Kinematics Based Joint-Torque Estimation Using Bayesian Particle Filters

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Abstract: The aim of this paper is to estimate unknown torque in a 7-DOF industrial robot using Bayesian approach by observing the kinematic quantities. This paper utilizes two PMCMC algorithms (Particle Gibbs and Particle MH algorithms) for estimating unknown parameters of Baxter manipulator including joint torques, measurement and noise errors. The SMC technique has been used to construct the proposal distribution at each time step. The results indicate that for the Baxter manipulator, both PG and PMH algorithms perform well, but PG performs better as the estimated parameters using this technique have less deviation from the true parameters value. And this is due to sampling from parameters conditional distributions.

1 INTRODUCTION

Many engineered and physical systems contain parameters that are time-varying and contain uncertainties. Various techniques have been proposed for parameter estimation in linear and nonlinear mathematical models, such as Neural Networks (Calderón, 2000), Kalman Filter (Van der Merwe,2001), nonlinear population Monte Carlo (Koblents,2016), Bayesian Approach (Bradley,1992) and Adaptive Sequential MCMC[1] (Wenk,1980). While numerous techniques have been proposed, the selection of an appropriate methodology is of significance, given its potential impact on both the accuracy of estimated parameters and the efficiency of computational processes (Bigoni,2012). The aim of this paper is to estimate unknown parameters in nonlinear state space models (SSM) using Bayesian approach by observing the kinematic quantities. Many of the parameter estimation techniques use optimization formulations such as linear least-squares, orthogonal least-squares, gradient weighted least-squares, bias-correlated renormalization and Kalman filtering techniques. While these techniques are efficient and reliable for linear mathematical models, their implementation for non-linear models does not guarantee a reliable parameter estimation (Beck,,1977). Techniques such as Sequential Bayesian methods and specifically Sequential MCMC has been introduced to cope with highly non-linear dynamic systems (Andrieu,2010).

The Gibbs sampler, an MCMC technique, draws samples from conditional distributions of model parameters, providing an accurate representation of marginal posterior parameter densities (Nemeth,2013). C. Andrieu et al. introduced a novel approach that blends SMC[2] and MH[3] sampling to estimate unknown parameters in nonlinear dynamic models (Andrieu,2010). They adopted Particle MCMC (PMCMC) algorithms, replacing regular MCMC due to unreliable performance resulting from weak convergence assumptions. In this paper we discuss how utilizing two main algorithms of PMCMC: Particle Gibbs (PG) sampler and Particle Metropolis Hastings (PMH) sampler, could accurately estimate the unknown joint torques of the Baxter manipulator by observing the kinematic quantities. This paper is organized as follow: section two describes important mathematical preliminaries and background, section three provides the detail of the SMC technique, PG and PMH algorithms. In section four, the detail of Baxter dynamic model in State Space form is discussed, Also, the detail of the simulation setup is explained. Section five shows the

[1] Markov Chain Monte Carlo
[2] Sequential Monte Carlo
[3] Metropolis Hastings

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results of the PG and PMH algorithms and analyses the effectiveness of PG and PMH samplers.

2 PMCMC APPROACH

In probabilistic systems, the SSM can be considered as a Markov Chain with a sequence of stochastic random variables (Andrieu,2010). In hidden Markov model, the system being modelled is assumed to be a Markov process with unobservable states. It can be also written in below form.

$$x_{t+1} = h_\theta(x_t, u_t) \tag{1}$$

$$y_t = g_\theta(x_t, v_t) \tag{2}$$

In this context, If T considered as period of interest in the SSM, a hidden Markov state process: $x_{1:T} \cong \{x_{1,}x_2, \ldots, x_T\}$ is characterize by its initial density and transition probability density$h_\theta(x_{t+1}|x_t)$, for some statistical parameter θ which might be multidimensional (Andrieu,2010). The state process of $x_{1:T}$ can be observed through process of observations as $y_{1:T} \cong \{y_{1,}y_2, \ldots, y_T\}$. These observations are assumed to be conditionally independent with probability density $g_\theta(y_t|x_t)$.

u_t is system noise and v_t is observation error.
In this paper, h_θ and g_θ in Eq. (1) and (2), considered as a pair of non-linear functions and model parameters θ are unknown and need to be estimated from the observed data. Also, two probability density functions, $p_\theta(.)$ and $p(\theta,.)$, corresponding to cases whose parameters are known and unknown, respectively. The posterior density of unknown parameter θ, based on the Bayes rules is as following:

$$p(x_{1:T}, \theta|y_{1:T}) \propto p(\theta)p_\theta(y_{1:T}|x_{1:T}) \tag{3}$$

Where $p(\theta)$ considered as prior density of θ and $p_\theta(y_{1:T}|x_{1:T})$ considered as a likelihood function and $p(x_{1:T}, \theta|y_{1:T})$ is the posterior density of unknown parameter θ.

