Dynamic Periodic Event-Triggered Control for Linear Systems Based on Partial State Information

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Abstract: We are interested in the design of satabilizing event-driven controllers for linear time-invariant systems. We assume that the plant state is partially known and the feedback signal is sent to the controller at discrete-time instants via a digital channel and we synthesize an event-triggered controller based solely on the available plant measurement. The event-triggering law that we construct is novel and only verified at periodic time instants, i.e., periodic event-triggering mechanism, which is more adapted to practical implementation. The proposed approach ensures a global asymptotic stability property for the closed-loop system under mild conditions. The overall model is developed as a hybrid dynamical system to truly describe the mixed continuous-time and discrete-time dynamics. The stability is studied using appropriate Lyapunov functions. The efficiency of the technique is illustrated on a numerical example.

1 INTRODUCTION

Event-triggered control (ETC) is an implementation technique in which the transmission instants of the feedback measurements are generated by a state-dependent rule instead of the traditional periodic sampling approach. This allows for more efficient utilization of the limited bandwidth of the shared communication channel in different domains of applications such as networked control systems (Zhang et al., 2017), sensors networks (Alajmi et al., 2022), cyber physical systems (Lu and Yang, 2020), multi-agent systems (Samy et al., 2022), (Filho et al., 2023) and distributed control systems (Ge et al., 2017).

A significant amount of research work on ETC is based on the continuous verification of the triggering condition to decide the next transmission instance, e.g., (Tabuada, 2007), (Abdelrahim et al., 2013), (Wu et al., 2022), (Yang et al., 2023). However, a major challenge in this type of continuous ETC is to prevent the accumulation of transmission instants, i.e., Zeno phenomenon (Borgers and Heemels, 2014). Alternatively, periodic event-triggered control (PETC) has been proposed such that the triggering rule is only checked at periodic time instants, which is more adapted to practical implementation and automatically rules out Zeno behaviour (Heemels et al., 2013b), (Postoyan et al., 2013), (Li et al., 2023), (Wang et al., 2020), (Sun and Zeng, 2022), (Yu et al., 2020), (Liu and Hao, 2015), (Borgers et al., 2018), (Abdelrahim et al., 2015).

In this paper, we consider the problem of periodic event-triggered control of output feedback linear time-invariant (LTI) systems. We assume that only an output of the plant is known and we construct an appropriate periodic ETC to decide whether to release a transmission at the next periodic instant. The proposed periodic ETC is novel and equipped by a dynamic variable, i.e., dynamic periodic ETC, to further reduce the amount of transmissions. Moreover, the periodic sampling interval is designed based on the approach of (Carnevale et al., 2007) to derive the maximally allowable transmission interval (MATI) for the case of time-triggered control. The proposed approach is designed by emulation where we first ignore the effect of network and stabilize the plant in continuous-time. Then, we consider the sampling due to the network and we construct a periodic ETC mechanism such that the closed-loop stability is preserved. The overall system is formulated as a hybrid dynamical system to truly describe the mixed continuous-time and discrete-time dynamics of the system. Sufficient conditions are provided in terms of a linear matrix inequality (LMI) to properly identify

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the parameters of the event-triggering mechanism in a systematic manner. The stability is investigated by using appropriate Lyapunov functions. The effectiveness of the approach is demonstrated via numerical simulations.

The problem of PETC synthesis has been studied in several works of the literature, see e.g., (Antunes et al., 2012), (Heemels et al., 2011), (Fu and Jr., 2018), (Wei et al., 2023), (Sun et al., 2023), (Heemels et al., 2013a), (Li et al., 2023), (Postoyan et al., 2013), (Sun and Zeng, 2022). It is noted that the majority of previous works are adapted to the case of state feedback control, which is not feasible in many practical situations. The proposed approach in this paper is adapted to the case of output feedback control, which is more challenging than when the full state measurement is available. Moreover, the setup that we consider and the obtained stability property are different from existing techniques of the literature.

The main contribution of this paper is summarized below

- we construct a novel PETC for linear systems based on partial state information;
- the sampling period is designed as the maximally allowable transmission interval;
- the closed-loop system is modelled as a hybrid dynamical system;
- sufficient conditions are formulated in terms of an LMI condition.

