Stochastic Estimation of Fundamental and Harmonic Signal Components

Chukwuemeka Aduba[a]
Naval Surface Warfare Center, Philadelphia Division Philadelphia, Pennsylvania 19112, U.S.A.

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Abstract: The paper investigates the estimation of fundamental and harmonic components in power system signal using stochastic estimator concept. The power system signal is approximated with a stochastic linear system model where the phase and amplitude components are estimated using a Kalman filter (KF) and an Ensemble Kalman filter (EnKF). The power system signal is modeled in both continuous and discrete form and then represented in state-space approach. Simulation results show that EnKF estimates converge to KF estimates as the ensemble size increases while reducing the computational complexity for highly-dimensional stochastic systems.

1 INTRODUCTION

Power systems have continued to evolve in recent history as designers and researchers propose different power system architecture for commercial and military shipboard applications. In shipboard application, an overwhelming power system architecture proposal has been one that integrates the propulsion (mobility) with all mission and support load to form an integrated power system (IPS) (McCoy, 2015). Further, the IPS is improved with zonal electrical distribution (ZED) model for increase flexibility and reliability as the system can source power from several directions without compromising performance due to unforeseen system faults. However, the integration of propulsion and service or mission load still presents power quality challenges in shipboard non-hybrid or hybrid-based micro-grid. These micro-grids while AC/DC in nature, are clusters of distributed generators, active and passive devices, energy storage system and linear/nonlinear loads (Wang et al., 2019).

Similarly, the widespread utilization of power electronics interfaces, proliferation of nonlinear loads and power-electronics-based industrial load devices have led to power quality concerns due to the resulting harmonic disturbances (Wang et al., 2014). Harmonics are introduced in the power system as a result of nonlinear behavior of power-electronic interfaces and power-electronic-based industrial load to sinusoidal current. This flow of sinusoidal current results in eventual non-sinusoidal periodic current. The non-sinusoidal periodic current propagates and interacts with the system impedance resulting in non-sinusoidal periodic load voltage otherwise referred to as voltage harmonics.

In frequency domain, harmonics are spectral components of a distorted periodic signal whose frequencies are integral multiples of the fundamental frequency. These harmonics are undesirable in power system networks due to the immediate and long-term detrimental effects on power quality such as degradation of electrical equipment, overheating of transformers and malfunction of metering devices (Farzanehrafat and Watson, 2013). To alleviate or mitigate or possibly eliminate harmonic distortion, it is imperative to be able detect, identify and classify them. In this work, harmonic parameter estimation for a linearly modeled system is explored to gain an understanding into possible harmonic identification process.

Earlier studies of power system harmonics estimation were reported in (Girgis et al., 1991), (Ma and Girgis, 1996) while the more recent studies in harmonics analysis and estimation have been described in (Farzanehrafat and Watson, 2013), (Medina et al., 2013) and (Wang et al., 2014). Girgis *et al.* (Girgis et al., 1991) considered harmonics in power systems based on optimal measurement scheme while the authors in (Ma and Girgis, 1996) considered the identification and tracking of harmonic sources using KF. In (Farzanehrafat and Watson, 2013), the researchers investigated power quality state and three-phase state transient state estimation. The authors in (Medina et al., 2013), presented an overview of frequency-domain, time-domain and hybrid frequency-time harmonic analysis. In addition, the constraints and drawbacks were highlighted for practical power network.

[a] https://orcid.org/0000-0001-6017-7287

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In (Wang et al., 2014), the modeling and analysis for harmonic stability problems in AC power electronics-based power systems were performed to gain insights into stability issues. Harmonic elimination or mitigation approaches have been through passive, active and hybrid filtering involving integrated passive/active filter combination. In addition, discrete fourier transforms (DFT) and fast fourier transforms (FFT) have been applied to harmonics filtering involving stationary signal. However, both transform techniques exhibit poor results in time-varying frequency systems (Girgis et al., 1991).

