Maximizing Particle Coverage with Fixed-Area Rectangles

Seung-Yeol Hong[a] and Yong-Hyuk Kim[b]
Department of Computer Science, Kwangwoon University, Seoul, Republic of Korea

Keywords: Genetic Algorithm, Optimization, Deployment.

Abstract: In this study, our primary focus is on enhancing particle coverage through the effective deployment of rectangles with fixed areas and flexible shapes within a two-dimensional space. These rectangles represent areas optimally managed by various units like CCTV cameras, police personnel, search and rescue units and robots. Particles are objects designated for processing by these units. To realize this objective, we presented an efficient technique for deploying rectangles in a two-dimensional space using a genetic algorithm (GA). The GA searches for the optimal deployment of rectangles that maximizes the sum of the particle densities represented int the heat map. We experimented by applying our problem to maritime search and rescue planning. The main application of our method in maritime search and rescue planning is the deployment of search and rescue units in the ocean. As a result, the GA outperformed the greedy method by up to 14%. The experimental outcomes demonstrate the superiority of our proposed method compared to existing techniques. Specifically, its effectiveness becomes more pronounced when the total area covered by the placed rectangles is smaller than the entire search area.

1 INTRODUCTION

Determining a maritime search and rescue (SAR) plan considers a variety of factors, including the expected location of the search target, the availability of search and rescue units (SRU). So, determining a maritime SAR plan is a challenging task.

With the advancement of science and technology, research is being conducted on decision-making systems for maritime SAR planning. One study improved the mathematical model used for SAR and used genetic simulated annealing algorithm (GSAA) to support SRU resource scheduling (Ai et al. 2019). They designed a search area allocation algorithm that considers spatio-temporal characteristics. Another study conducted a comprehensive review on decision support in maritime emergencies and proposed a decision support method using two optimization algorithms (Xiong et al. 2020). They are used differential evolution (DE) (Storn et al. 1997) and NSGA-II (Deb et al. 2000) to find appropriate SAR plans and support SRU resource scheduling.

Determining a search area for SRUs is an important part of SAR planning. Existing studies set up one large rectangle that represents the entire search area and then divide it to deploy SRUs. However, there are two problems with this approach. First, efficiency is reduced when SRUs cannot cover the entire search area. Second, SRUs may be wasted on unimportant areas because all search areas are explored.

In this paper, we present a generalization of the problem of placing SRUs in SAR planning. The generalized problem has broader applicability and can be extended to various other problems. Furthermore, we propose an optimal rectangle deployment method using a genetic algorithm (GA) that improves upon the existing studies. We deploy rectangles in the two-dimensional space where search targets represented by particle exists. The rectangles represent areas that can be explored by SRUs. These rectangles are deployed to contain as many particles as possible.

The problem of deploying rectangles to cover as many particles as possible can be applicable not only to SAR planning but also to various other problems. For example, police can be distributed on heavily travelled streets to clear traffic and prevent accidents. In another example, sensors can be deployed on most sensitive areas. One study (Yoon et al. 2022) used a

[a] https://orcid.org/0009-0006-5806-3401
[b] https://orcid.org/0000-0002-0492-0889

Hong, S. and Kim, Y.
Maximizing Particle Coverage with Fixed-Area Rectangles.
DOI: 10.5220/0012162900003595
In *Proceedings of the 15th International Joint Conference on Computational Intelligence (IJCCI 2023)*, pages 172-178
ISBN: 978-989-758-674-3; ISSN: 2184-3236

GA to deploy sensors and maximize space coverage, while another study (Seo et al. 2017) used a GA for the efficient deployment of CCTVs on streets. However, the limitation of previous studies is the representation of sensors as fixed-size circles or fans, restricting their applicability to diverse problem scenarios. In particular, configuring the search area as a curve in SAR planning presents disadvantages. During the path planning phase, when the search area contains curves, obtaining crucial variables like the probability of detection becomes challenging. Hence, the circle and fan shapes, as employed in prior studies, are not suitable for SAR planning, and rectangles prove to be a better fit for such scenarios.