The rest of the paper is organised as follows. Preliminaries are given in Section 2. The problem is formally stated in Section 3. The hybrid model is given in Section 4. We present the main results in Section 5. Numerical simulations are given in Section 6. Conclusions are provided in Section 7.

2 PRELIMINARIES

Let $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{\geqslant 0} := [0, \infty)$, $\mathbb{N} := \{0, 1, 2, \ldots\}$ and $\mathbb{N}_{\geqslant 0} := \{1, 2, \ldots\}$. Standard notation are ddopted in this paper.

We consider hybrid systems of the following form (Goebel et al., 2012; Cai and Teel, 2009)

$$\dot{x} = F(x) \quad x \in \mathcal{C}, \quad x^+ \in G(x) \quad x \in \mathcal{D}, \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $\mathcal{C}$ is the flow set, F is the flow map, $\mathcal{D}$ is the jump set and G is the jump map. Solutions to system (1) are defined on *hybrid time domains*, see (Goebel et al., 2012), (Cai and Teel, 2009) for more detail.

3 PROBLEM FORMULATION

We consider plant models with the following dynamis

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u \\ y &= C_p x_p, \end{aligned} \tag{2}$$

where $x_p \in \mathbb{R}^{n_p}$ is the plant state, $u \in \mathbb{R}^{n_u}$ is the control input, $y \in \mathbb{R}^{n_y}$ is the measured output, and A_p, B_p, C_p, E_p are matrices of appropriate dimensions. The plant is stabilized by the following dynamic controller

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c \hat{y} \\ u &= C_c x_c + D_c \hat{y} \end{aligned} \tag{3}$$

where $x_c \in \mathbb{R}^{n_c}$ is the controller state, $\hat{y} \in \mathbb{R}^{n_y}$ is the last transmitted value of y, and A_c, B_c, C_c, D_c are matrices of appropriate dimensions. The feedback law (3) is designed by emulation, that is we first stabilize the plant (2) in continuous-time assuming perfect communication, i.e., $\hat{y} = y$. Then, we take into account the sampling effects.

3.1 Implementation Scenario

The implementation scenario is shown in Figure 1. We consider the case where the controller is co-located with the plant while the sensors and the controller are communicating over a shared network. We assume that the plant state x_p is not available for measurement and only an output $y(t)$ can be transmitted to the controller.

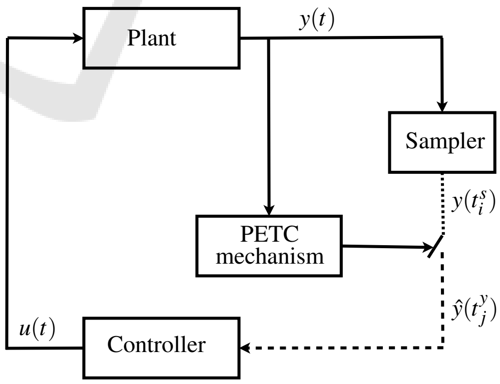

Figure 1: Periodic event-triggered output feedback control. (solid line) continuous-time; (dotted line) periodic instants; (dash line) event-triggered instants.

We consider that the output $y(t)$ is sampled at periodic sampling times $t_i^s, i \in \mathbb{N}$. Then, an event-triggering mechanism is employed to decide wether to submit the output value $y(t_i^s)$, where the time instants at which $y(t_i^s)$ is released are denoted by $t_j^y, j \in \mathbb{N}$, leading to the so-called periodic event-triggered control (PETC), and we refer by $\hat{y}(t_j^y)$ the most recent value of $y(t_i^s)$ at the controller at time $t_j^y, j \in \mathbb{N}$, see Figure 1. Hence, if we define

$$\begin{aligned} \mathcal{T}_s &= \{t_i^s\}, i \in \mathbb{N} \\ \mathcal{T}_y &= \{t_j^y\}, j \in \mathbb{N}, \end{aligned} \tag{4}$$

where $\mathcal{T}_s$ and $\mathcal{T}_y$ denote the increasing sequence of periodic time instants and transmission instants, respectively. Then, it hold that $\mathcal{T}_y \subseteq \mathcal{T}_s$.

