

# Learning-Based Energy Consumption Model of Machining Processes Using Gaussian Process Regression

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**Abstract:** Currently, the global energy mix is largely dominated by the use of fossil fuels, with the industrial sector accounting for a significant portion of this demand. This results in a significant carbon footprint. As such, the manufacturing industry must become active participants in reducing their impact on the environment through the realization of sustainable manufacturing practices. This study analyzes the performance of a data-driven model enhanced with machine learning techniques in order to build a digital twin that can update its parameters in real-time in response to dynamic changes in the energy consumption of a machining process. This type of model is suitable for the application of a higher-level controller, such as a model predictive controller to optimize the efficiency of the process operation. This paper proposes a digital twin modelling approach based on Gaussian process regression, which updates model parameters with closed-loop data from the process in real-time to retrain the model (evolving). The updating of the model online enables the model to maintain accuracy over time despite changes in the system's dynamics.

## 1 INTRODUCTION

The industrial sector accounts for more than 40% of the world's electricity consumption, and manufacturing firms consume nearly 50% of that energy (IEA, 2020). Energy efficiency plays an important role in transforming factories to become more sustainable and more environmentally friendly to address the key societal challenge of the depletion of energy resources and the deterioration of the environment. However, improving the energy efficiency in the manufacturing process is a non-trivial task. This is due to the complexity of flexible manufacturing systems and their power consumption dynamics.

The energy consumed by machining equipment in discrete manufacturing processes has typically been considered less significant compared to other manufacturing processes, e.g. the furnaces of steel industries, therefore, there is a lack of relevant research on the energy efficiency and modeling of machining processes despite the fact the overall impact on in-

dustrial consumption can be compelling (Gu et al., 2020). In (Moradnazard and Unver, 2017), it was highlighted that improving the energy efficiency of machine tools can be impactful, and more research is required to develop methods to improve real-time energy optimization beyond optimizing time and costs. In (Huang et al., 2023), an energy-saving control strategy was developed for multi-sleep states of machine tools considering component priority. The results show that the control strategy considering component priority (i.e. the order in which the components of a machine tool are started up or shut down) can obtain more stable productivity and a better energy-saving effect compared to other control strategies.

In enhancing the energy efficiency of machine tools, two approaches are typically employed. Firstly, developing machine tools that are energy and material-efficient. Secondly, optimizing the machining process to conserve energy. The first approach needs a significant monetary commitment towards modifying the machine. Whereas, the second approach entails maximizing the efficiency of machining operations through scheduling optimization and management of both primary components and support units to minimize the usage of redundant en-

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ergy (Yi et al., 2020; Bermeo and Ocampo-Martinez, 2019; Quinn et al., 2022). The latter can be carried out by applying control strategies, e.g. Model predictive control (MPC), which has been extensively utilized for industrial process optimization, resulting in highly favourable outcomes (e.g., manufacturing systems (Lanzetti et al., 2019; Huang et al., 2023), chemical industry (Shin et al., 2020; Wu et al., 2019b), and pharmaceutical industry (Wong et al., 2018)). Moreover, it has been more broadly implemented for managing energy efficiency (Bermeo and Ocampo-Martinez, 2019). As a model-based control method, MPC requires an accurate model of the controlled system to enhance its performance. When it comes to performance, MPC can outperform other control techniques since predictions of the process permit control actions to be calculated based on future evolutions, and it allows for preview information about references and disturbances to be considered. Consequently, the prediction model is a critical component of MPC (Carmacho, 2013).

Given the importance of an accurate model for the performance of MPCs, most of the research in learning-based MPC is focusing on improving the model quality (Hewing et al., 2020; Narciso and Martins, 2020). However the computation load required to equate the prediction model can make the application of the MPC in real time infeasible, e.g. computation fluid dynamics (CFD) is a powerful modelling tool, but its computation cost is large, making it prohibitive for a real-time optimisation application (Jeon et al., 2019). Therefore, recent research has been focused on developing accurate data-driven models suitable to be applied in real-time by an MPC. These methods include modeling the system's dynamics with machining learning (ML) techniques such as genetic algorithms (GA) (e.g. (Huang et al., 2023)), Gaussian process regression (GPR) (e.g. (Park et al., 2015; Maiworm et al., 2021)), decision trees, decision forests, logistic regression, support vector machine (SVM), neural network (NN) (e.g. (Shin et al., 2020; Lanzetti et al., 2019; Wu et al., 2019a)), and Bayesian classifiers (Jordan and Mitchell, 2015).