Meanwhile, research is also being conducted on a problem similar to SAR planning: planning paths for Unmanned Aerial Vehicles (UAVs) (Mansouri et al. 2017, Akshya et al. 2020). These problems involve deploying a rectangle that covers a specific polygon or object and devising a path around the rectangle's center point. However, these problems typically assume that the UAV can cover the entire area. If the UAV lacks the time to explore the entire region, it necessitates a strategy to explore the crucial sections first. Our suggested rectangle placement method positions rectangles exclusively in important areas, making it suitable for UAV path planning under time constraints.

This paper is organized as follows. Section 2 describes how SRUs have been deployed in previous studies of SAR. We modify this method to our problem and compare it to our proposed method. Section 3 describes our proposed method. Section 4 describes the environment we experimented in and the setup for our experimental methods. Section 5 discusses the results of our experiments. Finally, in Section 6, we outline potential directions for future work.

```
GA (offspring size n, max generation g_max) {
    P ← generate 200 random initial solutions
    for generation g ← 1 to g_max
        for i ← 1 to n
            p1, p2 ← select two parent chromosomes
                    using Roulette wheel
            oi ← generate offspring chromosome from
                    p1, p2 using uniform crossover
            oi ← mutation(oi)
            oi ← repair(oi)
        end for
        replace n chromosomes in P with o1 to on
    end for
    return best chromosome in P
}
```

Algorithm 1: Pseudo-code of our GA.

2 PREVIOUS METHOD

In this part, we introduce how previous studies (Ai et al. 2019 & Xiong et al. 2020) have deployed the search area of SRUs. In existing studies, the entire search area is divided into equal-sized cells, and then the following algorithm is performed.

Step 1: select a cell with the highest value, among the areas where the SRU is not placed.
Step 2: expand the selected area by one column or one row in the direction of the higher fitness value (up, down, right, and left).
Step 3: repeat process Step 2 until the fitness value does not increase.
Step 4: when Step 3 ends, place the SRU in the selected area.
Step 5: repeat process Step 1 through Step 4 until the entire search area is covered.

Previous studies have employed specific functions to evaluate fitness values. The utilized function takes into consideration the rectangle's dimensions and the cumulative values of the cells it encompasses. If only the cumulative cell values were considered, the rectangle would expand indefinitely. However, the function uses a ratio of the size of the rectangle to the sum of the cell values so that the evaluation value decreases after the rectangle is large enough.

3 PROPOSED METHOD

3.1 Data

In the two-dimensional space where the rectangles are deployed, there are search targets represented by particles. If there are too many particles, the subsequent process, GA deploying rectangles, will take a long time. To address this, we simplify the two-dimensional space into a heat map. The heat map consists of N-by-N cells. Each cell has as its value the density of the particles it contains. Figure 1 shows the heat map we used in our experiment. Higher values in the cells are colored as red, and lower values in the cells are colored as blue. Cells with very few particles and therefore not important at all are colored as white.

3.2 Genetic Algorithm

A genetic algorithm (GA) is one of the most famous evolutionary algorithms. It is an optimization algorithm developed to mimic the evolutionary process of living organism (Holland, 1975).

