(t)+\mathbf{v}(\mathbf{q})=\boldsymbol{\tau}_m+\boldsymbol{\tau}_{ext} \quad (1)$$

where $\mathbf{q}$ contains the $n$ joint angles, $\mathbf{M}$ is the mass matrix, $\mathbf{G}$ the matrix that contains the gyroscopic non-linear terms, $\mathbf{v}$ is the vector that compensates for gravity, and $\boldsymbol{\tau}_{ext}$ is the external torques vector. In order to impose and establish a compliant behavior (Villani and De Schutter, 2008) to the system, an impedance controller is established by setting $\tau_{\mathbf{m}}$ as $[-\mathbf{K}\mathbf{q}(t)-\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{v}(\mathbf{q})+\mathbf{G}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}(t)]$, so that the system becomes

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\boldsymbol{\tau}_{ext} \quad (2)$$

and a multidimensional mass $\mathbf{M}$, damping $\mathbf{C}$ and stiffness $\mathbf{K}$ relationship is obtained for the robot, where the stiffness and damping matrices can be mapped from the Cartesian task space through the Jacobian matrix $\mathbf{J}$

$$\mathbf{K}=\mathbf{J}^T\mathbf{K}_C\mathbf{J}+\mathbf{K}_G+\mathbf{K}_B \quad (3)$$

$$\mathbf{C}=\mathbf{J}^T\mathbf{C}_C\mathbf{J} \quad (4)$$

where $\mathbf{K}_B=\mathbf{J}^T\mathbf{C}_C\dot{\mathbf{J}}$ and

$$\mathbf{K}_G=\left[\left(\frac{\partial\mathbf{J}^T}{\partial q_1}\mathbf{f}\right)\left(\frac{\partial\mathbf{J}^T}{\partial q_2}\mathbf{f}\right)\dots\left(\frac{\partial\mathbf{J}^T}{\partial q_n}\mathbf{f}\right)\right]$$

An appropriate selection of the damping parameters is not a very straight forward job, especially for redundant robots. By generating the parameter study of the elements of the damping matrix $\mathbf{C}$ after removing or handling the redundant degree(s) of freedom (Saldarriaga et al., 2022), we can modulate the contribution of each mode of vibration $\lambda_i$ and corresponding natural frequency $\omega_{ni}$ of the system, for example, those previously identified by FFT tools results, in a sound, analytical manner.

In summary, and as illustrated in Figure 2, we intend to modulate the response of the mechanical system by the choice of the damping parameters, after the dominating frequencies are identified from the FFT results plots, in a systematic manner by the use of an analytical methodology that allows us to consider the case of unconstrained mechanical systems, or redundant robots without losing generality, which would not be possible without the analytical tool described.

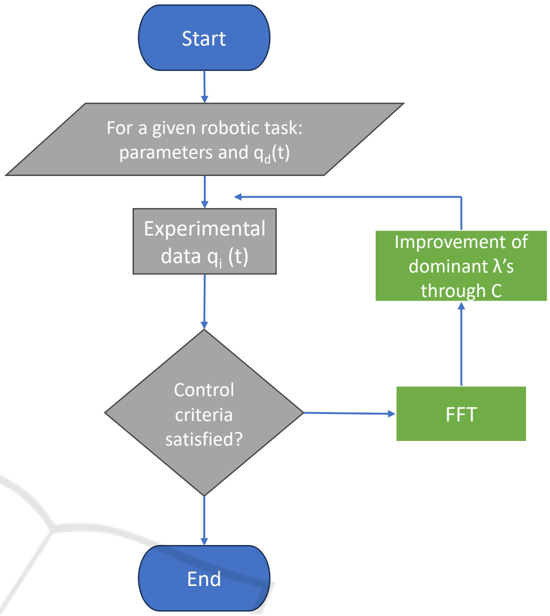



Figure 2: Flowchart of the methodology carried out in the experiments.

## 3 SIMULATIONS

In order to collect robot joint displacements data, the Franka Panda was modeled in Wolfram Mathematica, where the dynamic response of the system was obtained by numerically solving the differential equations. The NDSolve function was used to simulate and obtain an impedance behavior as the one in Equation 2 imposed through stiffness and damping parameters that were defined in the Cartesian space, and then mapped into the joint space through Equations 3 and 4; while the inertia mass matrix of the robot was obtained according to the robot structure and configuration as in (Murray et al., 1994).