It is important to note here that the periodic event-triggering mechanism is assumed to have access to both the actual output value, i.e., $y(t)$, and the last transmitted value $\hat{y}(t_j^k), j \in \mathbb{N}$.

The objectives of this paper include

- Synthesis of periodic sampling interval and periodic event-triggered controller by emulation;
- Derivation of hybrid dynamical model of the overall system;
- Providing sufficient conditions to ensure the closed-loop stability;
- Preventing the occurrence of Zeno behaviour.

4 HYBRID MODEL

In this section, we derive the dynamic behaviour of the closed-loop system and formulate it as a hybrid dynamical system. We define the sampling error $e_s(t) : \mathbb{R}^{n_y} \to \mathbb{R}^{n_y}$ and the network induced error $e_y(t) : \mathbb{R}^{n_y} \to \mathbb{R}^{n_y}$ between two transmission times as follows, for all $t \in [t_j^y, t_{j+1}^y)$

$$\begin{aligned} e_s(t) &:= y(t_i^s) - y(t) && \forall t \in [t_i^s, t_{i+1}^s), i \in \mathbb{N} \\ e_y(t) &:= \hat{y}(t_j^y) - y(t_i^s) && \forall t \in [t_j^y, t_{j+1}^y), i, j \in \mathbb{N}. \end{aligned} \tag{5}$$

Between two periodic sampling times $[t_i^s, t_{i+1}^s]$, the sampled output $y(t_i^s)$ is kept constants using ZOH. At each periodic sampling time $t_i^s, i \in \mathbb{N}$, the value of $y(t_i^s)$ is reset to $y(t)$. Moreover, between two transmission instants $[t_j^y, t_{j+1}^y]$, the last transmitted value of the output $y(t_j^y)$ is kept constants using ZOH and at each transmission instant $t_j^y, j \in \mathbb{N}$, $e_y(t)$ is reset to $y(t_i^s)$.

Define the total error $e(t)$ as the difference between the last transmitted value of the output $\hat{y}_q(t_j^y)$ and the current output measurement $y(t)$, that is

$$\begin{aligned} e(t) &:= \hat{y}(t_j^y) - y(t) \qquad \forall t \in [t_j^y, t_{j+1}^y) \\ &= e_s(t) + e_y(t). \end{aligned} \tag{6}$$

Then, it holds that

$$\begin{aligned} \dot{e}(t) &= -\dot{y} = -C_p \dot{x}_p && t \in [t_j^y, t_{j+1}^y) \\ e(t_j^{y^+}) &= e_s(t_j^{y^+}) + e_y(t_j^{y^+}) \\ &= 0. \end{aligned} \tag{7}$$

The last property implies that the total error $e(t)$ is reset to zero at each transmission instant $t_j^y, j \in \mathbb{N}$ since $y(t_i^s)$ is updated to $y(t)$ at each $t_j^y, j \in \mathbb{N}$.

Let $x = (x_p, x_c) \in \mathbb{R}^{n_x}$. Then, in view of (2), (3), (6), we obtain

$$\begin{aligned} \dot{x} &= \begin{bmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix} x + \begin{bmatrix} B_p D_c \\ B_c \end{bmatrix} e \\ &=: \mathcal{A}_1 x + \mathcal{B}_1 e \end{aligned} \tag{8}$$

and

$$\begin{aligned} \dot{e} &= \begin{bmatrix} -C_p(A_p + B_p D_c C_p) & -C_p B_p C_c \end{bmatrix} x + \begin{bmatrix} -C_p B_p D_c \end{bmatrix} e \\ &=: \mathcal{A}_2 x + \mathcal{B}_2 e. \end{aligned} \tag{9}$$

