Effects Study of Sensors' Placement on the Accuracy of a 3D TDOA-Based Localization System

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Abstract: Time Difference of Arrival (TDOA) based measurements are used for passive localization systems in various applications. While significant research has been performed on the development of TDOA measurement-based approaches, there has been relatively little focus on the sensor deployment geometry which significantly impacts the location estimation accuracy. Therefore, a study on the effects of four sensors' placement on location accuracy has been conducted. Several factors are considered in numerical simulations analysis which have an obvious effect on the localization accuracy. Based on the analysis of the Geometric Dilution of Precision (GDOP) performance metric, a comparison is conducted between square and star geometries. The results show that the star geometry gives better performance in terms of location estimation accuracy, especially when the main receiver is positioned within the polygon formed with baseline angles of 120°. Furthermore, the star geometry is used to study also the influence of sensor height and baseline length to achieve an optimum three-dimensional sensor placement with four sensors. The results can be applied to enhance the sensor deployment in 3D sensor geometry for TDOA-based localization systems.

1 INTRODUCTION

Recently, passive localization systems have assumed a significant role in various civilian and military applications, including radar, sonar and navigation. These systems commonly employ time measurement-based localization techniques, which can be classified as follows: Time of Arrival (TOA) and TDOA, also known as multilateration technique (Wan et al., 2018; Deng et al., 2019). The later use TDOA measurements observed at a set of spatially separated receivers. Each TDOA measurement defines a hyperbolic line, and the intersection of these lines gives the estimation of the source location.

The literature has mainly focused on developing TDOA measurements-based approaches to improve location estimation accuracy. Other works have aimed to enhance localization performance in terms of location estimation accuracy, which depends on various factors such as the number of sensor used, the choice of main or reference sensor used to generate the TDOA measurements, the multilateration approach, and sensor deployment geometry (Sun et al., 2016; Shehu and Sha'ameri, 2018a; Shehu, 2018). Although the latter factor significantly influences the location estimation accuracy, little research, as far as we know, has been conducted on it, as works presented in (Qin et al., 2016; Shehu and Sha'ameri, 2018b). Therefore, in order to enhance localization performance, it is necessary to investigate the optimal geometry.

In this paper, we focus on studying the impact of four sensors' deployment geometry on the location performance using TDOA measurements. Specifically, two geometries, namely square and star geometry, are evaluated to determine the optimal choice for location estimation accuracy. The selected geometry is then used to investigate the influence of other factors, namely baseline angle, sensor height and baseline length. In this context, a baseline refers to the line connecting the main sensor and one of the three auxiliary sensors. The analysis of the GDOP parameter (Li et al., 2011; Thompson et al., 2019) forms the basis of this study, enabling a comprehensive assess-

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ment of the localization system's performance under varying deployment geometries.

The remainder of the paper is organized as follows: the location system geometry is described in section 2, the hyperbolic position location (PL) approach and its theoretical model are shown in section 3, the theoretical derivation of the GDOP is given in section 4, the simulation result and discussion are detailed in section 5, which is followed by conclusion in section 6.

2 SYSTEM GEOMETRY

Let us consider the scenario illustrated in Figure 1, which presents a system geometry for TDOA-based location consisting of four sensors (receivers). We set sensor 1 as the main receiver and set others as the auxiliary receivers. Let us consider (X_i, Y_i, Z_i) the position coordinates of the sensor i, for $i = 1, \ldots, 4$. The source is on the position (x, y, z).

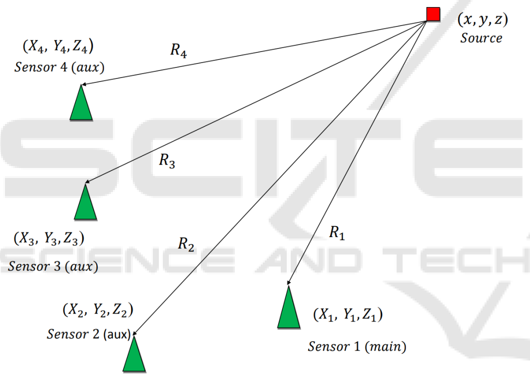

Figure 1: System geometry considered for TDOA based localization.