3 SMC AND PMCMC APPROACH

SMC methods are a class of algorithms used to sequentially approximate the posterior density $p_\theta(x_{1:T}|y_{1:T})$ by utilizing a set of N weighted random samples called particles through the Eq. (4). (Andrieu,2010). This posterior function is simply expressing the plausibility's of different parameter values for a given sample of data.

$$p_\theta(x_{1:T}|y_{1:T}) \approx \sum_{i=1}^{N} W_T^i \delta_{x_{1:T}^i}(dx_{1:T}) \tag{4}$$

Where, W_T^i is importance weight associated with particle $x_{1:T}^i$, $\delta_x(S)$ is a Dirac measure at given state x. Importance weight acts as a correction weight to balance the probability sampling from a different distribution. The SMC algorithm does state and posterior density estimation through propagating particles $x_{1:T}^i$ and updating the weights of each particle (samples) using Eq. (8)., normalizes them and computes W_t^i using Eq. (9). This approached is iterated using importance sampling technique and predetermined importance density$q_\theta(.|.)$. In SSM models, usually, transition probability density $h_\theta(x_{t+1}|x_t)$ will be used as importance density $q_\theta(.|.)$. The algorithm for SMC is described below (Andrieu,2010):

Step1: at time t=1, (Sample noted as upper case X_1^i, where superscript i denotes the i^{th} sample and 1 in the subscript notes as sample at step 1 or initial sample)

a) Draw samples $X_1^i \sim q_\theta(.|y_1)$ (importance density given observation y_1 at time t=1)
b) Compute and normalize the weights (for N samples)

$$w_1(X_1^i) := \frac{p_\theta(X_1^i|y_1)}{q_\theta(X_1^i|y_1)} \tag{5}$$

$$W_1^i := \frac{w_1(X_1^i)}{\sum_{i=1}^{N} w_1(X_1^i)} \tag{6}$$

Step2: at time t=2… T,

a) Draw a sample $A_{t-1}^i \sim F(.|\overrightarrow{W}_{t-1})$ (where $\overrightarrow{W}_{t-1} = (W_{t-1}^1, W_{t-1}^2, \ldots, W_{t-1}^N)$ (36)
b) Sample $X_t^i \sim q_\theta(.|y_t, X_{t-1}^{A_{t-1}^i})$ and set

$$X_{1:t}^i := (X_{1:t-1}^{A_{t-1}^i}, X_t^i) \tag{7}$$

c) Compute and normalize the weights.

$$w_t(X_{1:t}^i) := \frac{p_\theta(X_{1:t}^i|y_{1:t})}{p_\theta(X_{1:t-1}^{A_{t-1}^i}|y_{1:t-1})q_\theta(X_t^i|y_t, X_{t-1}^{A_{t-1}^i})} \tag{8}$$

$$W_t^i := \frac{w_t(X_{1:t}^i)}{\sum_{i=1}^{N} w_t(X_{1:t}^i)} \tag{9}$$

Where A_{t-1}^{i} indicate the index of sample i at time *t-1* of particle$X_{1:t}^{i}$. $w_t(X_{1:t}^{i})$ refers to the weight of particle $X_{1:t}^{i}$ before normalizing. $F(.|\overrightarrow{W}_{t-1})$ is discrete probability distribution of sample weights. W_t^i is associated with the normalized weights of particles$X_{1:t}^{i}$. Generally, the algorithm assigns higher weights to particles that are more likely to generate the observed value, denoted as y_t,recorded by the model. Subsequently, the algorithm normalizes these weights to ensure their sum equals 1.