We define two auxiliary time variables $\tau_s, \tau_y : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ as follows

$$\begin{aligned} \dot{\tau}_s(t) &= 1 && t \in [t_i^s, t_{i+1}^s) \\ \tau_s(t_i^{s^+}) &= 0 && t \in \{t_i^s\}_{i \in \mathbb{N}} \end{aligned} \tag{10}$$

and

$$\begin{aligned} \dot{\tau}_y(t) &= 1 && t \in [t_j^y, t_{j+1}^y) \\ \tau_y(t_j^{y^+}) &= 0 && t \in \{t_j^y\}_{j \in \mathbb{N}}. \end{aligned} \tag{11}$$

The time variable τ_s will be used to describe the time between two periodic sampling instants $[t_i^s, t_{i+1}^s]$ and it is reset to zero at each periodic instance $t_i^s, i \in \mathbb{N}$. Similarly, the time variable τ_y will be used to track the time between two transmission instants $[t_j^y, t_{j+1}^y]$ and it is reset to zero at each transmission instance $t_j^y, j \in \mathbb{N}$. These two time variables τ_s and τ_y will be helpful to construct the hybrid dynamical model of the system as explained in the sequel.

In order to complete the description of the overall system, we outline below the general structure of the proposed periodic event-triggering mechanism, which will be clearly developed in the next section. We synthesize a PETC based on a dynamic variable η, which has the following dynamics, see also (Girard, 2015; Dolk et al., 2017; Postoyan et al., 2015),

$$\begin{aligned} \dot{\eta}(t) &= \Psi(y, e, \eta) && t \in [t_i^s, t_{i+1}^s) \\ \eta(t^+) &= g_s(y, e, \eta) && t \in \mathcal{T}_s \setminus \mathcal{T}_y \\ \eta(t^+) &= g_y(y, e, \eta) && t \in \mathcal{T}_y, \end{aligned} \tag{12}$$

where the functions Ψ, g_s and g_y will be specified in Section 5. Note that the functions Ψ, η_s and η_y depend only on locally available information (y, e, η) at the event-triggering mechanism. The sequence of transmission instants are generated by the following mechanism

$$t_{j+1}^y = \min\{t > t_j^y \mid t \in \mathcal{T}_s \wedge g_s(y,e,\eta) \leqslant 0\}, \quad (13)$$

where $t_0^y = 0$.

In view of (7)-(12) we obtain the following impulsive model

$$\left.\begin{aligned}\dot{x} &= \mathcal{A}_1 x + \mathcal{B}_1 e\\ \dot{e} &= \mathcal{A}_2 x + \mathcal{B}_2 e\\ \dot{\eta}(t) &= \Psi(y,e,\eta)\\ \dot{\tau}_s(t) &= 1\\ \dot{\tau}_y(t) &= 1\end{aligned}\right\} \quad t \notin \mathcal{T}_s$$
$$u = C_c x_c + D_c \hat{y}_q$$
$$y = C_p x_p \quad (14)$$
$$\left.\begin{aligned}\tau_s(t^+) &= 0\\ \eta(t^+) &= g_s(y,e,\eta)\end{aligned}\right\} \quad t \in \mathcal{T}_s \setminus \mathcal{T}_y$$
$$\left.\begin{aligned}e(t^+) &= e_q(t)\\ \eta(t^+) &= g_y(y,e,\eta)\\ \tau_s(t^+) &= 0\\ \tau_y(t^+) &= 0\end{aligned}\right\} \quad t \in \mathcal{T}_y$$

Let $\xi := (x, e, \eta, \tau_s, \tau_y) \in \mathbb{X}$ be the concatenation of the state variables, with $\mathbb{X} = \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0}$. Then, we obtain the hybrid dynamical system

$$\begin{aligned}\dot{\xi} &= \mathcal{F}(\xi) & \xi &\in \mathcal{C}_s\\ \xi^+ &\in \mathcal{G}(\xi) & \xi &\in \mathcal{D}_s,\end{aligned} \quad (15)$$

where the flow set $\mathcal{C}_s$ and the jump $\mathcal{D}_s$ are defined as

$$\begin{aligned}\mathcal{C}_s &:= \Big\{\xi \in \mathbb{X} : \tau_s \in [0,T]\Big\}\\ \mathcal{D}_s &:= \Big\{\xi \in \mathbb{X} : \tau_s = T\Big\},\end{aligned} \quad (16)$$

where $T > 0$ is the periodic sampling interval and to be designed. We also define the jump set $\mathcal{D}_y \subset \mathcal{D}_s$ to identify the transmission instants as follows

$$\mathcal{D}_y := \Big\{\xi \in \mathbb{X} : \tau_s = T \text{ and } g_s(t) \leqslant 0\Big\}. \quad (17)$$