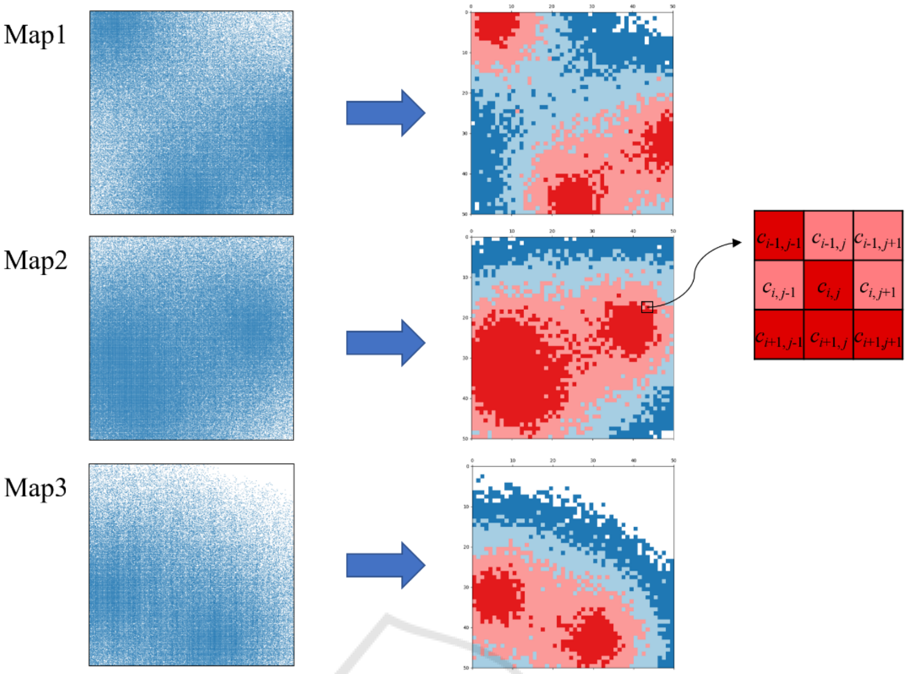


Figure 1: Heat map of particles in a given domain.

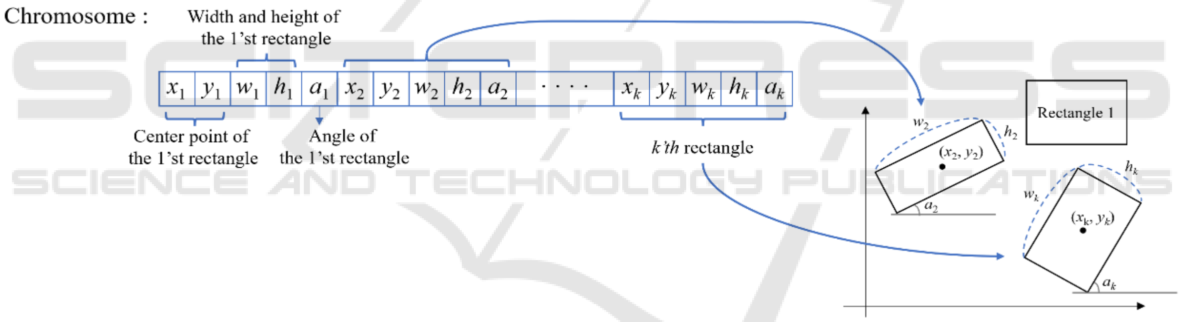


Figure 2: Example illustration of the genotype (left) and the phenotype (right) of a chromosome.

We propose using the GA to deploy rectangles on the data described in Section 3.1. We designed a method for representing rectangles in the two-dimensional space and evaluating their performance. Additionally, we designed a repair operator to maintain a fixed rectangle area, which is applied after the mutation operation. The structure of our proposed GA is shown in Algorithm 1.

3.2.1 Encoding

Rectangles are deployed in the heat map to indicate that the SRU is limited. The shape of the rectangles is free; however, its area (the product of width and height) must be fixed at a certain value. This is indicating that SRUs can only search a limited area.

Encoding defines the way how the solution is represented. In this study, a rectangle (gene) is represented by an array of {x-coordinate of the center , y-coordinate of the center, width (w), height (h), and angle(a)}. Thus, the chromosome with k rectangles is represented by a one-dimensional array with $5k$ elements. Figure 2 shows how we represent rectangles as chromosome. a is the angle the rectangle is rotated with respect to the x-axis. The number under each variable is the number of the rectangle. For example, x_1 is the x-coordinate of the center of the first rectangle.