The system was handled and solved in a very similar manner as a general case of free vibration of discrete systems (Meirovitch, 2001) with respect to an equilibrium joint configuration. An initial condition (displacement) of approximately $10cm$ was imposed to the end-effector in the $Y$ direction for every simulation, with zero initial velocity, or from rest. Only the damping matrix $\mathbf{C}_C$ was updated according to the intended dynamic response of the system and the analytical tool. The simulation data consisted of joint displacements with transients and steady state responses, after imposing an initial displacement to the end-effector (with corresponding joint initial conditions through inverse kinematics) and releasing from rest.

The initial robot configuration for every simulation was $\mathbf{q}_0 = [0.072; -0.379; 0.167; -2.548; 0.0748; 2.17; 0.185]^T$ *rad*, the chosen Cartesian stiffness matrix was $\mathbf{K}_C$ =diag(2000, 2000, 2000, 100, 100, 100) in SI units for both translation and rotation, respectively. An initial arbitrarily low damping matrix $\mathbf{C}_{C0}$ was chosen as $0.1\mathbf{I}_6$ to illustrate the methodology, after that, the procedure described in the previous Section was carried out to modulate the dynamic response of the system through the FFT tool and the damping parameters selection. The plot results of the most significant joints are shown in Figure 3, we can see how the response is underdamped with large settling time.

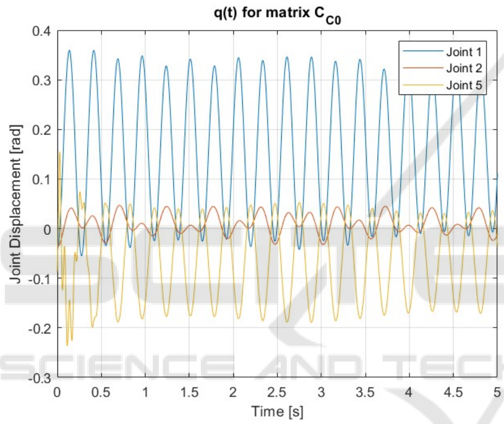
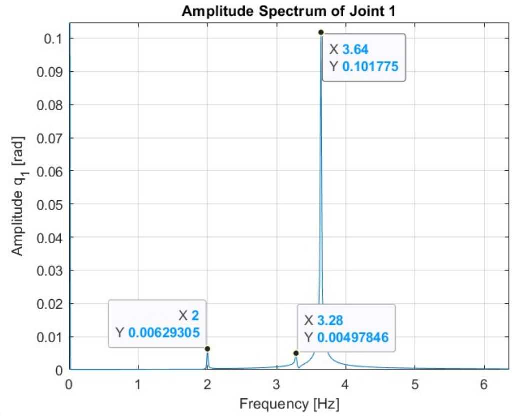



Figure 3: Joint Displacements for the arbitrarily low initial damping matrix $\mathbf{C}_{C0}$. Plots of the most significant joints.

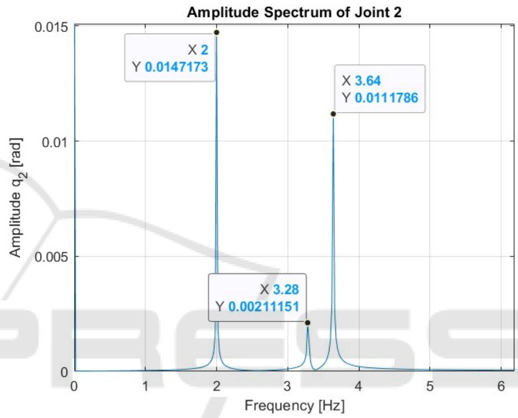
## 4 RESULTS AND DISCUSSION

After obtaining the joint displacements of free vibration response for the first set of parameters, the FFT tool showed that the dominant modes λ's were those corresponding to the natural frequencies 2, 3.2, and 3.6 *Hz* as shown in Figure 4, which after generating the parameter study for the starting configuration and the system, correspond to the lowest frequencies in the modal space, as shown in Figures 5 and 6, and that gives us a very good understanding on how to select a new damping matrix to improve the response.