It is evident from (16) and (17) that $mathcalD_y \subset \mathcal{D}_s$. The flow map $\mathcal{F}(\xi)$ and the jump map $\mathcal{G}(\xi)$ in (15) are given by

$$\mathcal{F}(\xi) = \begin{pmatrix}\mathcal{A}_1 x + \mathcal{B}_1 e\\ \mathcal{A}_2 x + \mathcal{B}_2 e\\ \Psi(y,e,\eta)\\ 1\\ 1\end{pmatrix} \quad (18)$$

and

$$\mathcal{G}(\xi) := \begin{cases}\Big\{G_s(\xi)\Big\} & \xi \in \mathcal{D}_s\\ \Big\{G_y(\xi)\Big\} & \xi \in \mathcal{D}_y\\ \Big\{G_s(\xi), G_y(\xi)\Big\} & \xi \in \mathcal{D}_s \cap \mathcal{D}_y\\ \emptyset & \xi \notin \mathcal{D}_s \cup \mathcal{D}_\mu\end{cases} \quad (19)$$

with

$$G_s(\xi) := \begin{pmatrix}x\\ e\\ g_s\\ 0\\ \tau_y\end{pmatrix}, \qquad G_y(\xi) := \begin{pmatrix}x\\ 0\\ g_y\\ 0\\ 0\end{pmatrix} \quad (20)$$

The system flows on $\mathcal{C}_s$ when $\tau_s \leqslant T$, i.e., between two periodic instants, otherwise the system experiences a jump. The jump map in (20) can be interpreted as follows. When $\xi \in \mathcal{D}_s \setminus \mathcal{D}_y$, only the variables τ_s and η are updated but no transmission is generated. When $\xi \in \mathcal{D}_y$, implying that the triggering condition is violated, the variables e, η, τ_s, τ_y are updated.

5 MAIN RESULT

We present here the main result. First we state the following lemma on system (15).

Lemma 1. *Consider system (15). If there exist* $\varepsilon_x, \varepsilon_y, \gamma > 0$ *and a positive definite symmetric real matrix P such that*

$$\begin{bmatrix}\mathcal{A}_1^T P + P\mathcal{A}_1 + \varepsilon_x \mathbb{I}_{n_x} + \mathcal{A}_2^T \mathcal{A}_2 + \varepsilon_y \widetilde{C}_p^T \widetilde{C}_p & P\mathcal{B}_1\\ \mathcal{B}_1^T P & -\gamma^2 \mathbb{I}_{n_e}\end{bmatrix} \leqslant 0, \quad (21)$$

where $\widetilde{C}_p := [C_p \quad 0]$, *then the Laypunov function candidate* $V(x) = x^T P x$ *satisfies, for all* $e \in \mathbb{R}^{n_e}$ *and almost all* $x \in \mathbb{R}^{n_x}$

$$\begin{aligned}\langle \nabla V(x), \mathcal{A}_1 x + \mathcal{B}_1 e\rangle \leq{}& -\varepsilon_x |x|^2 - |\mathcal{A}_2 x|^2 - \varepsilon_y |y|^2\\ &+ \gamma^2 |e|^2.\end{aligned} \quad (22)$$

■

Proof of Lemma 1.