3.2.2 Objective Function

As the objective function we compute a fitness score, which indicates how well the chromosome is suited

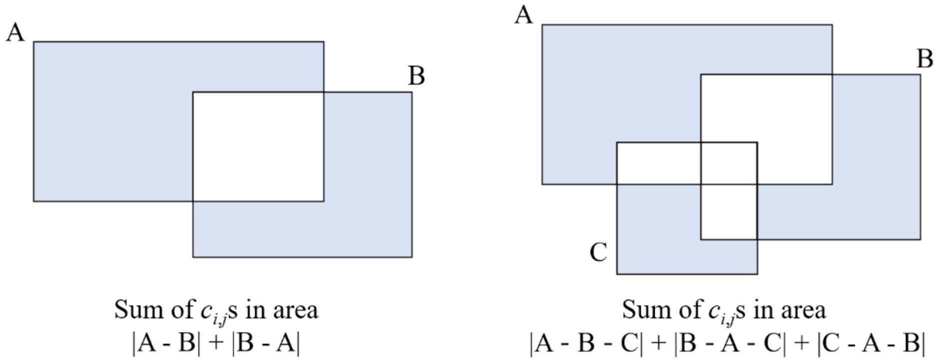


Figure 3: How to apply a penalty to the objective function to exclude overlapping regions.

to the problem. It evaluates the chromosomes by summing up the values of all cells within the heat map that are covered by the rectangle. The objective function considers a rectangle to contain a cell if the cell's center point falls within the rectangle.

To incentivize rectangles to cover as many cells as possible, we introduce a penalty for overlapping regions. If a rectangle overlaps with another, the overlapping area is excluded from the calculation of the objective function. Figure 3 illustrates the application of penalties in the objective function. The fitness score is computed by considering the values of the cells within the colored region. However, cells in the uncolored region, where the rectangles overlap, are not taken into account for the fitness score computation. This penalty is applied for the following two reasons.

Firstly, when rectangles overlap, it undermines our objective of maximizing cell coverage with a limited number of rectangles. If the rectangles overlap, it is natural that the number of cells they can cover will be reduced. So, by applying a penalty and excluding the overlapping areas within the objective function, our genetic algorithm can converge towards optimal solutions more effectively.

Secondly, in real-world scenarios, the overlap of rectangles can lead to negative consequences. In the context of SAR planning, when rectangles overlap, it indicates that the search areas of SRUs also overlap. This overlap can result in adverse outcomes, such as collisions or an increased scale of accidents. Thus, penalizing overlapping areas helps to avoid such undesirable situations.

Equation (1) represents our objective function. Let $c_{i,j}$ denote the value of the cell in row i and column j. A variable $w_{i,j}$ determines whether or not a penalty is applied. If only one rectangle contains the cell in row i and column j, $w_{i,j}$ is set to 1. On the other hand, if no rectangle contains the cell in row i and column j or if multiple rectangles contain the cell in row i and column j, $w_{i,j}$ is set to 0. A set of rectangles with no overlapping areas will receive a better evaluation when using this objective function.

$$f = \sum_{i=1}^{N} \sum_{j=1}^{N} c_{i,j} \times w_{i,j} \text{ ,}$$
where
$$w_{i,j} = \begin{cases} 1 \text{ if exatly one rectangle contain } c_{i,j} \\ 0 \text{ otherwise} \end{cases} \quad (1)$$

3.2.3 Repair Operator

The repair operator fixes the height or width using the fitness score. The repair operator divides the fixed-area by the height to get the modified width. Conversely, the modified height is obtained by dividing the fixed-area by the width. Then, the repair operator selects the one that has a higher fitness score when the modified height or modified width is applied. The pseudo-code for the repair operation is presented in Algorithm 2.