4 PG ALGORITHM

In PG algorithm, the target distribution is $p(x_{1:T},\theta|y_{1:T})$. To calculate this target distribution, the algorithm samples iteratively from $p(\theta|y_{1:T,}x_{1:T})$ and $p_\theta(x_{1:T}|y_{1:T})$ (Andrieu,2010). Since the posterior density$p_\theta(x_{1:T}|y_{1:T})$ becomes highly multidimensional in nonlinear dynamic systems, direct sampling from it becomes intractable. Consequently, the PG algorithm employs sampling from an SMC approach instead. In this algorithm, $X_{1:T}^{i}$ are sampled from $p_\theta(x_{1:T}|y_{1:T})$ by using conditional SMC. In conditional SMC algorithm, there is pre-specified path for particles $X_{1:T}^{i}$ and this path has pre-specified ancestral lineage $B_{1:T}^{i}$. In conditional SMC, in each iteration, the generated particles are conditional on particles of previous steps which means that if $X_1^i\sim q_\theta(.|y_1)$, then the next particle X_2^i will be sampled as below:

$X_2^i|X_1^i \sim q_\theta(.|y_1)$ for each *N* and path is updated in each iteration. The pseudocode of PG sampler is described as follow:

Step1: initialize Markov chain at *i*=0. For $\theta(0)$ sampling from its full conditional distribution $p(\theta|y_{1:T},x_{1:T}(0))$ .set $X_{1:T}(0)$ and ancestral lineage $B_{1:T}(0)$ arbitrarily.

Step2: for i=1…, M

a) Sample a new parameter $\theta(i)$ from the full conditional distribution $p(\theta|y_{1:T},x_{1:T}(i-1))$ which is conditional distribution of unknown parameter θ
b) Run conditional SMC to estimate the posterior density of $p_{\theta(i)}(x_{1:T}|y_{1:T})$ for parameter $\theta(i)$, conditional on particles of $X_{1:T}(i-1)$ and their ancestral lineage $B_{1:T}(i-1)$ (Particles of previous step)
c) Sample new particles $X_{1:T}(i)$ from estimated $p_{\theta(i)}(x_{1:T}|y_{1:T})$ and its ancestral lineage.

Step3: iterate step2 and record Markov Chain $\theta(i)$ and particles $X_{1:T}(i)$ for i=0, …, M

In summary, the algorithm first initialize value for $\theta(i=0)$ and $X_{1:T}(0)$ and its ancestral lineage at zero. In the next step the new sets of $\theta(i)$ for *i*=1, 2,…,M will be sampled from the full conditional distribution conditional on sampled$X_{1:T}(i-1)$ in previous step.

5 PMH ALGORITHM

This algorithm employs SMC method to estimate the posterior density $p(x_{1:T},\theta|y_{1:T})$ and samples from the updated posterior density to estimates the unknown parameter (Andrieu,2010). Unlike PG algorithm, PMH sampler jointly updates θ and particles $X_{1:T}$ and constructs the Markov Chain of $(x_{1:T},\theta)$.

To summarize, in each iteration of PMH algorithm, the algorithm draws a new parameter value from proposal density $q(.|x_{1:T},\theta)$, then, based on the posterior density generated by SMC algorithm and the prior distribution of the parameter, the PMH algorithm calculated the acceptance ratio of the parameter shown in Eq. (10). The PMH algorithm is as follow:

Step1: initialization, i=0 $q(.|\theta)$
Set $\theta(0)$ arbitrarily.
Run SMC algorithm targeting $p_{\theta(0)}(x_{1:T}|y_{1:T})$ and sample $X_{1:T}(0)$ from the resulting estimated distribution $\hat{p}_{\theta(0)}(.|y_{1:T})$.

Step2: for iteration i≥1,
Sample the new parameter θ^* from the proposal density $q(.|\theta(i-1))$
Run SMC algorithm targeting $p_{\theta^*}(x_{1:T}|y_{1:T})$. Sample new samples $X^*{}_{1:T}$ from its transition probability distribution $h_\theta(x_{t+1}|x_t)$
Let $p_\theta(y_{1:T})$ denote marginal likelihood estimate with probability.

$$min(1,\frac{p(x_{1:T}^*,\theta^*|y_{1:T})q(x_{1:T},\theta|x_{1:T}^*,\theta^*)}{p(x_{1:T},\theta|y_{1:T})q(x_{1:T}^*,\theta^*|x_{1:T},\theta)} = \frac{q(\theta|\theta^*)p_{\theta^*}(y_{1:T})p(\theta^*)}{q(\theta^*|\theta)p_\theta\ \ (y_{1:T})p(\theta)} \tag{10}$$

accept the new samples. We generate a random value between 0 and 1 and compare it with the acceptance ratio generated in Eq. (10). The new parameter θ^* will be accepted if the acceptance ratio is greater than the generated random number and set $\theta(i)=\theta^*$, $X_{1:T}(i)=X^*{}_{(1:T)}$; otherwise we reject the new sample and set $\theta(i)=\theta(i-1)$ and $X_{1:T}(i)=X_{1:T}(i-1)$