Let $V(x) = x^T P x$. Consequently, it holds that, for all $e \in \mathbb{R}^{n_e}$ and almost all $x \in \mathbb{R}^{n_x}$

$$\begin{aligned}\langle \nabla V(x), \mathcal{A}_1 x + \mathcal{B}_1 e\rangle ={}& x^T(\mathcal{A}_1^T P + P\mathcal{A}_1)x\\ &+ x^T P\mathcal{B}_1 e + e^T \mathcal{B}_1^T P x.\end{aligned} \quad (23)$$

By post- and pre-multiplying LMI (21) respectively by the state vector (x,e) and its transpose, we obtain

$$x^T(\mathcal{A}_1^T P + P\mathcal{A}_1)x + x^T P\mathcal{B}_1 e + e^T \mathcal{B}_1^T P x \leq -\varepsilon_x x^T x \\ -x^T \mathcal{A}_2^T \mathcal{A}_2 x - \varepsilon_y x^T \widetilde{C}_p^T \widetilde{C}_p x + \gamma^2 e^T e \tag{24}$$

which implies

$$\begin{aligned} x^T(\mathcal{A}_1^T P + P\mathcal{A}_1)x + x^T P\mathcal{B}_1 e + e^T \mathcal{B}_1^T P x \leq \\ -\varepsilon_x |x|^2 - |\mathcal{A}_2 x|^2 - \varepsilon_y |\widetilde{C}_p x|^2 + \gamma^2 |e|^2 \\ = -\varepsilon_x |x|^2 - |\mathcal{A}_2 x|^2 - \varepsilon_y |y|^2 + \gamma^2 |e|^2 \end{aligned} \tag{25}$$

and the conclusion of Lemma 1 holds. □

Lemma 1 establishes an $\mathcal{L}_2$-gain stability property for the system $\dot{x} = \mathcal{A}_1 x + \mathcal{B}_1 e$ from $|e|$ to $(|\mathcal{A}_2 x|, |y|)$, see also e.g. (Carnevale et al., 2007; **?**; Dolk et al., 2017).

5.1 Event-Triggering Mechanism

We define $W(e) := |e|$, then n view of (15), it holds that for all $x \in \mathbb{R}^{n_x}$ and almost all $e \in \mathbb{R}^{n_e}$

$$\langle \nabla W(e), \mathcal{A}_2 x + \mathcal{B}_2 e \rangle \leqslant |\mathcal{A}_2 x| + L|e|, \tag{26}$$

where $L := |\mathcal{B}_2|$.

The dynamics of the triggering function η in (12) is defined by the functions Ψ, η_s and η_y, which are given by

$$\begin{aligned} \Psi(y,e,\eta) &:= \varepsilon_y |y|^2 - \vartheta\eta \\ g_s(y,e,\eta) &:= \gamma(\lambda - \tfrac{1}{\lambda})|e|^2 + \eta \\ g_y(y,e,\eta) &:= \gamma\lambda|e|^2 + \eta \end{aligned} \tag{27}$$

where $\lambda \in (0,1)$, $\tilde{\lambda} \in [\lambda, \lambda^{-1})$, $\tilde{\gamma} := \gamma^2 + \gamma^2\tilde{\lambda}^2 + 2\gamma\tilde{\lambda}\tilde{L}$ with $\tilde{L} := L + \nu$ for any $\nu > 0$ and $L = |\mathcal{B}_2|$, and the constant γ comes from Lemma 1. The sampling period T is designed as the maximally allowable transmission interval (MATI) of time-triggered systems (Carnevale et al., 2007), which leads to

$$T(\lambda,\tilde{\lambda},\gamma,\tilde{L}) := \begin{cases} \frac{1}{\tilde{L}r}\arctan\left(\frac{r(1-\lambda\tilde{\lambda})}{\frac{\gamma}{\tilde{L}}(\lambda+\tilde{\lambda})+1+\lambda\tilde{\lambda}}\right) & \gamma > \tilde{L} \\ \frac{1}{\tilde{L}}\frac{1-\lambda\tilde{\lambda}}{\lambda\tilde{\lambda}+\lambda+\tilde{\lambda}+1} & \gamma = \tilde{L} \\ \frac{1}{\tilde{L}r}\operatorname{arctanh}\left(\frac{r(1-\lambda\tilde{\lambda})}{\frac{\gamma}{\tilde{L}}(\lambda+\tilde{\lambda})+1+\lambda\tilde{\lambda}}\right) & \gamma < \tilde{L} \end{cases} \tag{28}$$

with $r := \sqrt{\left|(\frac{\gamma}{\tilde{L}})^2 - 1\right|}$. Note that when $\tilde{\lambda} = \lambda$ in (28), we recover the MATI bound of time-triggered controllers in (Carnevale et al., 2007). By designing the sampling period T as the MATI bound, we opt to further reduce the amount of the transmissions by using the PETC mechanism.