The localization accuracy is dependent on the deployment of the sensors, which can be characterized by the GDOP or the Cramer-Rao Lower Band (CRLB) (Vankayalapati, 2014; Li et al., 2017; Zhang and Lu, 2020; Diez-Gonzalez et al., 2020; Wang et al., 2022). The CRLB is a concept widely used in positioning performance analysis. It determines the minimum achievable error of a locating system with independence from the positioning algorithm used (Álvarez et al., 2019).

In this work, the average values of GDOP for different sensors' deployment geometries are examined for optimization in a 3D TDOA-based location system. The following steps are taken

- Two geometries are chosen to deploy the four sensors, named star and square geometry (Figure 2).
- The geometry that will give the best localization performance will be selected to study the influence of other factors on localization performance.
- The factors that will be studied are the baseline angle, heights of the four sensors and baseline length.

The subsequent sections of this paper cover two main aspects. Firstly, we describe the hyperbolic position location system, followed by the development of its GDOP to analyze the optimal sensors' deployment geometry. Lastly, we present simulation results that evaluate the TDOA localization performance for various geometries.

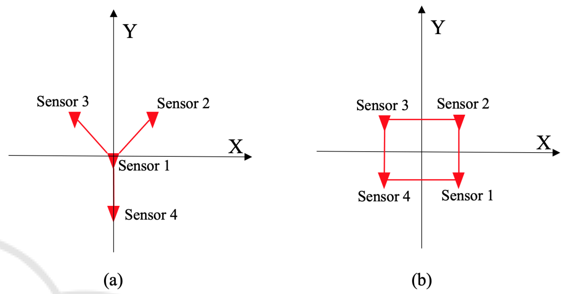

Figure 2: Geometries considered for evaluation. (a) star geometry. (b) square geometry.

3 HYPERBOLIC PL SYSTEM

TDOA measurements can be used to localize a target (source). Its location is estimated by the intersection of hyperboloids describing range difference measurements between three or more sensors. The foci of the hyperboloid are at the positions of the sensors i and j. In a system with N sensors, there are $N-1$ linearly independent TDOA measurements. The geometric approach uses intersecting hyperboloid surfaces created from the TDOA measurements made by the passive sensors to determine the target location (Wong et al., 2017).

The relationship between the Range Difference of Arrival (RDOA) and TDOA, is given by

$$R_{i,j} = c\tau_{i,j} = R_i - R_j \tag{1}$$

where R_i and R_j, are the distances from the source to the sensors i, j respectively, $R_{i,j}$ and $\tau_{i,j}$ are the RDOA and TDOA of a signal received by sensor pair i and j and c is signal propagation speed.

In a three-dimensional (3D) system, the hyperboloids that describe the range difference, $R_{i,j}$ between sensor positions are given by

$$R_{i,j} = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} - \sqrt{(X_j - x)^2 + (Y_j - y)^2 + (Z_j - z)^2} \tag{2}$$

where: (X_i, Y_i, Z_i) and (X_j, Y_j, Z_j) define the positions of sensor i and j respectively, and (x, y, z) is the source position.

The TDOA method offers a significant benefit as it does not require knowledge of the transmit time from the source, eliminating the need for strict clock synchronization between the source and receiver (Wang et al., 2019). Furthermore, unlike TOA methods, the hyperbolic position location method can reduce or eliminate common errors experienced at all sensors due to the channel.

Referring all TDOAs to the main sensor, which is assumed to be the reference for all sensors, let us use 'i' with $i = 2, ..., M$ to represent the auxiliary sensors. The range difference between sensors with respect to the main sensor, is

$$\begin{aligned} R_{i,1} &= c\tau_{i,1} = R_i - R_1 \\ &= \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} \\ &\quad - \sqrt{(X_1 - x)^2 + (Y_1 - y)^2 + (Z_1 - z)^2} \end{aligned} \tag{3}$$

where, $R_{i,1}$ and $\tau_{i,1}$ are the range and time difference measurements between the main sensor and the i^{th} sensor, and R_1 is the distance from the main sensor to the source. This defines the set of nonlinear hyperbolic equations whose solution gives the 3D source location. Numerical methods are needed to solve the nonlinear equations of (3) (Díez-González et al., 2022). Linearizing this set of equations is commonly performed using typical TDOA location algorithms which are: the Chan algorithm, Fang algorithm, and Taylor series expansion algorithm (Foy, 1976; Chan and Ho, 1994; Zhang and TAN, 2008; Al Harbi and Helgert, 2010). Fang algorithm and Chan algorithm are closed-form algorithms with analytic expressions. Taylor series expansion algorithm is an iterative algorithm without analytic expression.