6 SSM FOR BAXTER MANIPULATOR

The robotic platform utilized was a 7 DOF Baxter Research robot. In each joint, Series Elastic Actuators (SEAs) are the actuation mechanisms responsible for moving the robot links. Non-linear dynamic model of Baxter manipulator is described by a second-order differential equation is shown as following:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \tag{11}$$

Where q denotes the vector of joint angles, which in our case is 7×1 vector; $M(q) \in \Re^{7\times 7}$ is the symmetric, bounded, positive definite inertia matrix (including mass and moment of inertia), $C(q,\dot{q})\dot{q} \in \Re^{7\times 7}$ denotes the Coriolis and Centrifugal force; $G(q) \in \Re^{7}$ is the gravitational force, and $\tau \in \Re^{7}$ is the vector of actuator torques which in our case is 7×1 vector. A Euler discretization of the differential equation of the robot manipulator model yields:

$$q_{1,t+1} = q_{1,t} + hq_{2,t} \tag{12}$$

$$\begin{aligned} q_{2,t+1} = q_{2,t} - hM^{-1}(q_{1,t})C(q_{1,t},q_{2,t})q_{2,t} \\ - hM^{-1}(q_{1,t})G(q_{1,t}) \\ + hM^{-1}(q_{1,t})\tau \end{aligned} \tag{13}$$

Where $q_{1,t}$and $q_{2,t}$ are 7×1 vector and h is the step size. We assume that the manipulator model is influenced by the disturbance which is a stochastic white noise with zero mean and a covariance matrix $\Sigma_1, \Sigma_2 \in \Re^{7\times 7}$.

Considering these disturbances in Eq. (12). and Eq. (13). yields:

$$\begin{aligned} q_{1,t+1} &= q_{1,t} + hq_{2,t} + u_{1,t} \\ q_{2,t+1} &= q_{2,t} - hM^{-1}(q_{1,t})C(q_{1,t},q_{2,t})q_{2,t} \\ &\quad - hM^{-1}(q_{1,t})G(q_{1,t}) \\ &\quad + hM^{-1}(q_{1,t})\tau + u_{2,t} \end{aligned} \tag{14}$$

$$y_t = q_{2,t} + v_t \tag{15}$$

Where noise is defined as vector $u_t = (u_{1t}, u_{2t})$ and measurement error considered as vector $v_t = u_{3t}$. As we assumed to have a white noise in the dynamic system, the noise and measurement distributions considered as follow:

$$u_{1t} \approx N(0,\Sigma_1) \tag{16}$$

$$u_{2t} \approx N(0,\Sigma_2) \tag{17}$$

$$u_{3t} \approx N(0,\Sigma_3) \tag{18}$$

$N(.,.)$represents normal distribution.

Also, we assume that measurement error is in form of additive white noise with zero mean and a covariance matrix Σ_3. $\Sigma_1, \Sigma_2, \Sigma_3$ are 7×7 positive definite matrices corresponding to variances of u_{1t}, u_{2t}, u_{3t}, respectively. The goal is to estimate unknown parameters of vector θ where $\theta = (\tau, \Sigma_1, \Sigma_2, \Sigma_3)$ by using two algorithms: the PG algorithm and PMH algorithm, based on system kinematic data. In the PG algorithm, first $\theta(0)$ is initialized and then a new sample θ is drawn from the full conditional distribution of θ. The SMC algorithm is run to estimate the posterior density and obtain samples $\{q_{1:T}^{(i)}\}_{i=1}^{N}$. In the Baxter, the lower and upper bound for the parameter τ is given and because the probability of all torque values within this boundary is equal, the proper prior distributions for the unknown parameter τ is a multivariate uniform distribution:

$$\tau \approx U(a,b) \tag{19}$$

$$a = (0,0,0,0,0,0,0)^T \tag{20}$$

$$b = (50,50,50,50,15,15,15)^T \tag{21}$$

Multivariate uniform distribution is a generalization of one-dimensional uniform to higher dimensions (Wackerly,2016). Values of these vectors came from the Baxter manipulator joint torques limits. The prior distribution for the parameters of the measurement error and the noise considered as multivariate inverse gammas distribution which is also called inverse Wishart. As $\Sigma_1, \Sigma_2, \Sigma_3$ are the parameters of the measurement error and the noise and they are coming from multivariate normal distributions and they are covariance matrices, inverse Wishart distribution, represented as $IW(Q,p)$, with scale matrix *'Q'* and degrees of freedom *'p'*, is the conjugate prior distribution for them. The priors for the parameters measurement error and the noise considered as follow:

$$\Sigma_1 \approx IW(Q_1,p_1) \tag{22}$$

$$\Sigma_2 \approx IW(Q_2,p_2) \tag{23}$$

$$\Sigma_3 \approx IW(Q_3,p_3) \tag{24}$$

Where Q_1, Q_2, Q_3 are symmetric positive definite scale matrices and p_1, p_2, p_3 are degrees of freedom. As the full conditional distributions of each unknown parameters are needed for PG algorithm to sample the new parameters from them, these full conditional distributions have been derived and the derivation results are shown below:

$$\begin{aligned} &f(\tau|-\tau, q_{1:T}, y_{1:T}) \sim \\ &N_{[c,d]}\Big(\sum_{t=1}^{T-1} A_t^T \sum{}^{-1}_{2} A_t)^{-1}\left(\sum_{t=1}^{T-1} A_t^T \sum{}^{-1}_{2} B_t\right), \\ &\textstyle\sum_{t=1}^{T-1} A_t^T \sum{}^{-1}_{2} A_t)\Big) \end{aligned} \tag{25}$$

$$f(\Sigma_1 | -\Sigma_1, q_{1:T}, y_{1:T}) \sim IW(p_1 + T - 1, Q_1 + \sum_{t=1}^{T-1} u_{1t}\, u_{1t}^T) \tag{26}$$

$$f(\Sigma_2 | -\Sigma_2, q_{1:T}, y_{1:T}) \sim IW(p_2 + T - 1, Q_2 + \sum_{t=1}^{T-1} u_{2t}\, u_{2t}^T) \tag{27}$$

$$f(\Sigma_3 | -\Sigma_3, q_{1:T}, y_{1:T}) \sim IW(p_3 + T, Q_3 + \sum_{t=1}^{T} u_{3t}\, u_{3t}^T) \tag{28}$$

Where, $N_{[c,d]}(.,.)$is a truncated normal distribution within interval [c,d] and the minus before a parameter means this parameter in not in the parameter set θ.

Other terms in Eq. (25). are:

$$A_t = hM^{-1}(q_{1,t}) \tag{29}$$

$$B_t = q_{2t+1} - q_{2t} + h\varpi(q_{1t}, q_{2t}) \tag{30}$$

Where:

$$\varpi(q_{1,t}, q_{2,t}) = M^{-1}(q_{1,t})C(q_{1,t}, q_{2,t})q_{2,t} + M^{-1}(q_{1,t})G(q_{1,t}) \tag{31}$$

In the PMH algorithm, to run the SMC algorithm and updating the state variables and their weights, transition density and observation density is needed. Regarding to assumptions described in (Dahlin,2019) for the PMH algorithm, we considered below multivariate normal densities as the probability transition density $h_\theta(q_{t+1}|q_t)$ and observation density $g_\theta(y_t|q_t)$

$$h_\theta(q_{1,t+1}|q_t) \sim N(q_{1,t} + hq_{2,t}, u_{1t})$$

$$h_\theta(q_{2,t+1}|q_t) \sim N(q_{2,t} - hM^{-1}(q_{1,t})C(q_{1,t}, q_{2,t})q_{2,t} - \tag{32}$$

$$hM^{-1}(q_{1,t})G(q_{1,t}) + hM^{-1}(q_{1,t})\tau, u_{1t}) \tag{33}$$

$$g_\theta(y_t|q_t) \sim N(q_{2,t}, u_{3t}) \tag{34}$$

In PMH algorithm, we need to define a proposal distribution. In our case, due to considering multiple unknown parameters in the system and because θ is multidimensional, a multivariate normal distribution has been considered for proposal distribution.