4 EXPERIMENTAL SETUP

We conduct experiments using three different methods, all share the following two settings.

First, to represent a resource scarce situation, we set the number of rectangles, k, to 3. Thus, the chromosome length in our experiment is 15 (= 3 × 5). Second, we set the size of the heat map, N, to 50.

4.1 GA Parameters

The GA selection operation uses a Roulette wheel. The variable to control the selection pressure of the Roulette wheel is set to 3. For the crossover operation, uniform crossover and a mutation probability of 0.05 are applied. When mutations occur, different ranges of random values are applied to each gene. For the genes x and y, which represent

Table 1: Experiment results of 30 runs.

	Greedy method		Random multi-start				GA			
	-		Generation = 1000		Generation = 1500		Generation = 1000		Generation = 1500	
	Run 1 time[1]	O.A.[2]	Run 30 times	O.A.[2]	Run 30 times	O.A.[2]	Run 30 times	O.A.[2]	Run 30 times	O.A.[2]
Map1	Best 887.0	0	Best 806.6	17.8	Best 812.6	12.6	Best 917.9	0.9	Best 923.5	1.2
	Ave. -		Ave. 790.6		Ave. 794.7		Ave. 887.4		Ave. 893.8	
	Std. -		Std. 8.3		Std. 8.6		Std. 16.4		Std. 18.8	
Map2	Best 942.1	0	Best 984.1	28.6	Best 991.6	22.6	Best 1071.2	4.1	Best 1077.1	3.4
	Ave. -		Ave. 966.7		Ave. 973.2		Ave. 1035.3		Ave. 1042.4	
	Std. -		Std. 8.9		Std. 9.6		Std. 18.8		Std. 18.4	
Map3	Best 827.6	0	Best 808.2	51.6	Best 808.2	48.9	Best 892.1	11.2	Best 897.2	8.5
	Ave. -		Ave. 782.1		Ave. 786.1		Ave. 865.4		Ave. 869.0	
	Std. -		Std. 10.7		Std. 10.2		Std. 14.6		Std. 20.7	

[1] The greedy method always produces the same result in the same data.

[2] Average of the area where the rectangles overlap.

the coordinates of the rectangle's center point, a real number between 0 and N is randomly assigned. The genes w and h, representing the width and height of the rectangle, are assigned a real number between 1 and the value of fixed-area. The changes of the area are not considered at this stage, as the repair operation is carried out after the mutation operation. The gene a, representing the angle, is randomly assigned a real number between 0 and 180 degrees. After the mutation operation, the chromosome is repaired to fix the rectangular area. Fifty offspring chromosomes are generated and replace the parent chromosomes with lower fitness scores. This process is repeated for 1,000 and 1,500 generations with a population of 200 per generation.

4.2 Greedy Method

The greedy method is designed as a variation of a previous method. The difference from previous method is that the present study does not cover all highest value in the heat map as the initial area. The selected area is then extended in the direction that could have a higher fitness score. The rectangle cannot encroach on another rectangular area. The rectangle can also extend outside of the heat map, and the outside of the heat map does not affect the rectangle's fitness score. The rectangle does not expand further after filling a fixed size. After the rectangle is deployed, the chromosome is repaired.

```
Repair (chromosome = {r1, r2, … , rk}) {
    for each rectangle ri = {xi, yi, wi, hi, di}
        ci ← maximum coverage area of ri
        hfix ← {xi, yi, wi, ci / wi, di}
        wfix ← {xi, yi, ci / hi, hi, di}
        if fitness(hfix) > fitness(wfix) then
            ri ← hfix
        if fitness(hfix) < fitness(wfix) then
            ri ← wfix
    end for
    return repaired chromosome
}
```

Algorithm 2: Pseudo-code of repair operation.