Traditionally, linear system filtering application has employed Kalman filters as optimal estimators. EnKF has been proposed for signal processing of harmonics in (Ray and Subudhi, 2012). The main application for EnKF has been in geophysical science for weather forecasting, hydrological modeling (Evensen, 1994) and more recently in medical science such as epidemiology modeling (Lal et al., 2021). EnKF is a random sampling implementation of KF for high-dimensional, possibly nonlinear and non-gaussian state estimation problem. It is related to particle filter where the particle is the ensemble. The starting point for EnKF is choosing a set of sample point (that represent an ensemble of the state estimates) to capture the initial probability distribution of the state. The final states are estimated by assimilating the observation into the state model. For a tutorial on EnKF including application areas, see (Evensen, 2009), (Roth et al., 2017).

The contributions in this study include formulation of harmonic signal in continuous and discrete fourier series form, detailed formalized equations for the linear estimators and performance evaluation of the estimators. The remainder of this paper is organized as follows: Section 2 models the harmonic signal in both discrete and continuous form. In addition, the signal system is formulated in state space approach. Section 3 describes the KF and EnKF including the recursive equations and the algorithm while Section 4 illustrates a simulation example as a case study. Finally, the conclusions including future directions are given in Section 5.

2 HARMONIC SIGNAL MODEL

2.1 Signal Model

The power system harmonic signal is modeled as a distorted waveform in both continuous and discrete form.

2.1.1 Continuous Harmonic Signal

Let the power system harmonic signal $Y(t)$ at time t be modeled as a distorted waveform in continuous form as:

$$Y(t) = \sum_{i=1}^{N} A_i \sin\left[w_i t + \theta_i\right] + \sum_{i=1}^{N} \omega_i(t), \qquad (1)$$

where $i = 1, 2, \ldots, N$ with N as the harmonic order. The angular frequency, amplitude and phase angle of the harmonic signal are given by w_i, A_i and θ_i respectively. The fundamental frequency is given as f with $w_i = i2\pi f$. For instance, $w_1 = 2\pi f$, $w_2 = 4\pi f$ and $w_3 = 6\pi f$. The $\omega_i(t)$ is the additive noise component with $\omega_1 = \omega_2 = \omega_3 = \ldots = \omega_N$.

2.1.2 Discrete Harmonic Signal

Similarly, let the power system harmonic signal Y_k at k time step be modeled as a distorted waveform in discrete form as:

$$Y_k = \sum_{i=1}^{N} A_i \sin\left[w_i k T_s + \theta_i\right] + \sum_{i=1}^{N} \omega_{ik}, \qquad (2)$$

where T_s is the sampling time. The ω_{ik} is the additive noise component. Similarly, the angular frequency, amplitude and phase angle of the harmonic signal are given by w_i, A_i and θ_i similar to the continuous signal case. The signal is nonlinear in phase and linear in amplitude. In parametric form, the signal in (2) can be converted to linear form as shown below by applying trigonometric identities giving:

$$Y_k = \sum_{i=1}^{N} A_i \sin(w_i k T_s)\cos(\theta_i) + A_i \cos(w_i k T_s)\sin(\theta_i) + \sum_{i=1}^{N} \omega_{ik}. \qquad (3)$$

In compact form, the signal depicted in (3) can be represented as:

$$Y_k = M_k L + \sum_{i=1}^{N} \omega_{ik}, \qquad (4)$$

$$M_k = \Big[\sin(w_1 k T_s)\ \cos(w_1 k T_s)\ \ldots\ldots\ \ldots\ldots\ \sin(w_N k T_s)\ \cos(w_N k T_s)\Big]_k, \qquad (5)$$

$$L = \Big[A_1\cos(\theta_1)\ A_1\sin(\theta_1)\ A_2\cos(\theta_2)\ A_2\sin(\theta_2)\ \ldots\ldots\ A_N\cos(\theta_N)\ A_N\sin(\theta_N)\Big]^T, \qquad (6)$$

with L as the vector of in-phase and quadrature phase components. The state equation $(\mathbf{x}_k)$ in linear form for the signal system (3) can be modeled as:

$$\mathbf{x}_k = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2N-1} \\ x_{2N} \end{bmatrix}_k + W_k, \quad (7)$$

where $x_1 = A_1 \cos(\theta_1)$, $x_2 = A_1 \sin(\theta_1)$ and $x_{2N-1} = A_N \cos(\theta_N)$, $x_{2N} = A_N \sin(\theta_N)$. The W_k is the additive noise component. In addition, the measurement equation $(\mathbf{z}_k)$ in linear form for the signal system (3) can be modeled as:

$$\mathbf{z}_k = \begin{bmatrix} \sin(w_1 k T_s) \\ \cos(w_1 k T_s) \\ \vdots \\ \sin(w_N k T_s) \\ \cos(w_N k T_s) \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2N-1} \\ x_{2N} \end{bmatrix}_k + V_k, \quad (8)$$

where V_k is the additive noise component.