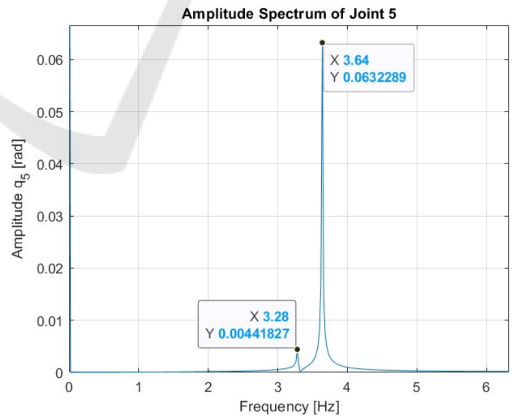
Thanks to the information provided by both the FFT results and the parameter study, a new damping matrix that improves the response of the system was chosen as $\mathbf{C}_{C1}$ =diag(70.3; 125.4; 43; 0.6; 1; 0.1), which was used to generate and obtain a new simulation and FFT plots, as shown in Figure 8. Now the




(a) FFT Joint 1.




(b) FFT Joint 2.




(c) FFT Joint 3.

Figure 4: FFT results for the arbitrarily low initial damping matrix $\mathbf{C}_{C0}$. Plots of the most significant joints.

dominant natural frequencies have changed, are not as prominent as before and depend on the intended joint to be analyzed. This new damping matrix $\mathbf{C}_{C1}$ generates the following damping ratios and natural fre-

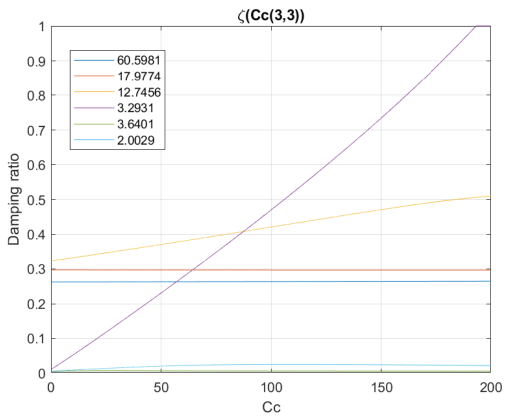



Figure 5: Parameter study of element (3,3) of the damping matrix showing the effect on each mode.

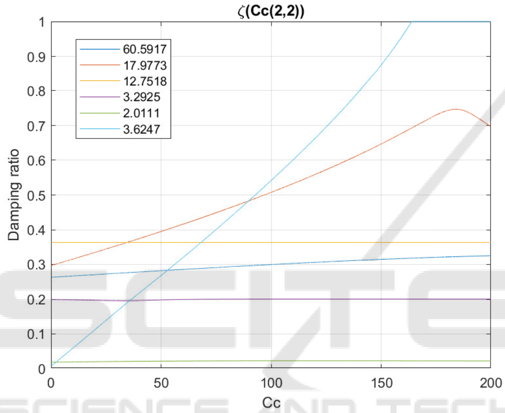



Figure 6: Parameter study of element (2,2) of the damping matrix showing the effect on each mode.

quencies on the system in the modal space

$$\zeta = \begin{bmatrix} 0.2 \\ 0.21 \\ 0.70 \\ 0.57 \\ 0.57 \\ 0.31 \end{bmatrix} ; \ \omega_n = \begin{bmatrix} 2.01 \\ 3.31 \\ 3.91 \\ 12.67 \\ 17.13 \\ 58.7 \end{bmatrix} Hz$$

Note that none of the modes are overdamped, and that the lowest frequencies have damping ratios greater or equal to 0.2.

Through the parameter study on the element (1,1) of the damping matrix $\mathbf{C}_C$ shown in Figure 9, generated after a value for element (3,3) was chosen to be 126 following the guidelines of Figure 5, and maintaining the value for element (2,2), we were able to modulate or damp out the response even further. After the second parameter study a new damping matrix was chosen as $\mathbf{C}_{C2}$ =diag(219; 125.4; 126; 0.6; 1; 0.1)

Figure 10 shows us the joint displacements of the system when using the $\mathbf{C}_{C2}$ damping matrix. As it

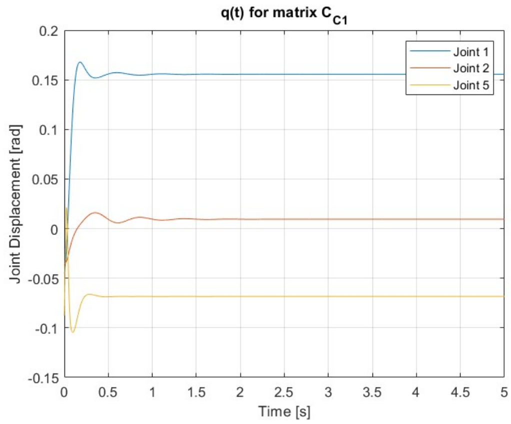



Figure 7: Joint Displacements for the $\mathbf{C}_{C1}$ damping matrix after a parameter study. Plots of the most significant joints.

can be seen, most of the transients have disappeared as expected from theory.

