Trajectory Planning for Multiple Vehicles Using Motion Primitives: A Moving Horizon Approach Under Uncertainty

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Abstract: Planning motion in cluttered and uncertain environments for autonomous systems remains a daunting challenge, especially when prioritizing safety and efficiency. This paper introduces an innovative method of melding motion primitives with a moving horizon strategy, drawing on the principles of model predictive control. Using motion primitives, we achieve simplified, high-level depictions of vehicle movement using various linear time-invariant models for each mode. This significantly cuts computational complexity in the subsequent planning stages. We integrate constraint backoff based on system uncertainties to ensure that generated trajectories are robust, collision-free, and adhere to all necessary constraints. This comprehensive framework produces optimal, safe trajectories that cater to environmental uncertainties and are suitable for real-time applications. Our simulation outcomes robustly highlight the strengths and distinctiveness of the proposed approach.

1 INTRODUCTION

Motion planning for autonomous vehicles operating in cluttered environments is complex and demanding. It involves finding an optimal trajectory for the vehicle to safely navigate from its current location to a goal point, considering the presence of static and moving obstacles, the physical limitations of the vehicle, uncertainties, and disturbance while being real-time capable (LaValle, 2006). The problem becomes even more challenging when multiple vehicles are involved, as one vehicle's motion can impact others' motion, and guaranteed collision avoidance becomes a critical concern. Therefore, there is a significant need to develop robust and efficient motion planning algorithms that can handle the uncertainties and complexities of real-world scenarios, such as fleet administration, search and rescue operations, and agriculture operations in harsh conditions (Wong et al., 2017).

Several approaches have been proposed for solving the motion planning problem for autonomous vehicles, e.g., graph-based approaches (Stahl et al., 2019), probabilistic planning approaches (Wang et al., 2022), and optimization-based approaches (Kulathunga and Klimchik, 2022), depend on the specific requirements and constraints of the problem. For a comprehensive overview, we refer to (Laumond et al., 1998; LaValle, 2006; Goerzen et al., 2010; Latombe, 2012). While these approaches offer valuable insights, they often have limitations regarding scalability, robustness, and adaptability to changing environments. When selecting an approach, it is essential to consider the trade-offs between accuracy and computational complexity (LaValle, 2006).

Recently, model predictive control (MPC) has emerged as a powerful motion planning framework involving multiple autonomous vehicles (Zuo et al., 2020; Liu et al., 2017). The basic concept of MPC is based on the repeated solution of optimization problems taking the system dynamics and obstacles via constraints into account. Only the first path segment is applied, and then the process is repeated until the goal is reached (Findeisen and Allgöwer, 2002; Mattingley et al., 2011; Rawlings et al., 2017a). MPC has several advantages over traditional methods, including handling complex constraints, incorporating multiple objectives, and adapting to changing environments (Rawlings et al., 2017b).

However, solving the resulting optimization planning problem is often computationally challenging (Reiter and Reiter, 2020). Therefore, often different

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approximations and simplifications are used. However, generating a safe trajectory using a simplified vehicle model is affected by uncertainty in the vehicle's dynamics and the environment. It may lead to suboptimal or unsafe control sequences (Borrelli et al., 2017). Various robust MPC approaches, such as stochastic, robust, set-theoretic, and adaptive approaches, have been proposed to overcome these drawbacks. However, these methods can be computationally expensive, challenging to implement, or prone to instability (Magdici and Althoff, 2016; Rawlings et al., 2017b).

Motivated by robust predictive control, we propose an approach that combines contract-based model predictive control (CB-MPC) (Ibrahim et al., 2020; Kögel et al., 2022) with Point-to-Point motion primitives (PTP) to handle uncertainty in motion planning while improving performance and reducing computational time complexity. The combination results in a mixed-integer linear programming optimization problem, which generates safe, optimal, and smooth reference trajectories while avoiding obstacles even in uncertain conditions. The simulation results show the method's effectiveness in maximizing vehicle performance while handling the impact of uncertainty.

The remainder of the paper is structured as follows. Section 2 presents the problem formulation for a single vehicle and its extension to multiple vehicles. Section 3 illustrates the planning strategy and how to integrate motion primitives in the hybrid robust receding horizon planning formulation. Section 4 presents the mathematical structure of the proposed point-to-point motion primitives integration with a contract-based MPC framework for real-time applications. The simulations are presented in Section 5 before concluding the paper with final remarks in Section 6.