3 ESTIMATOR MODEL

In this section, the estimators will be modeled based on the harmonic signal structures in earlier section.

3.1 Kalman Filter

The Kalman filter is an optimal estimator for a linear stochastic system corrupted by gaussian noise. The details of the Kalman filter in discrete-time format are given as follows. Let the linear state and measurement dynamics be given as:

$$\begin{aligned} x_k &= \Phi_{k-1} x_{k-1} + w_{k-1}, \\ z_k &= H_{k-1} x_{k-1} + v_{k-1}, \end{aligned} \quad (9)$$

where Φ_{k-1} is the state transition matrix, H_{k-1} is the observation matrix, $x_k \in \mathbb{R}^{2n}$ is the state vector, w_k and v_k are gaussian random processes defined on a probability space (Ω_0, F, P) where Ω_0 is a nonempty set, F is a σ-algebra of Ω_0 and P is a probability measure on (Ω_0, F). The initial state vector $\hat{x}_0 = E(x_0)$ has initial covariance matrix $P_0 = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right]$ and z_0 is the initial output vector. The gaussian random process w_k has zero mean and covariance of $E(w_k w_k^T) = Q_k$. Similarly, the gaussian random process v_k has zero mean and covariance of $E(v_k v_k^T) = R_k$. The noise processes w_k, v_k and x_0 are assumed uncorrelated. The following Φ_{k-1}, H_{k-1} are assumed to have appropriate dimensions. The sequential recursive computation steps for the Kalman estimator is summarized as follows (Crassidis and Junkins, 2012):

$$\begin{aligned} &\hat{x}_{k/k-1} = \Phi_{k-1} \hat{x}_{k-1/k-1}, \\ &P_{k/k-1} = \Phi_{k-1} P_{k-1/k-1} \Phi_{k-1}^T + Q_k, \\ &K_k = P_{k/k-1} H_{k-1}^T \left(H_{k-1} P_{k/k-1} H_{k-1}^T + R_{k-1}\right)^{-1}, \\ &\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k \left(z_k - H_k \hat{x}_{k/k-1}\right), \\ &P_{k/k} = \left(I - K_k H_{k-1}\right) P_{k/k-1}. \end{aligned} \quad (10)$$

The first-two lines in (10) are the prediction or time update steps while the last-three lines in (10) are the analysis or measurement update steps. The analysis steps adjust the prediction steps. In this sub-section, the estimator described was envisaged for linear application. However the nonlinear equivalent of (9) is given as:

$$\begin{aligned} x_k &= F(x_{k-1}) + w_{k-1}, \\ z_k &= G(x_{k-1}) + v_{k-1}, \end{aligned} \quad (11)$$

where $F(.)$ and $G(.)$ are nonlinear functions of state.
Remark: In general, extended Kalman Filter (EKF) is applied for nonlinear gaussian systems estimation while KF remains the optimal estimator for linear gaussian system.

3.1.1 Algorithm 1

Data: Actual signal
Result: Amplitude and Phase
initialize the state;
initialize the covariance;
while *for a fixed output size N* **do**
  estimate the state;
  estimate the covariance;
  compute the filter gain;
  estimate the output error;
  update the state with measurement;
  update the covariance with measurement;
end

Algorithm 1: KF Estimator Algorithm.

3.2 Ensemble Kalman Filter

The Ensemble Kalman filter is an estimator with application to large-dimensional nonlinear system. For large states, maintaining the state covariance matrix can lead to computational instabilities, thus EnKF allows sample covariance instead of state covariance to be utilized in the process. In addition, the state distribution in EnKF is represented by ensemble of a fixed size forecasted state estimates with random noise.