7 RESULTS

To test the PG algorithm, initial settings and prior distributions for the system parameters have been used. This algorithm first initialized $\theta(0)$ using its full conditional distributions and the new parameter $\theta(i)$ sampled from full conditional distributions. PG algorithm were run for 10,000 steps and the first 2,500 steps are discarded as a burn-in step. The number of the particles were chosen as *N*= 1000 for the SMC algorithm. The bigger number of particles results in better estimation (Elvira,2016) but increasing the number of particles over a certain value may not significantly improve the quality of the approximation while decreasing the number of particles can dramatically affect the performance of the filter (Elvira,2016). The posterior distributions of the unknown parameters $\theta = (\tau, \Sigma_1, \Sigma_2, \Sigma_3)$ including seven joint torques, observation error and noise generated by the PG algorithm are shown in Fig.1. As mentioned earlier, $\Sigma_1, \Sigma_2, \Sigma_3$ are 7×7 diagonal matrices which diagonal elements are $\sigma1, \sigma2, \ldots, \sigma7$. The same initial settings have been considered for the PMH algorithm. The proposal density $q(.|\theta) \sim N(\theta, C)$ where all elements of C, are 10^{-5} has been considered. Same as PG algorithm, the number of the particles in SMC algorithms, were chosen as N=1000 particles. In practical applications, the convergence of the algorithms has been checked to ensure that the samples drawn from the sequential Markov Chain are sampled from correct target distributions. The algorithms ran for 10000 steps, and the first 2500 steps are discarded as burn-in steps. Table 1 and Table 2 compares the true and estimated

Table 1: True and estimated parameter values for Baxter manipulator system using PG algorithm.

	Parameters	True values	Estimated values		Parameters	True values	Estimated values
	τ_1	0.7	0.70007317130		σ_1	2e-7	2.6164e-7
	τ_2	0.6	0.60503887197		σ_2	2e-7	1.9921e-7
	τ_3	1.8	1.79937309232		σ_3	2e-7	2.6292e-7
τ	τ_4	1	0.99945268679	Σ_2	σ_4	2e-7	2.20990e-7
	τ_5	0.6	0.59990970609		σ_5	2e-7	2.1428e-7
	τ_6	0.25	0.24996085840		σ_6	2e-7	3.5965e-7
	τ_7	0.085	0.08499487445		σ_7	2e-7	2.8616e-7
	σ_1	2e-7	2.7761 e-7		σ_1	2e-7	2.4346e-7
	σ_2	2e-7	2.8072 e-7		σ_2	2e-7	6.6409 e-7
	σ_3	2e-7	2.9171 e-7		σ_3	2e-7	2.6719 e-7
Σ_1	σ_4	2e-7	2.9808 e-7	Σ_3	σ_4	2e-7	5.197 e-7
	σ_5	2e-7	2.7667 e-7		σ_5	2e-7	5.7298 e-7
	σ_6	2e-7	2.9012 e-7		σ_6	2e-7	6.0577 e-7
	σ_7	2e-7	2.8435 e-7		σ_7	2e-7	1.1084 e-7

Table 2: True and estimated parameter values for Baxter manipulator system using PMH algorithm.

	Parameters	True values	Estimated values		Parameters	True values	Estimated values
	τ_1	0.7	0.75166502867		σ_1	2e-7	2.3663 e-7
	τ_2	0.6	0.67385067623		σ_2	2e-7	1.1683 e-6
	τ_3	1.8	1.72964483008		σ_3	2e-7	3.2121 e-7
τ	τ_4	1	1.20124443732	Σ_2	σ_4	2e-7	2.6992 e-7
	τ_5	0.6	0.70002775868		σ_5	2e-7	1.3277 e-7
	τ_6	0.25	0.28008934183		σ_6	2e-7	5.0762 e-7
	τ_7	0.085	0.09099767778		σ_7	2e-7	3.3479 e-7
	σ_1	2e-7	3.0841 e-7		σ_1	2e-7	6.3016e-7
	σ_2	2e-7	3.0069 e-7		σ_2	2e-7	7.2581e-02
	σ_3	2e-7	2.8161 e-7		σ_3	2e-7	1.4294 e-5
Σ_1	σ_4	2e-7	2.7359 e-7	Σ_3	σ_4	2e-7	3.8908 e-6
	σ_5	2e-7	2.8486 e-7		σ_5	2e-7	3.7973 e-7
	σ_6	2e-7	3.4936 e-7		σ_6	2e-7	3.5284 e-7
	σ_7	2e-7	2.7719 e-7		σ_7	2e-7	1.7562 e-7

parameter values using a PG and PMH algorithms, respectively. The generated PMH algorithm is not sensitive to the initial values of parameters.