Remark 1. *It is important to note that in view of (27), we have that* $\dot{\eta}(t) = \varepsilon_y|y|^2 - \vartheta\eta \geqslant -\vartheta\eta$. *Moreover, since* $\eta(t)$ *is reset to* $g_s(y,e,\eta)$ *when* $g_s(y,e,\eta) > 0$ *and* $\eta(t)$ *is reset to* $g_y(y,e,\eta)$*, which is strictly positive, when* $g_s(y,e,\eta) \leqslant 0$. *Consequently, by using the comparison principle, it hold that* $\eta(t) \geqslant 0$ *for all* $t \in \mathbb{R}_{\geqslant 0}$. *This property is crucial in establishing the stability of the closed-loop system as will be shown later.*

5.2 Stability Result

We obtain the following result.

Theorem 1. *Consider system (15) with the flow and the jump sets as in (16). Suppose that the LMI (21) in Lemma 1 is satisfied. Then, there exists a* $\mathcal{KL}$ *function* β *such that any solution* $\xi(t,j) \in \mathbb{X}$ *satisfies*

$$|\xi(t,j)| \leqslant \beta(|\xi(0,0))|, t+j). \tag{29}$$

■

The proof is omitted due to space limit. Theorem 1 implies that the closed-loop system (15) is globally asymptotically stable under the proposed PETC.

Remark 2. *It is clear from the proposed PETM (13) that there exists a tradeoff between the periodic sampling interval* T *and the generated amount of transmissions. That is, when the value of* T *is enlarged, the generated number of transmissions will be increased and vice versa. This tradeoff can be adjusted by the user to satisfy desirable performance of the PETC.*

6 ILLUSTRATIVE EXAMPLE

Consider the following LTI control system

$$\begin{aligned} \dot{x}_p &= \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u, \quad y = x_{p_1} \\ \dot{x}_c &= \begin{bmatrix} 0 & -2 \\ 0 & -3 \end{bmatrix} x_c + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{y}_q, \quad u = \begin{bmatrix} -1 & -2 \end{bmatrix} x_c \end{aligned} \tag{30}$$

We develop the hybrid model (15) as described in Section 4. We check the required conditions of Lemma 1 and found that the LMI condition (21) is feasible and we obtain the following values $\varepsilon_y = 0.7861$, $L = 0$, $\gamma = 3.7634$. By setting $\lambda = 0.5$, $\tilde{\lambda} = 0.6$ and $\nu = 0.01$ and substituting in (28), we get $T = 0.1204$ and $\tilde{\gamma} = 23.778$. Finally, we pick $\vartheta = 0.01$ and thus all parameters of the PETM (27) are set.

We examine the approach on MATLAB simulation with the initial conditions $x_p(0,0) = (-20, 20)$,

$x_c(0,0) = (10,-10)$, $e(0,0) = 0$, $\eta(0,0) = 0$, $\tau_s(0,0) = 0$, $\tau_y(0,0) = 0$, $\mu(0,0) = 0.35$, consequently, the output magnitude $|y(0,0)|$ initially is within the range $(\ell_{\text{in}}\mu(0,0), \ell_{\text{out}}\mu(0,0))$.

By running the simulation for 40 seconds, the obtained minimum and average inter-transmission times were found to be $\tau_{\min} = 0.1295$ and $\tau_{\text{avg}} = 0.4298$, respectively. As expected, the minimum inter-transmission time $\tau_{\min}$ is typically equal to the periodic sampling interval T, however, the average inter-transmission time τ_{avg} is larger than T, which supports our analysis and justifies the benefit of the approach compared to periodic sampling. The closed-loop response is shown in the figures below.