Let the ensemble Z_0 be represented as:

$$Z_0 = \left[x^{(1)},\, x^{(2)},\, \dots,\, x^{(N-2)},\, x^{(N-1)},\, x^{(N)}\right], \quad (12)$$

where Z_0 is an $n \times N$ matrix, whose columns of $\mathbb{R}^n$ are the samples from prior distribution. In terms of a harmonic signal X_k,

$$X_k = \left[x_k^{(1)},\ x_k^{(2)},\ \ldots,\ x_k^{(2N-2)},\ x_k^{(2N-1)},\ x_k^{(2N)}\right], \tag{13}$$

with X_k defined as $n \times 2N$ matrix, whose column are $\mathbb{R}^n$. As in equation (10), the sequential recursive computation steps for the EnKF is summarized as follows (Crassidis and Junkins, 2012), (Gillijns et al., 2006):

$$\begin{aligned}
\hat{x}^{(i)}_{k/k-1} &= \Phi_{k-1}\hat{x}_{k-1/k-1} + w^{(i)}_{k-1},\\
\bar{x}_{k/k-1} &= \frac{1}{2N}\sum_{i=1}^{2N}\hat{x}^{(i)}_{k/k-1},\\
z^{(i)}_{k/k-1} &= H_{k-1}\hat{x}^{(i)}_{k/k-1},\\
\bar{z}_{k/k-1} &= \frac{1}{2N}\sum_{i=1}^{2N} z^{(i)}_{k/k-1},\\
E_{x_k} &= \left[\hat{x}^{(1)}_{k/k-1} - \bar{x}_{k/k-1}\ ,\ldots,\hat{x}^{(2N)}_{k/k-1} - \bar{x}_{k/k-1}\right],\\
E_{z_k} &= \left[z^{(1)}_{k/k-1} - \bar{z}_{k/k-1}\ ,\ldots,z^{(2N)}_{k/k-1} - \bar{z}_{k/k-1}\right],\\
\hat{P}_{xz_k} &= \frac{1}{2N-1}E_{x_k}(E_{z_k})^T,\\
\hat{P}_{zz_k} &= \frac{1}{2N-1}E_{z_k}(E_{z_k})^T,\\
K_k &= \hat{P}_{xz_k}(\hat{P}_{zz_k})^{-1},\\
\hat{z}^{(i)}_{k/k-1} &= z_{k/k-1} + v^{(i)}_k,\\
x^{(i)}_{k/k} &= \hat{x}^{(i)}_{k/k-1} + K_k\left(\hat{z}^{(i)}_{k/k-1} - z^{(i)}_{k/k-1}\right),
\end{aligned} \tag{14}$$

where the ensemble mean for the input and output are given as $\bar{x}_{k/k-1}$ and $\bar{z}_{k/k-1}$. The ensemble cross covariance and covariance are given as $\hat{P}_{xz_k}$ and $\hat{P}_{zz_k}$. The perturbed measured signal is $\hat{z}^{(i)}_{k/k-1}$ while the measured signal is $z_{k/k-1}$. The first-four lines in (14) are the prediction or time update steps while the last-seven lines in (14) are the analysis or measurement update steps. The analysis steps adjust the prediction steps through the use of new measurement.

For additional signal parameter analysis, the amplitude $A_{i,k}$ and phase $\theta_{i,k}$ at any time step k can be computed following trigonometric identities as:

$$\begin{aligned}
A_{i,k} &= \sqrt{\left(\hat{x}^2_{2i-1,k} + \hat{x}^2_{2i,k}\right)},\\
\theta_{i,k} &= \arctan\left(\frac{\hat{x}_{2i,k}}{\hat{x}_{2i-1,k}}\right),
\end{aligned} \tag{15}$$

where $i = 1,2,3\ldots,N-2,N-1,N$.

Remark: For linear gaussian systems, EnKF estimates should converge to KF estimates with increasing ensemble size.

3.2.1 Algorithm 2

Data: Actual signal
Result: Amplitude and Phase
initialize the state;
initialize the covariance;
initialize the ensemble;
while *for a fixed output size N* **do**
  estimate the ensemble;
  estimate the perturbed output;
  estimate the cross covariance;
  estimate the covariance;
  compute the filter gain;
  update the ensemble;
end

Algorithm 2: EnKF Estimator Algorithm.