4.3 Random Multi-Start

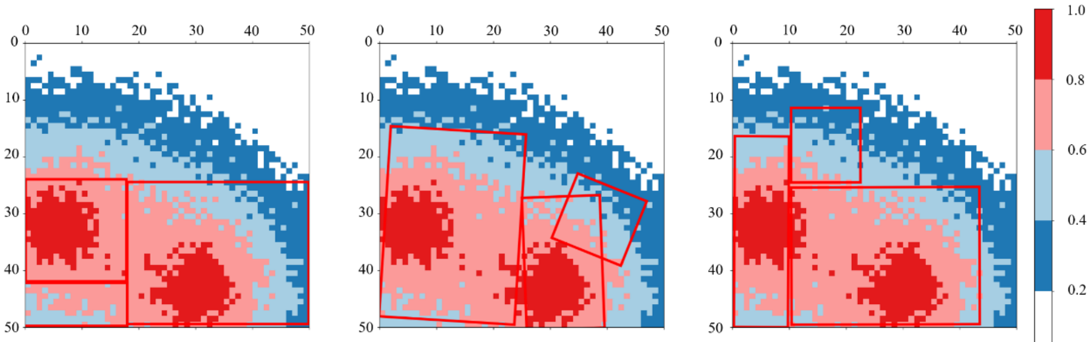
Random multi-start randomly generates as many chromosomes as the number of chromosomes generated by the GA, and it repairs them. It selects the best chromosome among them. The pseudo-code for the random multi-start is presented in Algorithm 3.

5 RESULTS

The rectangles are deployed on a heat map using the greedy method, GA, and random multi-start. Table 1

```
RandomMultiStart (offspring size n,
                  max generation gmax,
                  population P) {
    R ← | P | + n × gmax
    for i ← 1 to R
        oi ← randomly-generated chromosome
        oi ← repair(oi)
    end for
    return best in O = {o1, o2, o3, …, oR }
}
```

Algorithm 3: Pseudo-code of random multi-start.



Fitness score : 827.6 | Fitness score : 808.2 | Fitness score : 897.2

(a) Greedy Method | (b) Random Multi-Start | (c) GA

Figure 4: Best solutions in Map 3.

shows the highest, average, and standard deviation of the fitness score, and the average overlap area for 30 runs for each method. The greedy method exhibits significant performance differences depending on the characteristics of the data. In Map1 (left of Figure 1), the dense regions are separated; thus, the greedy method performs well. In contrast, in Map2 (middle of Figure 1), the dense regions are clustered together, and the greedy method does not perform well. On average, the GA exhibits a stable performance. The GA outperforms the greedy method by an average of 5% and a maximum of 14 %. The GA also reduces the average overlap area by 86% on average and 95% compared to random multi-start. Figure 4 shows the best solutions obtained from the greedy method, random multi-start, and GA in Map3.

The GA, executed on an Intel i7-7700 3.60GHz CPU with 16GB of memory, Microsoft Windows 10 operating system, and implemented using the C++ programming language, achieves an average completion time of 90 seconds. In contrast, the random multi-start approach takes approximately 58 seconds on average, while the Greedy Method exhibits a runtime of around 1 second.

6 CONCLUSIONS

In this study, we presented an efficient approach for deploying rectangles with flexible shapes and fixed sizes. While the problem we addressed has potential applications in various fields, our main goal was to apply it to SAR planning. Existing researches about SAR planning typically assumes that the search area can be completely covered. However, real-world scenarios may involve limitations in number of SRU or time availability. We successfully deployed rectangles within a heat map that represent the significance of search areas. This approach was particularly effective when not all regions could be thoroughly explored due to constraints in SRU availability or time constraints, thereby mitigating the drawbacks of prior methodologies.

7 FUTURE WORK

Future research can be conducted as follows. First, accommodating changes in the area of rectangles is important. In the real world, there is a travel time for resources to reach the deployed area. As resources are active for a fixed duration, the coverage area they can handle may decrease or p depending on their travel time. Therefore, it would be more realistic to adjust the size of the rectangles based on their deployment location.