It is also worth pointing out that due to the redundancy of the system, there are zero-potential-energy (ZP) motions that belong to the null space of the joint stiffness matrix $\mathbf{K}$, and move or make the robot go into a new equilibrium configuration, different from the starting one, as it can also be seen in Figures 3 and 7, and cannot be removed unless the redundancy is taken care of for the task.

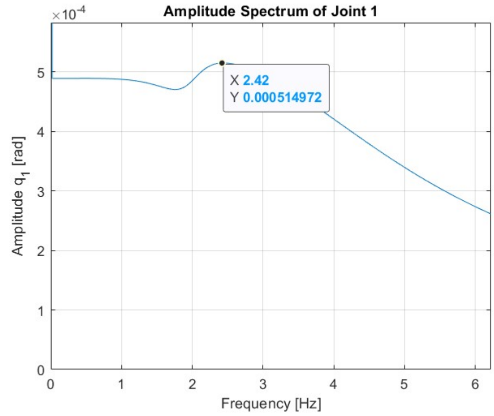
Once the responses have been damped out, the results from the FFT tool are not as visible or notorious as for the case for underdamped systems, which makes sense according to theory.

In a more practical or experimental sense, if we compare the numerical damping values of $\mathbf{C}_{C1}$ and $\mathbf{C}_{C2}$, they are significantly different, but if we compare the responses, the differences are small. This may incur in an unnecessary larger control effort when using $\mathbf{C}_{C2}$ that would probably generate out of range torques, strangely slow motions, or instabilities. Our theoretically sound methodology can help us avoid all these situations.

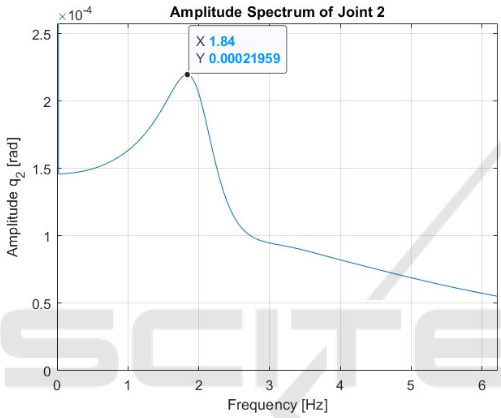
## 5 CONCLUSIONS

Through simulations on a 7-degree-of-freedom redundant Panda robotic manipulator, we have demonstrated the effectiveness of our approach and achieved the desired outcomes fulfilling the objectives of accurately identifying the natural frequencies and optimizing the system's response behavior through appropriate damping modification.

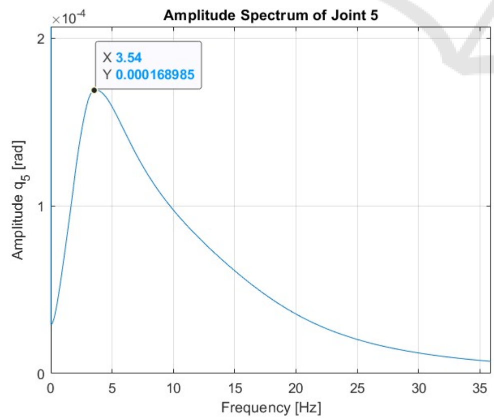
By strategically introducing damping at specific frequencies, we were able to modify the system's response behavior and enhance its stability and perfor-




(a) FFT Joint 1.




(b) FFT Joint 2.




(c) FFT Joint 3.

Figure 8: FFT results for the $\mathbf{C}_{C1}$ damping matrix after a parameter study. Plots of the most significant joints.

mance. Our methodology considers the relative magnitudes of the peaks in the frequency spectrum to identify the frequencies that require additional damping.