4 SIMULATION EXAMPLE

Consider the following signal harmonic which consists of sinusoidal continuous signal plus random noise given as:

$$\begin{aligned}
X(t) = 5\Big[&\sin(2\times\pi\times f_1\times t + 70^\circ)\\
&+ 0.2\sin(2\times\pi\times f_3\times t + 50^\circ)\\
&+ 0.12\sin(2\times\pi\times f_5\times t + 45^\circ)\\
&+ 0.07\sin(2\times\pi\times f_7\times t + 30^\circ)\\
&+ 0.04\sin(2\times\pi\times f_9\times t + 25^\circ)\Big]\\
&+ 0.002\mu(t),
\end{aligned} \tag{16}$$

where the gaussian random signal $\mu(t)$ has a mean of zero with covariances $R > 0$, $Q > 0$. The problem is to estimate the harmonic signal amplitudes and phases using the KF and EnKF estimators. The following parameters are defined:

$$\begin{aligned}
&E(w_k) = 0,\ E(w_k w_k^T) = Q_k,\\
&E(w_k w_j^T) = 0; (j \neq k),\ E(v_k) = 0,\\
&E(v_k v_k^T) = R_k,\ E(v_k v_j^T) = 0; (j \neq k),\\
&E(w_k v_j^T) = 0; (j \forall k),\ f_s = 3.0\ \text{kHz},\\
&f_1 = 60\ \text{Hz},\ f_3 = 180\ \text{Hz},\ f_5 = 300\ \text{Hz},\\
&f_7 = 420\ \text{Hz},\ f_9 = 540\ \text{Hz},\\
&P_{0/0} = 0.002 I_{10x10},\\
&x_0 = 5[0.34\ 0.93\ 0.12\ 0.15\ 0.08\ 0.08\ 0.06\\
&\quad 0.03\ 0.03\ 0.01]'.\\
&R = 3.6e-3,\ Q = 3.6e-3 I_{10x10}.
\end{aligned} \tag{17}$$

Figures 1, 2 and 3 show the amplitude and phase estimation for Kalman filter-based algorithm while

Figures 4, 5 and 6 show the corresponding amplitude and phase estimation for the Ensemble Kalman filter-based algorithm for the fundamental and harmonic components of the given signal. For linear systems, EnKF-based algorithm results converges to KF-based algorithm results as expected for increased number of ensembles (Roth et al., 2017).

Figures 7, 8 and 9 show the estimated signal for both filters against the true signal and the signal frequency spectrum. The frequencies can be estimated by adjusting the formulation in (5) and (6) respectively. Figures 10, 11 and 12, the mean square error (MSE) for the signal estimation was captured for different ensemble sizes. From the MSE figures, it can be observed that as the ensemble size increased, the estimated signal mirrors the true signal.

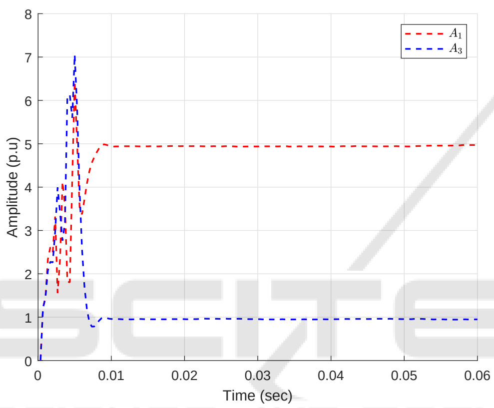

Figure 1: Kalman Filter Amplitude Estimate for A_1, A_3.

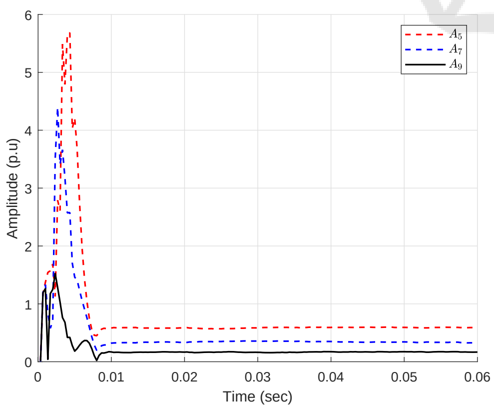

Figure 2: Kalman Filter Amplitude Estimate for A_5, A_7, A_9.