In (Shin et al., 2020), an MPC framework using a NN to model the system's dynamics was developed. The aim was to increase the speed of optimization and accuracy of the model. The adoption of the NN model instead of using the existing linearized model enhances the operational efficiency of the process industry. In (Wu et al., 2019a), a machine learning-based predictive control system was developed for nonlinear processes using an ensemble of recurrent neural network (RNN) models. Their Lyapunov-MPC formulation employs machine learning ensemble re-

gression modelling tools to improve the prediction accuracy of RNN models and overall closed-loop performance while parallel computing is utilized to reduce computation time. In (Lanzetti et al., 2019), a tailored RNN model for system identification is presented. It is scalable and flexible for handling complex systems encountered in industrial applications. The proposed framework is applied in an industrial simulation case study, showing good performance in dealing with challenging practical conditions such as multiple-input multiple-output (MIMO) control, nonlinearities, noise, and time delays. Which makes this method scalable to machining processes.

Degradation of machines and dynamic production environments can result in variations of energy consumption. In these uncertain situations, it is proposed that optimizing the machining process using real-time data is the most appropriate method. To achieve online optimization, it is necessary to have an accurate energy model that can cope with uncertainty related to changes in machine components and production processes. Developing a new real-time predictive model or digital twin using ML techniques can address this challenge since it has the potential to capture inherent dynamics and update parameters continually during operation. Unlike traditional system identification methods, most of which are suitable for offline processes, this technology allows for real-time operation. In (Hewing et al., 2020), the advantages of learning-based MPC were explored, e.g. including the ability to exploit the abundance of data in a reliable manner, particularly while taking safety constraints into account. The proposed method addressed the automated and data-driven generation or adaptation of elements of the MPC formulation such that the control performance with respect to the desired closed-loop system behavior is improved. The setup in which this learning takes place can be diverse. For instance, offline learning considers the adaptation of the controller between different trials or episodes of a control task, during which data are collected. In methods that learn online, the controller is adjusted during closed-loop operation or using the data collected during one task execution.

In (Park et al., 2015), energy prediction models are developed for different subprocesses of a CNC milling machine using a GPR model. The study investigates the effects of machining parameters on energy consumption and identifies the optimum input features for the model of each different subprocess. An uncertainty analysis is also presented to develop confidence bounds for the prediction model. In addition, GPR can refine the model online during operation. GPR models are capable of efficient on-

line learning and can reduce the computation load by limiting the number of training data points while maintaining good performance. Gaussian processes are also flexible and can handle non-linear and non-Gaussian systems (Maiworm et al., 2021).

Given the lack of research on the energy efficiency of machining processes and the requirement of MPC to have a sufficiently accurate model that enables real-time operation, this work proposes a modeling framework based on GPR to capture the energy consumption dynamics of machining processes. In addition, the proposed method is capable of online retraining of the model (evolving), hence the accuracy of the model is maintained even when the system's dynamics vary. Most of the studies in this field employ modelling techniques such as GPR to model the system, however, these models are built for prediction purposes and are not applied in MPC applications. In this work, a framework is provided for the development of a GPR model that can be applied in MPC applications for optimizing the energy efficiency of machining processes.

## 2 MODELING AND EVOLVING GAUSSIAN PROCESS REGRESSION MODEL

In this work, a model using GPR to capture the system's energy consumption dynamics of a machining process is built. The model is updated with new data when changes in the physical properties of the process are detected. This enables for the capture of the uncertainties and any variation of the system's dynamics due to aging or any environmental change that affects the system's dynamics. This allows the model to improve and maintain accuracy over time. The online training will be performed using new measurements obtained in closed-loop operation. With this, the model can be used as a control model for MPC in order to optimize the energy consumption and the control performance can be improved over time by utilizing the measured data. In what follows, the fundamentals of GPR are introduced. Then the evolving GPR concept is described that will be applied to iteratively update the energy consumption model.