Second, considering overlapping rectangles under specific conditions can be beneficial. The types of robots used in search and rescue operations (ground, air, marine, etc.) are becoming increasingly diverse (Queralta et al. 2020). When different types of robots are involved, overlapping search areas may not pose a risk of accidents. By incorporating this consideration, our approach can be applied to a wider range of problems by designing an objective function that cancels the penalty for overlapping rectangles under certain conditions.

Third, more experimentation is required. Although we have enhanced the performance of rectangular deployment compared to existing methods, we have not yet established a diverse comparison group. It would hold more significance to compare the effectiveness of GA with metaheuristic algorithms and other optimization techniques like NSGA-II.

In last, expanding the objective function to include additional variables is worth exploring. Our study focuses on a simple objective of placing rectangles to cover as many particles as possible. However, in real-world scenarios, multiple environmental factors need to be taken into account. For instance, in maritime SAR planning, it is crucial to consider factors such as the probability of coverage (POC) and the probability of detection (POD) of a rescue target (Ototoe et al. 2019). Furthermore, various factors like the marine environment and the likelihood of the rescue target's survival should also be taken into account. Therefore, designing the objective function to incorporate more variables would enable more accurate experiments.

ACKNOWLEDGEMENTS

This research was supported by Korea Institute of Marine Science & Technology Promotion (KIMST) funded by Korea Coast Guard (20220463)."

REFERENCES

B. Ai, B. Li, S. Gao, J. Xu and H. Shang, (2019). An intelligent decision algorithm for the generation of maritime search and rescue emergency response plans. In *IEEE Access*, vol. 7, pp. 155835-155850.

W. Xiong, P.H.A.J.M. van Gelder, K. Yang, (2020). A decision support method for design and operationalization of search and rescue in maritime emergency. In *Ocean Engineering*, vol. 207, 107399.

R. Storn, K. Price, (1997). Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. In *Journal of Global Optimization 11*, 341–359.

K. Deb, S. Agrawal, A. Pratap, T. Meyarivan, (2000). A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In *Parallel Problem Solving from Nature PPSN VI*, pp 849-858.

Y. Yoon and Y. H. Kim, (2022). Maximizing the coverage of sensor deployments using a memetic algorithm and fast coverage estimation. In *IEEE Transactions on Cybernetics,* vol. 52, no. 7, pp. 6531-6542.

J.H. Seo, Y. Yoon, Y.H. Kim, (2017). Coverage problem in camera-based sensor networks using the CUDA platform. In *International Journal of Distributed Sensor Networks*, vol. 13, no. 12.

S.S. Mansouri, G. Georgoulas, T. Gustafsson and G. Nikolakopoulos, (2017). On the covering of a polygonal region with fixed size rectangles with an application towards aerial inspection. In *2017 25th Mediterranean Conference on Control and Automation (MED)*, pp. 1219-1224.

J. Akshya, P.L.K. Priyadarsini, (2020). Graph-based path planning for intelligent UAVs in area coverage applications. In *Journal of Intelligent & Fuzzy Systems*, vol. 39, issue 6, pp. 8191-8203.

J. H. Holland, (1975). Adaptation in Natural and Artificial Systems. University of Michigan Press, Ann Arbor, MI. second edition 1992.

J.P. Queralta, J. Taipalmaa, B.C. Pullinen, V.K. Sarker, T.N. Gia, H. Tenhunen, M. Gabbouj, J. Raitoharju, T. Westerlund, (2020). Collaborative multi-robot search and rescue: planning, coordination, perception, and active vision. In *IEEE Access*, vol. 8, pp. 191617-191643.

D.A. Otote, B. Li, B. Ai, S. Gao, J. Xu, X. Chen, G. Lv, (2019). A decision-making algorithm for maritime search and rescue plan. In *Sustainability* 2019, 11, 2084.