5 CONCLUSION

In this paper, an application of Kalman filter and Ensemble Kalman filter estimator to power system harmonic analysis was performed to get insights into signal detection in the presence of noise harmonics. A representative signal of the distorted harmonic signal was modeled and utilized in the simulation. The simulation study results show the effectiveness of the filter algorithm. It was observed that the error covariance parameter adjustments improved estimation results.

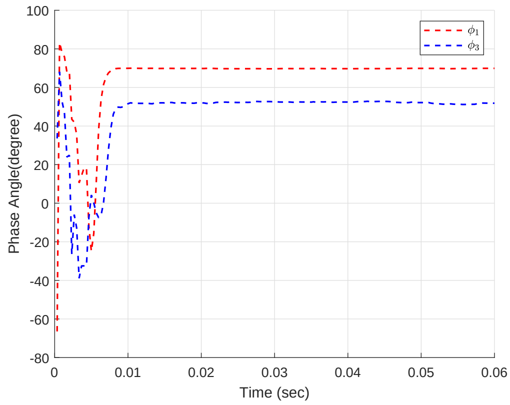

Figure 3: Kalman Filter Phase Estimate for ϕ_1, ϕ_3.

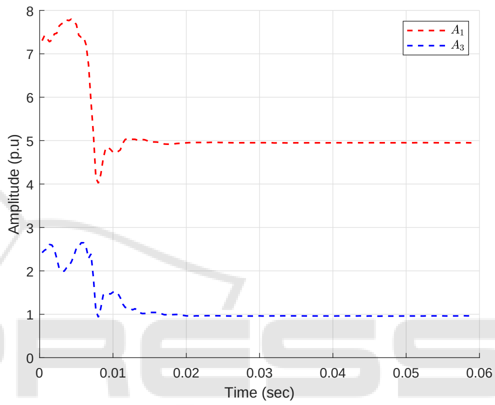

Figure 4: Ensemble Kalman Filter Amplitude Estimate for A_1, A_3.

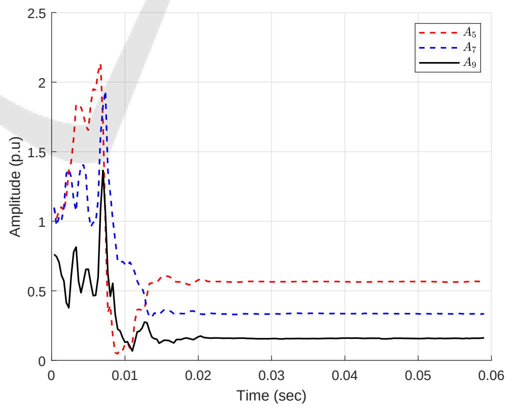

Figure 5: Ensemble Kalman Filter Amplitude Estimate for A_5, A_7, A_9.

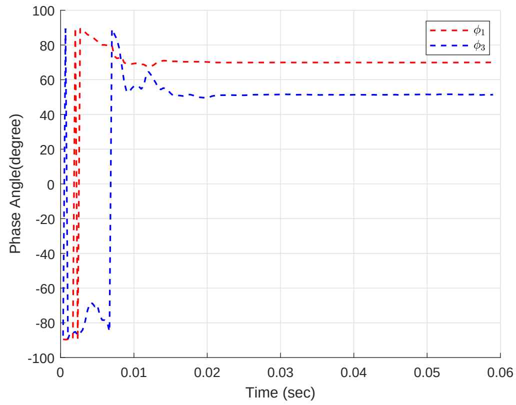

Figure 6: Ensemble Kalman Filter Phase Estimate for ϕ_1, ϕ_3.

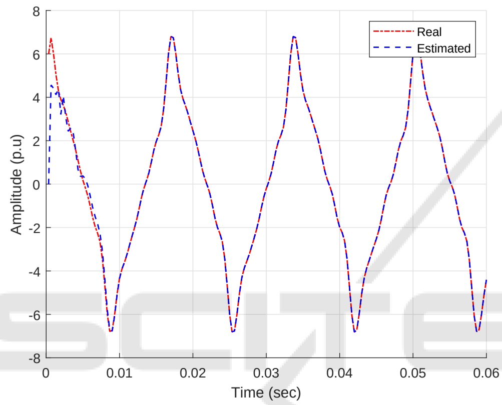

Figure 7: Kalman Filter Signal of Actual (Real) and Estimate.