8 CONCLUSIONS

This paper employs two Particle Markov Chain Monte Carlo (PMCMC) methods to estimate unknown parameters of the Baxter robotic manipulator, including joint torques, noise, and measurement errors within a nonlinear dynamic system. Accurate estimates of true state variables are achieved by estimating state variables within the State-Space Model. In this study, SMC technique is employed to estimate the states, and based on these estimates, the system parameters are further estimated using both PG and PMH algorithms.

SMC's capability to construct high-dimensional proposal distributions in each iteration enhances the reliability of PG and PMH algorithms in estimating joint torques, noise, and measurement errors. This contrasts with regular MCMC algorithms, which rely on lower-dimensional proposal distributions.

Consequently, implementing these methods enables the precise estimation of unknown robotic parameters, providing more realistic data for subsequent investigations.

The results indicate that for the Baxter manipulator, both PG and PMH algorithms perform

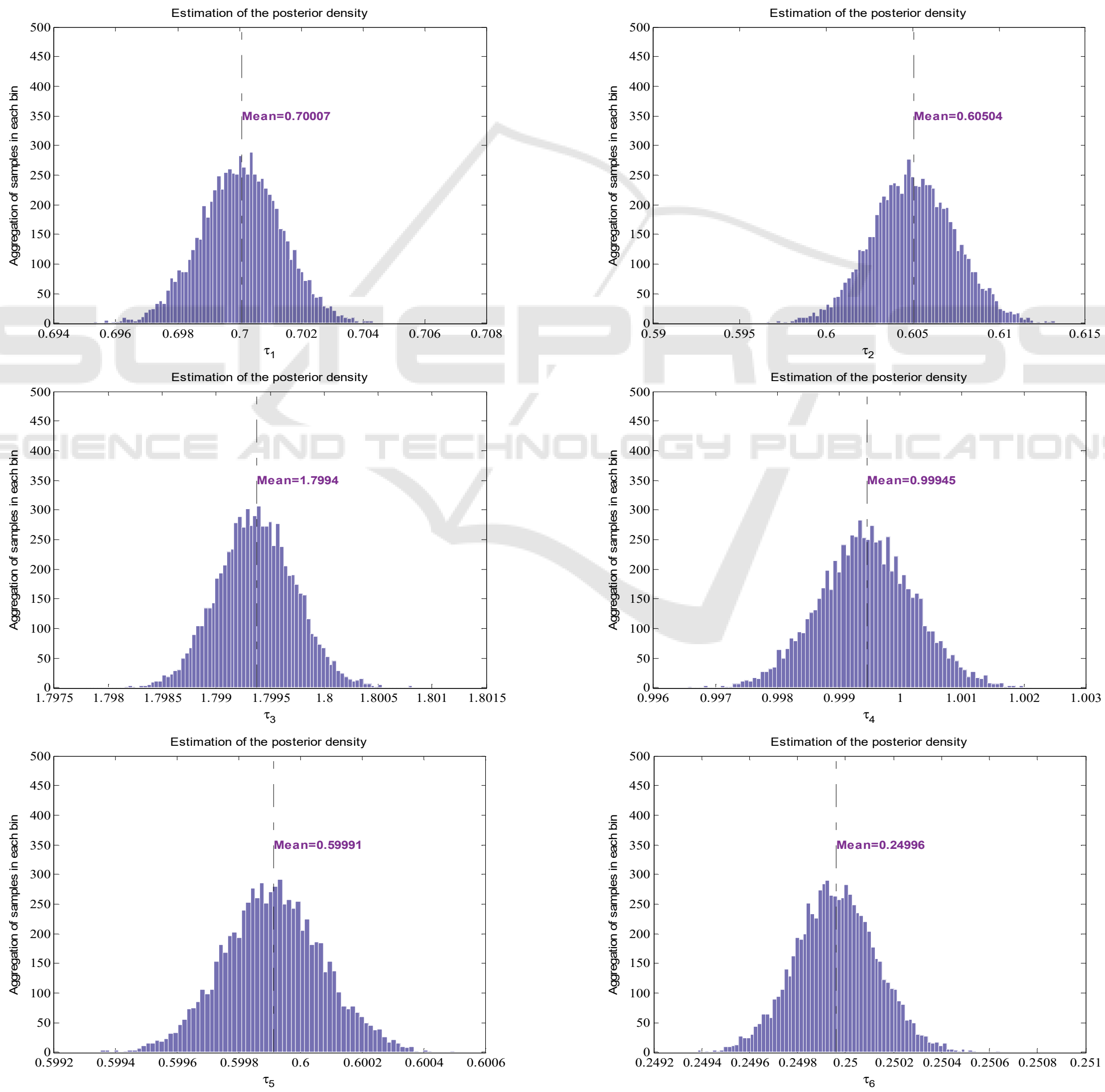

Figure 1: Histogram approximation of posterior densities of parameters τ1, τ2, τ3, τ4, τ5, τ6 based on output of the PG algorithm.