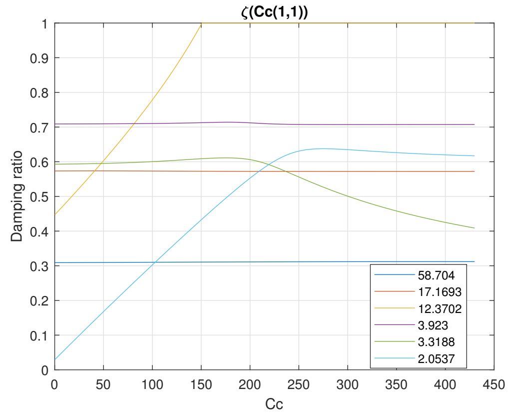



Figure 9: Second parameter study of element (2,2) of the damping matrix showing the effect on each mode.

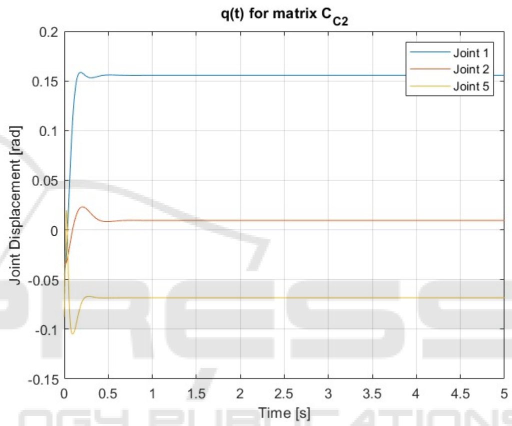
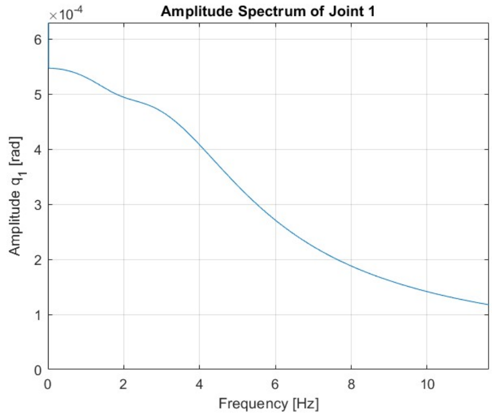


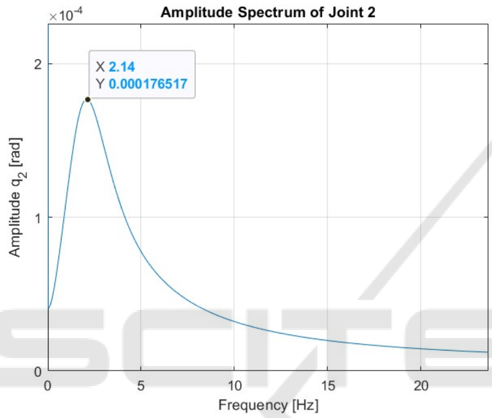

Figure 10: Joint Displacements for the $\mathbf{C}_{C2}$ damping matrix after a second parameter study. Plots of the most significant joints.

The significance of our methodology lies in its practicality, efficiency, and adaptability. Using computational tools and numerical simulations eliminates the need for extensive experimental modal analysis, saving time and resources. Moreover, the approach can be applied to a wide range of redundant robotic systems, offering flexibility in its implementation.

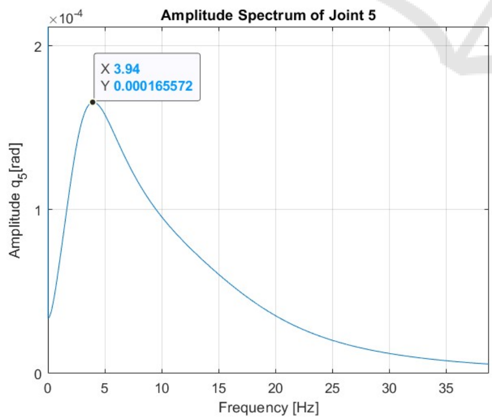
This approach can significantly contribute to the field of robotics by enabling the optimization of dynamic performance in complex systems. Further research and experimentation can build upon this methodology to address more sophisticated robotic applications and propel advancements in the field.




(a) FFT Joint 1.




(b) FFT Joint 2.




(c) FFT Joint 3.

Figure 11: FFT results for the $\mathbf{C}_{C2}$ damping matrix after a second parameter study. Plots of the most significant joints.

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