### 2.1 Gaussian Process Regression

A Gaussian process regression is a non-parametric model with uncertainty predictions (Särkkä, 2019). The GP prior distribution  $GPf(u) \sim GP(m(u), k(u, u'))$  is defined by the

mean function  $m(u) = E[f(u)]$  and the covariance or kernel function  $k(u, z') = cov[f(u), f(u')] = E[(f(u) - m(u))(f(u') - m(u'))]$ . Where  $u$  is the input called regressor and  $E$  is the expected value. The mean and the covariance functions along with their hyperparameters  $\theta$  define the GP. The GP is then trained with a set of  $n$  measured input  $u$  and output  $z$  data points defined as the training dataset  $D = \{u, z\}$  that will be used to infer the posterior Gaussian distribution (Jeon et al., 2019),

$$f(u|D) \sim GP(m(u|D), \sigma^2(u|D)). \quad (1)$$

The mean function mostly used is a constant zero prior mean  $m(u|\theta) = 0$ . The covariance function defines the smoothness property of the functions, which is usually selected to be the squared exponential covariance function (Särkkä, 2019). Then, the hyper-parameters  $\theta$  are determined maximizing the log marginal likelihood for the training data set  $D = \{u, z\}$ ,

$$\log(p(z|\omega, \theta)) = -\frac{1}{2}z^T k^{-1}z - \frac{1}{2}\log|K| - \frac{1}{2}\log(2\pi). \quad (2)$$

One advantage of this modeling technique is that it gives a regression mean of the prediction along with upper and lower error bars for the predicted values, as shown in Figure 1, which can be used as an estimate of prediction uncertainty (Särkkä, 2019).

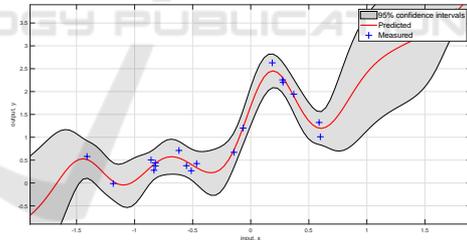


Figure 1: Illustration of a GPR prediction.

### 2.2 Evolving Gaussian Process Regression

Any change in the environment of the system, such as humidity, temperature, noise, or aging may vary the system's dynamic and if the model is not updated to capture those changes in the dynamics the accuracy of the predictions will be affected. For that reason, the training dataset  $D = \{u, z\}$  needs to be updated so the GP evolves with the real system (Maiworm et al., 2021).

There are different criteria to decide if a new data point needs to be included in  $D = \{u, z\}$ . In (Maiworm et al., 2021), it is determined if a new data point

is included if the prediction error  $e^p$  and the variance  $\sigma^2$  are higher than the determined thresholds  $\bar{e}$  and  $\bar{\sigma}^2$  respectively, i.e.

$$e^p > \bar{e} \quad (3)$$

$$\sigma^2 > \bar{\sigma}^2. \quad (4)$$

Increasing the number of data points will increase the computation time for the prediction, which could, at some point, make the model unfeasible for a real-time optimization application such as MPC. Therefore, there is a need to maintain the number of data points  $n$  when evolving the GPR. Different methods can be used to determine which data point should be replaced with a new one, e.g.

- Erasing the oldest data point.
- Erasing the data point that provides less information to the model (i.e. the one with the highest value of accuracy when used for prediction).

The first method has less computational cost, while the second method would need to predict the accuracy of the training dataset at every time  $k$  the model is updated (Maiworm et al., 2021). In some cases, the second method could be applied without affecting the real-time performance of the model, e.g. when the model is updated offline or if the computation time is less than the required prediction time-step. In Section 5, the accuracy and computation time of both methods is evaluated.