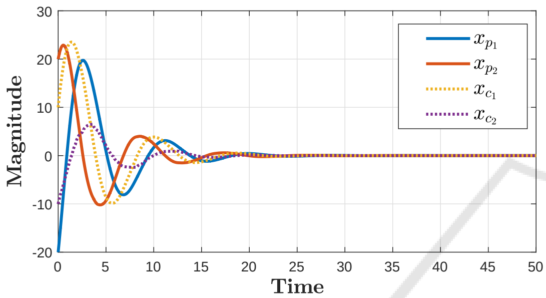

Figure 2: State trajectories of the plant and the controller.

Figure 2 shows that the plant and the controller states converge asymptotically to the origin as expected.

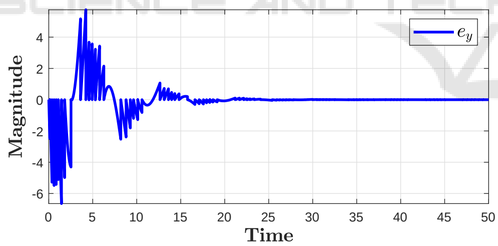

Figure 3: Sampling induced errors.

Figure 3 shows the evolutions of sampling induced-errors $e(t)$, which is reset to zero at each transmission instant as explained.

Figure 4 shows the trajectory of $\eta(t)$, where we note that $\eta(t) > 0$ as stated in Remark 1.

The periodic time instants and the transmission instants are shown in Figure 5. We note that the transmission instants generated by the PETC is much less than the periodic sampling instants, which supports the effectiveness of the proposed approach.

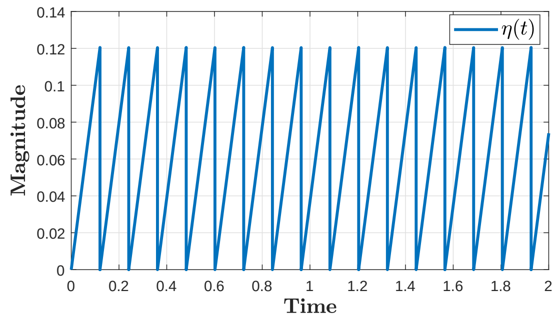

Figure 4: Evolution of $\eta(t)$ for first 3 sec.

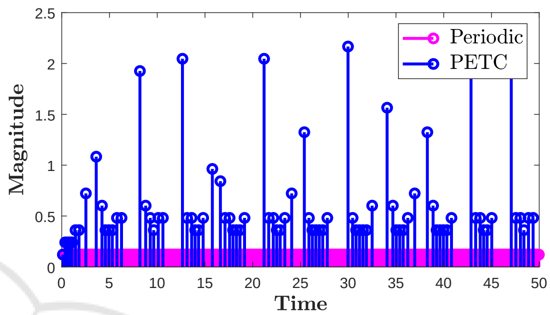

Figure 5: Periodic and transmission instants.

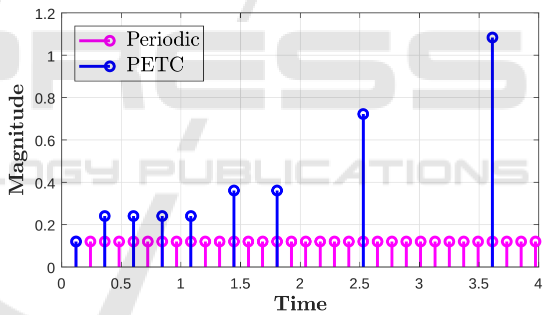

Figure 6: Periodic and transmission instants for first 4 sec.

A zoom in on the first 4 seconds for the periodic and the transmission instants is shown in Figure 6 to clearly highlight the fact the event-triggering condition is only verified at periodic sampling instants and not in continuous-time.

7 CONCLUSION

We studied the problem of periodic event-triggered control for linear systems based only on the output measurement. The proposed solution is well adapted to practical implementation since the event-triggering mechanism is checked only at periodic time instants rather than continuous-time verification. The problem is formulated as a hybrid dynamical system to truly describe the dynamic behaviour. By using appropriate Lyapunov function, we show that the closed-loop stability is ensured while automatically ruling out the Zeno phenomenon. The effectiveness of the approach was proven by numerical simulation.

Future work includes extending this approach to nonlinear plant models and the investigation of different implementation scenarios such as multi-agent systems and distributed control architectures.

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