2 PROBLEM FORMULATION

Motion planning for autonomous vehicles in cluttered environments aims to find a safe and optimal trajectory to reach its goal while considering various constraints. Physical limitations of the vehicle, such as maximum speed and acceleration, must be considered to ensure safety, stability, and performance. The presence of static and moving obstacles in the environment limits the motion options and requires the motion planner to continuously update its plan to prevent collisions. Due to the unpredictable motion of obstacles, the uncertainty in the environment requires the solution to be robust. Real-time motion planning presents a complex optimization problem as it requires balancing conflicting objectives, such as task objectives, collision avoidance, and obstacle avoidance while considering the uncertainty in the environment. This requires the solution to be computed quickly enough to meet the real-time demands of the vehicle's motion. Furthermore, a new layer of complexity arises when extending motion planning algorithms to scenarios involving multiple vehicles navigating cluttered environments. The presence of multiple vehicles increases the complexity of the optimization problem. It requires the motion planner to consider not only the avoidance of static obstacles but also dynamic obstacles in the form of other vehicles. The motion planning algorithm must handle these complexities while satisfying the safety and stability requirements.

3 PLANNING STRATEGY

This paper addresses the challenge of devising a feasible and optimal trajectory for one or more autonomous vehicles (denoted as V) to navigate from an initial point (x_{start}) to a destination (x_{goal}) in a cluttered environment. This task involves considering uncertainties and adhering to various constraints, including dynamics, kinematics, and collision avoidance, while simultaneously optimizing objectives like energy consumption and minimizing the distance to the goal point at each time step, as illustrated in Fig. 2.

To tackle this complex problem, we propose a robust motion planning approach that leverages a Contract-based Model Predictive Control (CB-MPC) framework and integrates it with the Point-to-Point (PTP) primitives approach, as depicted in Fig. 1. The resulting optimization problem is formulated as a Mixed-Integer Linear Programming (MILP) problem. MILP offers several advantages, notably its ability to handle continuous and discrete variables. This is essential for addressing challenges where discrete decisions, such as mode changes due to environmental alterations, task objectives, or the vehicle's capabilities, are required (Schouwenaars et al., 2004; Schouwenaars, 2006).

To facilitate the understanding of the proposed approach, we will start by considering its application to a single vehicle and present the extension to multiple vehicles in Section 4.

3.1 Contract-Based MPC Approach

In (Ibrahim et al., 2020), a contract-based Model Predictive Control (MPC) framework is introduced to handle deterministic bounded uncertainty sets by

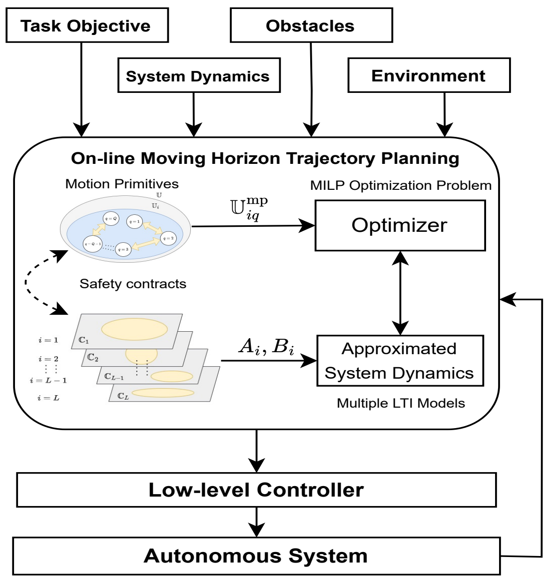

Figure 1: Motion primitives integration with safety contract receding horizon planner structure for online trajectory generation of autonomous vehicles under uncertainty.

defining feasible states and controlling input sets. This MPC framework predicts future vehicle motion and optimizes control inputs to achieve desired performance.

The contract-based approach formulates the underlying optimization problem as a Mixed-Integer Linear Programming (MILP) problem. This MILP problem considers various constraints and accounts for environmental uncertainty while capturing vehicle capabilities through contract sets and precision-uncertainty specifications. These contracts ensure the satisfaction of various properties, including constraint compliance.