### 3 DATA ACQUISITION

This section describes the acquisition of training and validation data. More details can be found in (Bhinge et al., 2014). The data was used in (Park et al., 2015; Bhinge et al., 2014; Ferguson and Park, a)<sup>1</sup>.

#### 3.1 Training Data

The experiments were run using a Mori Seiki NVD 1500 – Micro NC Milling machine with a 9.525 mm diameter solid carbide tool to machine a 63.5 mm x 63.5 mm x 56 mm cold finish mild steel 1018 material. It includes six basic cutting subprocesses: face milling, counterboring, pocketing, slotting, spiraling, and drilling, and three non-cutting subprocesses: air-cut in x and y direction, air-cut in z-direction and rapid motion (Bhinge et al., 2014).

Eighteen sample parts were machined. The face-milling operations on the first nine parts were carried

out in the y-direction, while the remaining nine parts were milled in the x-direction. This ensures a better prediction performance of the model since the data covers both axes equally. In each part, the parameters that affect the energy consumption the most are varied so that every combination of them is applied for each sample part. The spindle speed is measured in revolutions per minute (RPM), the values used can be seen in Table 1. The feed rate measured in millimeters per minute is then obtained as the product of the spindle speed in RPM, the chip load measured in millimeters per tooth, and the number of tool teeth (Bhinge et al., 2014).

#### 3.2 Validation Data

Three test datasets were generated for validation including six basic cutting subprocesses: face milling, pocketing, and drilling, and three non-cutting subprocesses: air-cut in x and y direction, air-cut in z-direction, and rapid motion. In this case, the spindle speed (RPM) for each test data set were varied as it can be seen in Table 1 (Bhinge et al., 2014).

Table 1: Parameters of the training and validation data.

Dataset	Spindle Speed (RPM)
Training	{1500, 3000, 4500}
Test 1	{1500, 3000, 4500}
Test 2	{1700, 2800, 4300}
Test 3	{2125, 2400, 3750}

### 4 GAUSSIAN PROCESS REGRESSION BASED ENERGY CONSUMPTION MODELS

In this section, four different GPR-based energy consumption models for machining processes are presented. The performances are compared in Section 5:

- Baseline model.
- Reduced training data set.
- Reduced training data set and evolved offline.
- Reduced training data set and evolved online.

The hyperparameters for the evolved models are fixed with the values obtained with the reduced model, i.e. only the training dataset  $D = \{u, z\}$  is modified when retraining. This means that after retraining the model, there is no need to maximize the log marginal likelihood again, which saves most of

<sup>1</sup>Database at: <http://lma.berkeley.edu/raunak.html>

the computation load of the retraining process. MATLAB packages: PMML (Ferguson and Park, b) and GPML (Rasmussen and Nickisch, 2020) were used to generate, train, optimize, store, and use for prediction.

#### 4.1 Baseline Model

The baseline model corresponds to the model proposed in (Park et al., 2015). Each of the 9 subprocesses has a corresponding GPR model. The input features or regressors for each GPR model vary, as the impact of input parameters on the energy consumption dynamics varies depending on the subprocess. Table 2 shows which inputs were used for each subprocess's GPR model listed in order of importance, i.e.

- Feature 1 is the feed rate (RPM).
- Feature 2 is the spindle speed (mm/min).
- Feature 3 is the depth of cut (mm).
- Feature 4 is the active tool axis ID. It is derived from the length of the cut in each direction  $x$ ,  $y$ , and  $z$ .
- Feature 5 is the cutting strategy ID. It is the method for removing material.

Table 2: Input features of the GPR models by subprocess.

Feed	Spindle speed (RPM)	Input features
Cut	Facemilling	{1, 2, 3, 4, 5}
	Countouring	{4, 1, 3, 2}
	Slotting	{4, 1, 2, 5}
	Pocketing	{4, 1, 2}
	Spiraling	{1, 4, 3}
	Driling	{1, 2, 4, 3}
No cut	Air cut in x and y	{1, 2, 4}
	Air cut in z	{4, 1, 2}
	Rapid motion	{2, 4}

#### 4.2 Reduced Training Data Set Model

In this model, the number of data points used in the baseline model was reduced by erasing data points from the training dataset  $D = \{u, z\}$ . The erased data points were less meaningful data points for the model e.g. the data points that were predicted with the highest accuracy. Table 3 shows the difference in the number of training data points between the baseline and the reduced models for each subprocess.