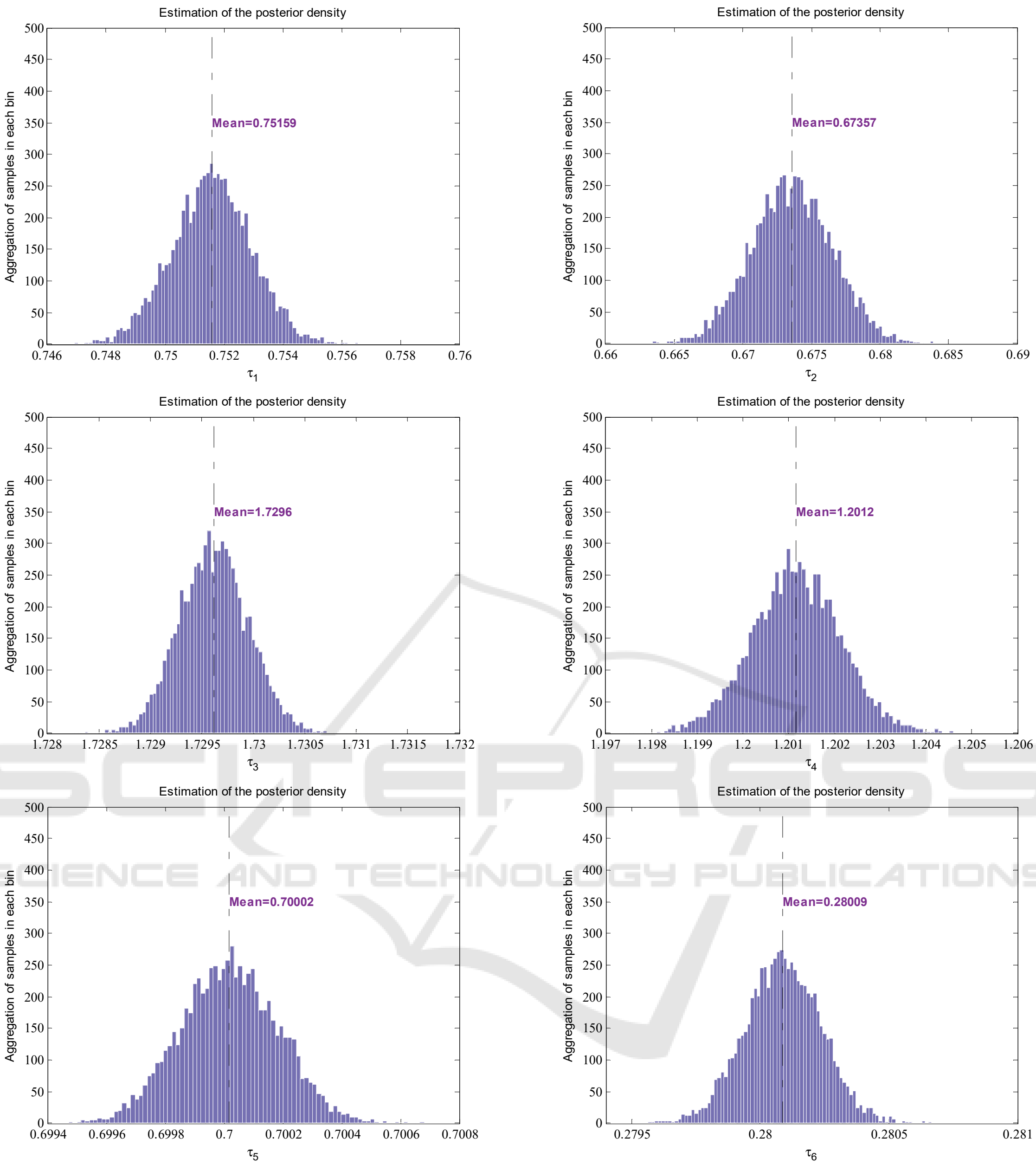

Figure 2: Histogram approximation of posterior densities of parameters $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6$ based on output of PMH algorithm.

satisfactorily, with PG demonstrating superior performance owing to its utilization of parameters' conditional distributions.

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