By employing the contract-based approach, a clear separation is achieved between the planning phase and the execution of the navigation objective. This separation enhances the overall efficiency and reliability of the motion planning process (Ibrahim et al., 2020; Kögel et al., 2022).

To illustrate this contract-based approach, let's consider that linear time-invariant models represent the dynamics of different modes for an autonomous vehicle.

$$\forall i \in \{1,\dots,L\}$$
$$x(k+1) = A_i x(k) + B_i u(k) + \omega(k), \quad (1a)$$
$$y(k) = C_i x(k), \quad (1b)$$
$$x(k) \in \mathbb{X}_i,\ u(k) \in \mathbb{U}_i, \quad (1c)$$
$$\omega(k) \in \mathbb{W}_i. \quad (1d)$$

Here L is the number of modes, $x(k) \in \mathbb{R}^n, u(k) \in \mathbb{R}^m$ are the vehicle's states and inputs, while $y(k) \in \mathbb{R}^p$ are the vehicle's outputs (positions). $A_i, B_i, C_i, \forall i \in \{1,\dots,L\}$ are matrices of corresponding dimensions for each mode i. Note that the dynamics are subject to bounded uncertainties $\omega(k)$ given by:

$$\forall i \in \{1,\dots,L\}$$
$$\text{If } x(k) \in \mathbb{X}_i \subseteq \mathbb{X}, \text{and } u(k) \in \mathbb{U}_i \subseteq \mathbb{U},$$
$$\text{Then } w(k) \in \mathbb{W}_i \subseteq \mathbb{W}.$$

We assume that $\mathbb{X}_i$, $\mathbb{U}_i$, and $\mathbb{W}_i$ are convex compact polytopes. The modes and uncertainty sets allow capturing mode-dependent disturbances. For example, if the vehicle moves slowly, the disturbance might be smaller; see Fig. 2. Fast operation leads to a larger disturbance set, e.g., capture unmodeled system dynamics. Note that the sets $\mathbb{X}_i$ and/or $\mathbb{U}_i$ can overlap

$$\forall i \neq j, \mathbb{X}_i \cap \mathbb{X}_j \neq \emptyset, \mathbb{U}_i \cap \mathbb{U}_j \neq \emptyset.$$

Obstacle Avoidance with Operation Mode Dependent Obstacle Enlargements:
The vehicles should navigate safely despite the disturbances and model uncertainties; see Fig. 2, i.e., the vehicle's position $y(k)$ should be outside the obstacle set; see Fig. 3

$$y(k) \notin \mathbb{O}_i$$

To avoid obstacle collisions, we use safety margins δ_i according to the "active" operation mode $\mathbb{W}_i$, see Fig. 3. This formulation provides a modular and easily constructed simple low-level velocity controller to drive the vehicle to its final destination while efficiently satisfying the constraints and rejecting external disturbance.

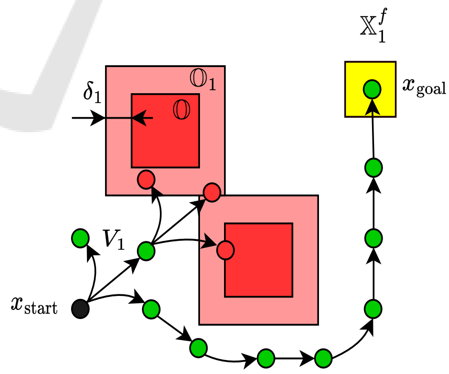

Figure 2: Conservative obstacle enlargement without a feasible solution or a very long solution path due to short horizon.