#### 4.3 Offline Evolved Model

The reduced model is then evolved offline with data from one of the validation datasets offline. This

Table 3: Number of training data points of the GPR models by subprocess.

Subprocess	Baseline model	Reduced model
1	1466	733
2	425	212
3	134	67
4	168	134
5	16	8
6	18	14
7	122	120
8	140	139
9	24	24

method enables maintaining accuracy with less computation time than the baseline model while it also enables the updating of the model in order to adapt the model to the changing dynamics of the system. Both methods were evaluated to decide which data points should be substituted, i.e.

- Erasing the oldest data point.
- Erasing the data point that provides less information to the model.

#### 4.4 Online Evolved Model

Our aim is to create an online learning GPR model suitable to be applied by an MPC in real-time to optimize the operations of a machining process. Therefore, the threshold for substituting one of the training data points with a new one is evaluated every time step. Both methods used for the offline evolved model to decide which data points are going to be substituted are also benchmarked for this model.

## 5 RESULTS

The model has been trained and validated on a laptop machine equipped with an Intel Core i7-10610U 1.8GHz and 32GB RAM running MATLAB 2022b 64-bit with the PMML (Ferguson and Park, b) and GPML (Rasmussen and Nickisch, 2020) packages.

Table 4 shows the accuracy of the baseline and reduced models. Note that the accuracy of the validation is higher than the accuracy of the training since the training dataset includes 9 machining subprocesses, while the validation datasets only include 6 of the 9 subprocesses. It also shows that the accuracy of the baseline model is maintained when the reduction of data points for training is carried out to create the reduced model.

Table 5 shows the computation time for training and the validation (prediction) processes, which are significantly reduced with the reduced model.

Table 4: Accuracy of the baseline and reduced models.

Dataset	Baseline	Reduced
	NMRSE(%)	
Training	74.0121	72.2632
Test 1	84.4659	83.5576
Test 2	80.4416	79.4065
Test 3	68.4466	68.0077

Table 5: Computation time of the baseline and reduced models.

Dataset	Baseline	Reduced
	Time (s)	
Training	28.27641	0.003509
Test 1	50.34184	22.30064
Test 2	48.66615	21.81061
Test 3	50.06636	22.19655

Tables 6 and 7 show that both evolved methods in each method *a)* and *b)* maintain and even increase the accuracy of the reduced model when predicting the validation dataset. The accuracy when predicting the training dataset is the one fixed in the reduced model.

Table 6: Accuracy of the evolved offline models.

Dataset	Evolve offline	
	a)	b)
	NMRSE (%)	
Training	72.2632	72.2632
Test 1	88.3044	88.1600
Test 2	87.5672	86.8094
Test 3	86.6200	86.8189

Table 7: Accuracy of the evolved online models.

Dataset	Evolve online	
	a)	b)
	NMRSE (%)	
Training	72.2632	72.2632
Test 1	86.1598	84.6490
Test 2	85.4315	81.0116
Test 3	76.3391	72.7208

In Table 8 and 9 it can be observed that the computation time for both evolving methods is similar. The computational time shown in Table 5 represents the computational time for predicting the full process, while the computational time shown in Table 8 and Table 9 represent the average computational time for predicting over an NC block.

Figure 2 shows the predictions of the baseline model for Test 1, while Figures 3 and 4 show the predictions of the evolved online model *b)* for Test 1. Figure 4 also shows at which NC block a training data point is substituted, e. i. when the thresholds described by equations (3) and (4) are crossed.

Table 8: Computation time of the evolved offline models.

Dataset	Evolve offline	
	a)	b)
	Time (s)	
Training	0.0033509	0.003509
Test 1	0.0045	0.0043
Test 2	0.0048	0.0045
Test 3	0.0052	0.0043

Table 9: Computation time of the evolved online models.