4 GDOP FOR TDOA BASED LOCALIZATION

GDOP is a metric used to describe the effect of geometry on the relationship between the measurement and position error. It is a measure of the quality of the geometric configuration of the sensor array and is used to evaluate the accuracy of TDOA-based localization systems (Elgamoudi et al., 2021). The lower the GDOP, the better the geometric configuration. The optimal placement of the sensors is the one that minimizes the GDOP.

A range measurement can be expressed as

$$L = f(x, y, z) \tag{4}$$

where L is a measured value, and (x, y, z) are unknown coordinates of the source. We use the Taylor series expansion algorithm to linearize the equation (4) by developing the function $f(x, y, z)$ to the first order

$$L \approx f(x_0, y_0, z_0) + \frac{(\frac{\partial L}{\partial x})_0 dx}{1!} + \frac{(\frac{\partial L}{\partial y})_0 dy}{1!} + \frac{(\frac{\partial L}{\partial z})_0 dz}{1!} \tag{5}$$

where (x_0, y_0, z_0) is the initial estimation of the source coordinates, and $(\frac{\partial L}{\partial x})_0$, $(\frac{\partial L}{\partial y})_0$, and $(\frac{\partial L}{\partial z})_0$ are the partial derivatives of the measured value L evaluated at the initial estimation.

Assume there are n observations, equation (5) can be written in the following matrix form

$$\boldsymbol{H}\Delta\mathbf{x} = \Delta\mathbf{r} \tag{6}$$

$$\Delta\mathbf{x} = (\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T\Delta\mathbf{r} \tag{7}$$

where $\Delta\mathbf{x}$ is the vector offset of the true source position from the linearization point, $\Delta\mathbf{r}$ is the vector offset of the true range to the range values corresponding to the linearization point, and $\boldsymbol{H}$ is a matrix that can be presented as

$$\boldsymbol{H} = \begin{bmatrix} (\frac{\partial L_1}{\partial x})_0 & (\frac{\partial L_1}{\partial y})_0 & (\frac{\partial L_1}{\partial z})_0 \\ (\frac{\partial L_2}{\partial x})_0 & (\frac{\partial L_2}{\partial y})_0 & (\frac{\partial L_2}{\partial z})_0 \\ & \vdots & \\ (\frac{\partial L_n}{\partial x})_0 & (\frac{\partial L_n}{\partial y})_0 & (\frac{\partial L_n}{\partial z})_0 \end{bmatrix} \tag{8}$$

In the case of hyperbolic multilateration system, the measured values are TDOAs, so

$$\begin{aligned} L &= R_{i,1} \\ &= \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} \\ &\quad - \sqrt{(X_1 - x)^2 + (Y_1 - y)^2 + (Z_1 - z)^2} \end{aligned} \tag{9}$$

with : $i = 2, ..., M$.

As previously mentioned, the considered TDOA multilateration positioning system consists of a main and three auxiliary sensors ($M = 4$). After calculating the various partial derivatives, matrix $\boldsymbol{H}$ can be expressed as

$$\boldsymbol{H} = \begin{bmatrix} \frac{x_0 - X_2}{R_2} - \frac{x_0 - X_1}{R_1}, \frac{y_0 - Y_2}{R_2} - \frac{y_0 - Y_1}{R_1}, \frac{z_0 - Z_2}{R_2} - \frac{z_0 - Z_1}{R_1} \\ \frac{x_0 - X_3}{R_3} - \frac{x_0 - X_1}{R_1}, \frac{y_0 - Y_3}{R_3} - \frac{y_0 - Y_1}{R_1}, \frac{z_0 - Z_3}{R_3} - \frac{z_0 - Z_1}{R_1} \\ \frac{x_0 - X_4}{R_4} - \frac{x_0 - X_1}{R_1}, \frac{y_0 - Y_4}{R_4} - \frac{y_0 - Y_1}{R_1}, \frac{z_0 - Z_4}{R_4} - \frac{z_0 - Z_1}{R_1} \end{bmatrix} \tag{10}$$

GDOP is defined as the ratio of the Root Mean Square (RMS) position error to the RMS ranging error (Elgamoudi et al., 2021)

$$\text{GDOP} = \frac{\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}}{\sigma_r} \tag{11}$$

with σ_r^2 the variance of the measurement error on the range.