3.2 Motion Primitives to Limit Planning Complexity

PTP motion primitives are fundamental in autonomous motion planning, facilitating real-time,

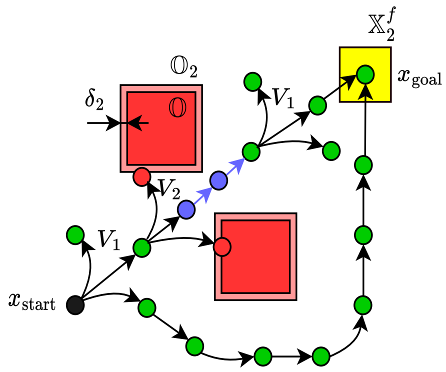

Figure 3: Smaller obstacle enlargement considering the slow motion, which allows the planner to find a feasible solution by switching between different velocity modes.

smooth trajectory generation (LaValle, 2006; Siciliano et al., 2008; Mahulea et al., 2020). They compile a pre-computed table of feasible control inputs, adapting to the vehicle's mode of operation for swift execution and responsive path adjustments. This reduces computational complexity (Frazzoli, 2001). PTP motion primitives ensure continuous and seamless vehicle motion (Bottasso et al., 2008; Pivtoraiko and Kelly, 2011). These trajectories are generated by simulating the vehicle's dynamics with various control inputs, allowing simultaneous consideration of multiple objectives such as goal attainment, energy efficiency, and collision avoidance. The approach remains flexible and modular, requiring no alterations to the generalized motion primitives set, while infeasible primitives can be excluded during optimization (Mahulea et al., 2020) (Fig. 2).

Control inputs at each time step, $u(k) \in \mathbb{U}^{\text{mp}}$, are expressed as a linear combination of motion primitives, P, using a binary selection variable, b_i. The optimization problem selects the most suitable motion primitive while respecting constraints and objectives, where i is the index of the motion primitive:

$$u(k) = \sum_{j=1}^{J} b_j P \tag{3}$$

Where J is the number of motion primitives, the binary variable, b_i, is defined such that only one motion primitive can be selected at each time step, i.e., $\sum_{j=1}^{n} b_j = 1$ and $b_j \in 0,1$. The optimization problem can be formulated to select the most suitable motion primitive at each time step while considering the constraints and objectives of the motion planning problem.

3.3 Combining Motion Primitives and Contract-Based MPC

The proposed method provides a more modular and efficient solution by formulating the motion planning problem as a MILP optimization and creating a contract between the high-level planner and low-level controller. CB-MPC is a framework that uses contracts, in other cases in the form of sets or constraints, to ensure system safety while still optimizing the desired performance objective. The combination of PTP with CB-MPC provides a robust solution to the motion planning problem, as the contracts can be updated in real-time based on the system's current state and the environment's status.

4 ROBUST MILP FORMULATION

To obtain a computationally feasible optimization problem, we consider simplified, discretized vehicle and moving obstacle dynamics. By fusing motion primitives and contracts, we obtain the following optimization problem:

$$\begin{aligned}
\underset{\{x\},\{u\},i}{\text{minimize}} \quad & J\big(x_p(k), u_p(k)\big) && \text{(4a)}\\
\text{subject to} \quad & \forall p \in \{1,\dots,V\}, \\
& \forall k \in \{0,\dots,N-1\}, \forall i \in \{1,\dots,L\}, \\
& x_p(k+1) = A_p^i x_p(k) + B_p^i u_p(k) + \omega_p^i(k), && \text{(4b)}\\
& y_p(k) = C_p^i x(k), && \text{(4c)}\\
& x_p(k) \in \mathbb{X}_p^i \subseteq \mathbb{X}_{\text{free},p}^i \subset \mathbb{X}_p, && \text{(4d)}\\
& u_p(k) \in \mathbb{U}_p^{\text{mp},i} \in \mathbb{U}_p^i \subseteq \mathbb{U}_p, && \text{(4e)}\\
& y_p(k) \notin \mathbb{O}_p^i \to y_p(k) \in \mathbb{Y}_{\text{free},p}, && \text{(4f)}\\
& \mathbb{Y}_{\text{free},p} = \mathbb{Y}_p \setminus \mathbb{Y}_{\text{obs},p}, && \text{(4g)}\\
& \omega_p(k) \in \mathbb{W}_p^i \subseteq \mathbb{W}_p, && \text{(4h)}\\
& x_p(N) \in \mathbb{X}_p^{f,i} \subseteq \mathbb{X}_{\text{free},p}, && \text{(4i)}
\end{aligned}$$

where V is the number of the vehicles, N is the planning horizon, i represents the operation mode, and L is the modes number. A_p^i, B_p^i, C_p^i are the model of the vehicle p at the mode i. The optimal solution $x^*(k)$ minimizes the path cost J, which we assume to be given by:

$$J(x,u) = \sum_{p=0}^{V} \big(||x_{\text{goal},p} - x_p(N)||_\infty + \sum_{k=0}^{N-1} ||u_p(k)||_\infty\big) \tag{5}$$

Here $||p_v(k)||_\infty$ penalize the control input, while the terminal cost penalizes the distance to the goal point x_{goal} at the end of the planning horizon for each vehicle.