Dataset	Evolve online	
	a)	b)
	Time (s)	
Training	0.003509	0.003509
Test 1	0.00469	0.0075
Test 2	0.00486	0.0074
Test 3	0.00596	0.0088

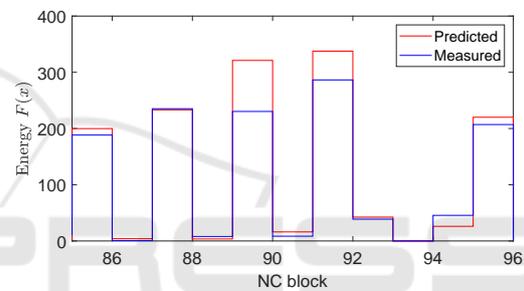
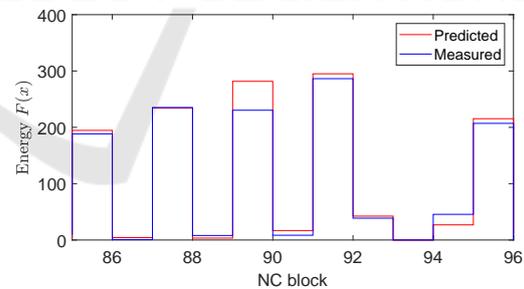


Figure 2: Measured values for Test 1 and predicted values using the baseline model.


 Figure 3: Measured values for Test 1 and predicted values using the evolved online model *b)*.

For example, between NC blocks 85-95, the training data points are substituted in consecutively NC blocks. Comparing Figures 2 and 3 it can be observed that the predicted signal is closer to the measured one when using the evolved online model *b)*. In Figure 4 it can be observed that between NC blocks 85-95 the retraining of the model is carried out every two NC blocks.

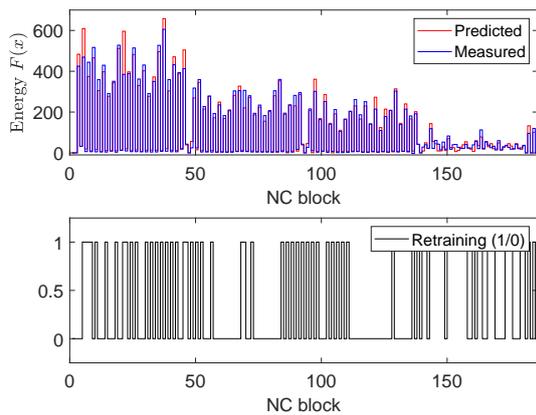


Figure 4: Measured values for Test 1 and predicted values using the evolved online model  $b$ ) and retraining signal.

## 6 CONCLUSION

The aim of this work is to build a digital twin that uses a non-parametric regression model, i.e. Gaussian process regression, suitable to be updated online allowing the model to sustain its accuracy over time despite any alterations that might occur in the system's dynamics. The performance of four different GPR models was analysed. The baseline model demonstrates the Gaussian process regression can be used to model the energy consumption of a CNC machine. The reduced model results demonstrate that the computation time could be reduced when less relevant data points are erased from the training data set while maintaining accuracy. The offline evolved model results show the reduced data points GP model can be retrained with new data, so the model changes along with the real system. The online evolved model demonstrates that the retraining of the model of the energy consumption of a CNC machine can be done online. Both evolved models, offline and online, have similar accuracy and computation time, but the model evolved online will include real-time changes in the system's dynamics. These results determine that online retraining of the model to capture the changes in the behaviour of the energy consumption of a machining process in real time is feasible.

A test bed is being built in order to collect new data to analyse the performance of the evolved online models. The computation time and accuracy of the evolved online models meet the characteristics required to build a DT-MPC framework to model the energy consumption of machining processes enhanced with real-time adaptive learning of the model and real-time optimization to reduce the energy consumption of the system. The literature review

on energy efficiency in machining typically employs alternative techniques for system modelling or GPR, but does not incorporate MPC for energy efficiency optimization. Thus, this study's primary contribution is the proposed GPR model's suitability for future MPC applications in optimizing energy efficiency in machining processes.

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