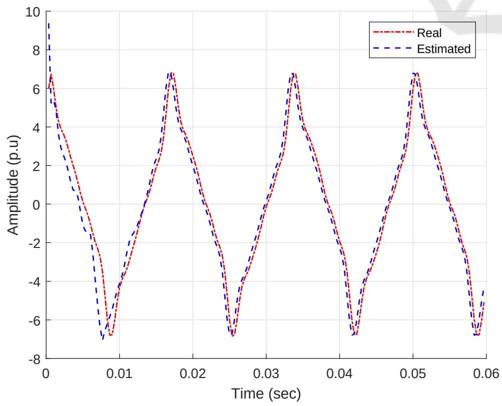

Figure 8: Ensemble Kalman Filter Signal of Actual (Real) and Estimate.

A further investigation is to consider time-varying harmonic signal in amplitude and phase with additional estimators for evaluation. Another area of research is analyzing captured data from a system implementation to compare with simulation results.

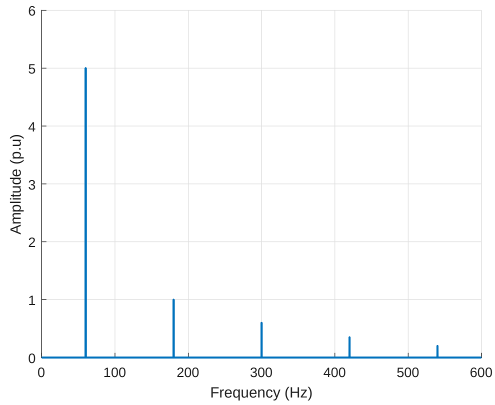

Figure 9: Single-Sided Amplitude Spectrum for the Actual Signal.

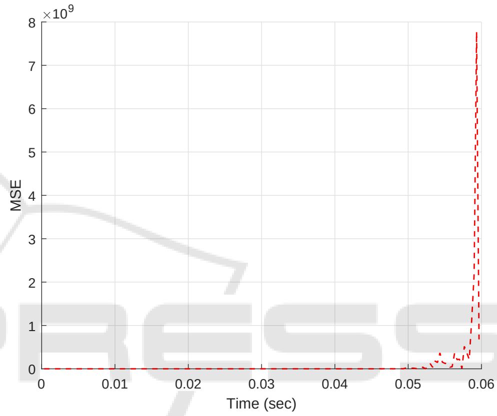

Figure 10: EnKF MSE for five (5) Ensembles.

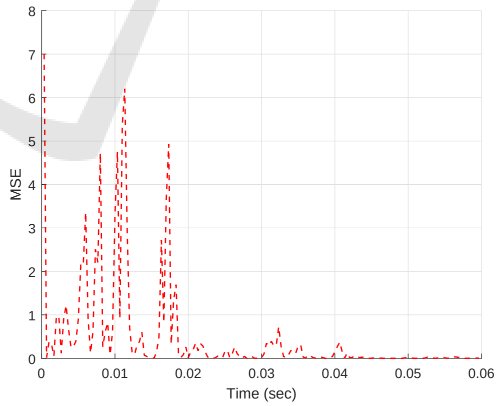

Figure 11: EnKF MSE for ten (10) Ensembles.

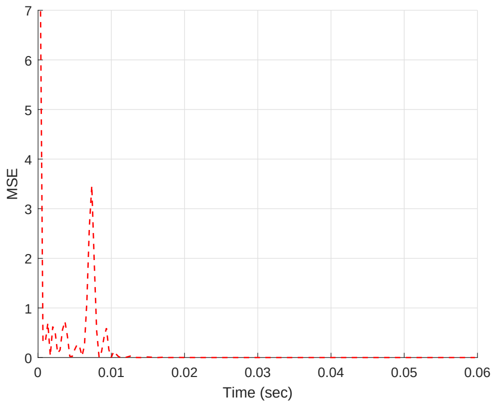

Figure 12: EnKF MSE for fifty (50) Ensembles.

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