4.1 Obstacle Avoidance

To simplify the problem's complexity, we consider that the vehicles operate in a two-dimensional space and that the obstacles are rectangular. The position of the obstacle is defined by its lower left and upper right corner points, denoted by (x_{min}, y_{min}) and (x_{max}, y_{max}), respectively.

At each time step k, the position of each vehicle must be located outside of the obstacle. The moving object's size is considered, and the obstacles are accordingly enlarged, reducing each vehicle to a moving point. This approach ensures safe and efficient navigation for all vehicles in the same workspace while avoiding collisions with the same obstacle. This can be formulated as:

$$\begin{aligned}
&\forall p \in \{1,\dots,V\}, \forall k \in \{0,\dots,N-1\}: \\
&\qquad x_{p,k} \leq x_{\text{min}} \\
&\text{or} \quad x_{p,k} \geq x_{\text{max}} \\
&\text{or} \quad y_{p,k} \leq y_{\text{min}} \\
&\text{or} \quad y_{p,k} \geq y_{\text{max}}
\end{aligned}$$

4.2 Collision Avoidance

When multiple vehicles are travelling to different destinations, avoiding collisions between them can be approached like avoiding stationary obstacles. At each time step, a minimum distance must be maintained between each pair of vehicles. For example, considering the position and the velocity for any pair of vehicles p and q at the k^{th} time step as $(x_{p,k}, y_{p,k})$ and $(x_{q,k}, y_{q,k})$ respectively. Then the safety distance between them can be denoted by d_{safety} for each axis as the following:

$$\begin{aligned}
\forall k \in \{0,\dots,N-1\} : \forall p,q \mid q > p : \\
|x_{p,k} - x_{q,k}| \geq d_{\text{safety}} \\
|y_{p,k} - y_{q,k}| \geq d_{\text{safety}}
\end{aligned}$$

The resulting non-convex constraint can be approximated by convex polygons using the Big-M method, introducing extra binary variables in the optimization problem. For further details, we refer to (Ibrahim et al., 2019; Schouwenaars, 2006; Frazzoli, 2001).

5 SIMULATION RESULTS

To demonstrate the efficacy of the proposed combination of point-to-point motion primitives and a robust contract-based receding horizon planning approach, we consider a simulation scenario in which vehicles operate in an environment filled with obstacles to reach their goal optimally. The vehicle model has four states, $x = \begin{bmatrix} p_x & p_y & v_x & v_y \end{bmatrix}^\top$, where p_x and p_y are positions, and v_x, v_y are the velocities. $u(k) = \begin{bmatrix} a_x & a_y \end{bmatrix}^\top$ are the inputs, i.e. the accelerations in x and y direction.

We consider a group of four homogeneous vehicles, $V = 4$, each with its start and goal points, operating simultaneously in the cluttered environment. Each vehicle can operate in two modes, each defined by a different set of state constraints based on different velocity bounds. Mode 1 has a velocity bound from 0 to 1 [m/s], while Mode 2 has a velocity bound from 1 to V_{max} [m/s]. Based on the current velocity, each vehicle is subject to bounded uncertainty, and therefore, higher safety margins are required for operating in mode 2 compared to mode 1. Additionally, a safety margin is added to prevent the collision between any two pairs of vehicles during operation, $d_{\text{safety}} = 1$ [m/s]. A finite set of motion primitives in the input space $\mathcal{U}^{\text{mp}} = [-1, 1]$ with sample step size $\Delta u = 0.1$ is used.

The resulting optimization problems are formulated via YALMIP (Löfberg, 2004) and solved via Gurobi (Optimization, 2014). To illustrate the effectiveness of the proposed approach, we consider two different scenarios. First, we assume that we have four vehicles in the corners of the square area, and their final goal is to reach the opposite corner. In this

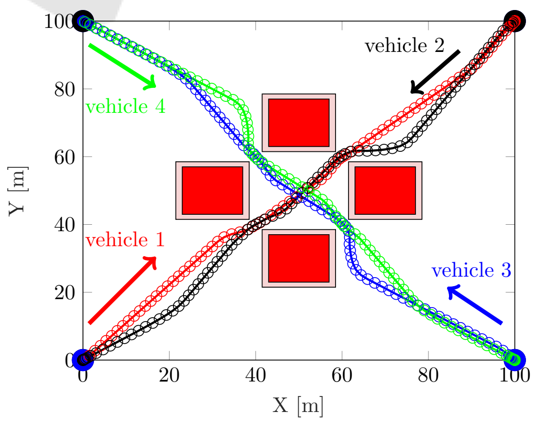

Figure 4: Scenario 1: Trajectory Generation of the Autonomous Vehicle without motion primitives with symmetric paths.

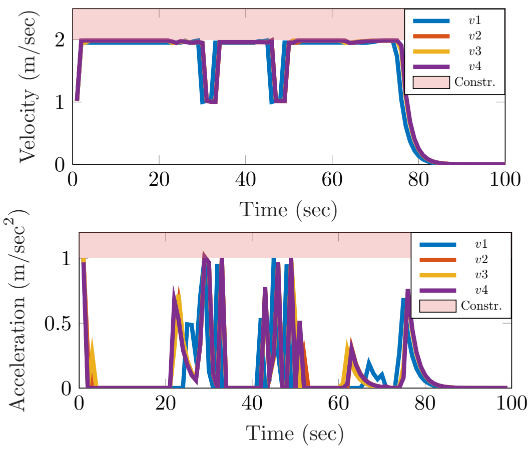

Figure 5: Scenario 1: Velocity and acceleration profile without motion primitives with symmetric path.

scenario, each vehicle is expected to move the same distance while satisfying obstacle and collision avoidance constraints, optimizing energy consumption, and minimizing the distance to the goal points at each time step. The second scenario selects arbitrary random start and goal points in a high-density, cluttered environment.

5.1 Results Discussions

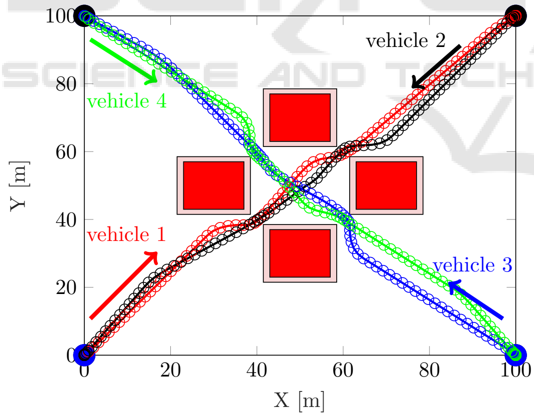

Figure 6: Scenario 2: with motion primitives with symmetric path.

The simulation results demonstrate the capability of the proposed approach in finding an optimal and collision-free trajectory for the vehicle to reach its goal.

As shown in Fig. 4,6,9, the planner can adapt to different modes of operation, including switching to a low-velocity mode, to handle uncertainties effectively. However, it is noted that if the vehicle operates at maximum velocity, the planner may not be able to find a feasible solution because the obstacles are enlarged with a large safety margin. Hence, the proposed structure provides additional flexibility and optimizes the vehicle's potential to achieve the task objectives with satisfying constraints, as seen in Fig. 5,7, and 10.

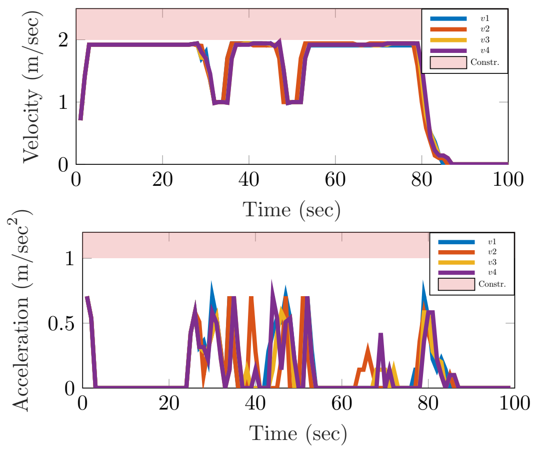

Figure 7: Scenario 2: Velocity and acceleration profile with motion primitives with symmetric path.

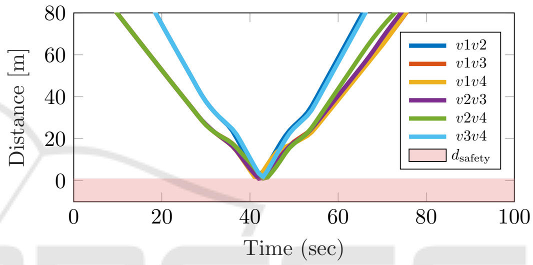

Figure 8: Scenario 2: The relative distance between any two pairs of vehicles during operation.

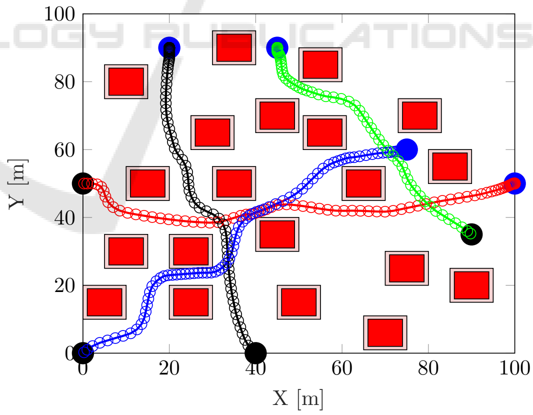

Figure 9: Scenario 3: Autonomous vehicles trajectory planning with motion primitives for arbitrary start and goal points.

Moreover, collision avoidance was effectively maintained throughout the operation, as evidenced by Fig 8 and 11, where the distance between any pair

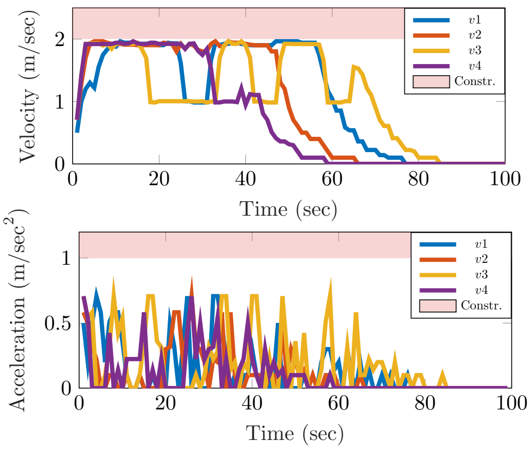

Figure 10: Scenario 3: Velocity and acceleration profile with motion primitives.

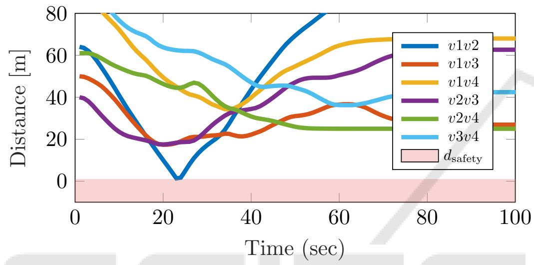

Figure 11: Scenario 3: The relative distance between any two pairs of vehicles during operation.

of vehicles remained greater than the safety margin of the vehicle. Another aspect to consider is the impact of Point-to-Point motion primitives on reducing the computational time complexity of the problem, as depicted in Figures 5 and 7. The proposed approach generates smoother paths with low control effort compared to other approaches without motion primitives.

6 CONCLUSION

We propose a moving horizon approach for multi-vehicle motion planning in complex and uncertain environments. Our method combines motion primitives with principles from model predictive control (MPC). By employing motion primitives, we create simplified representations of vehicle movement, each associated with a specific linear time-invariant model, significantly reducing computational complexity during planning. We also integrate a constraint back-off mechanism that accounts for model and external uncertainties, ensuring the generated trajectories are robust, collision-free, and adhere to critical constraints. Simulation results validate the method's effectiveness, showcasing its ability to generate safe and optimal trajectories for multiple vehicles in real time.

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