The provided mathematical expression for GDOP is a general form applicable to various scenarios. In the following discussion, we will specifically derive an explicit expression for GDOP in the context of a TDOA location system with four receivers.

Assuming the errors in the measurements on the range are random, independent, have zero mean, and an identical rms σ_r. The estimated distance is

$$\hat{R}_j = R_j + dr_j, \quad j = 1, \ldots, 4 \tag{12}$$

By performing the necessary calculations, we can determine the variances and covariances of the errors on range differences. The outcome of this calculation is presented in the subsequent error covariance matrix for RDOA

$$\mathbf{Q} = \sigma_r^2 \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 2 \end{bmatrix} \tag{13}$$

The covariance of $\Delta\mathbf{x}$ is calculated by using equation (7) as follows

$$\begin{aligned} \text{cov}(\Delta\mathbf{x}) &= E[\Delta\mathbf{x}\Delta\mathbf{x}^T] \\ &= E[(\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T\Delta\mathbf{r}\Delta\mathbf{r}^T\boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{H})^{-1}] \\ &= \boldsymbol{H}^{-1}(\boldsymbol{H}^T)^{-1}\boldsymbol{H}^T\mathbf{Q}\boldsymbol{H}\boldsymbol{H}^{-1}(\boldsymbol{H}^T)^{-1} \\ &= \boldsymbol{H}^{-1}\mathbf{Q}(\boldsymbol{H}^T)^{-1} \\ &= (\boldsymbol{H}^T\mathbf{Q}^{-1}\boldsymbol{H})^{-1} \end{aligned} \tag{14}$$

Finally, the expression for GDOP is presented as follows

$$\begin{aligned} \text{GDOP} &= \frac{\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}}{\sigma_r} \\ &= \sqrt{\sum_{i=1}^{3} ((H^T Q^{-1} H)^{-1})_{i,i}} \end{aligned} \tag{15}$$

The GDOP can be obtained by deriving it from the CRLB in the following manner (Thompson et al., 2019):

$$\text{GDOP} = \frac{1}{\sigma_r}\sqrt{\text{trace}(\text{CRLB}(\mathbf{P}))} \tag{16}$$

where $\mathbf{P}$ is the source position (x, y, z).

5 SIMULATION AND ANALYSIS

We consider the depicted location system geometry in Figure 1 for our analysis. The area of interest is a surface $[200 \times 200]\ Km^2$, in which we calculate the GDOP for a target at a height of 7000 m. The sensor coordinates (X_i, Y_i, Z_i), $i \in 1, \ldots 4$ are chosen as follows for the square geometry: $\{(0,0,0)$, $(20,0,0)$, $(20,20,0)$, and $(0,20,0)\}$ Km. Similarly, for the star geometry, the sensor coordinates are selected as:$\{(0,0,0)$,$(20,0,0)$, $(-20,-20,0)$, and $(0,20,0)\}$ Km.

The measurement noise was assumed to be Gaussian white with zero mean and standard deviation (RMS ranging error) $\sigma_r = 1\ m$.

The results of the GDOP calculation for the two sensor configurations, square and star geometry, are depicted in Figures 3 and 4, respectively. The values are presented in meters.

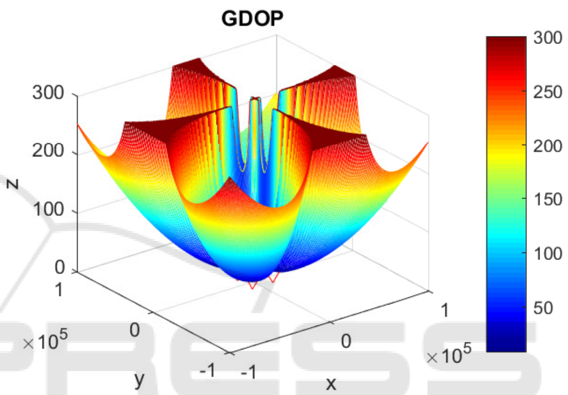

Figure 3: 3D GDOP calculation for square geometry.

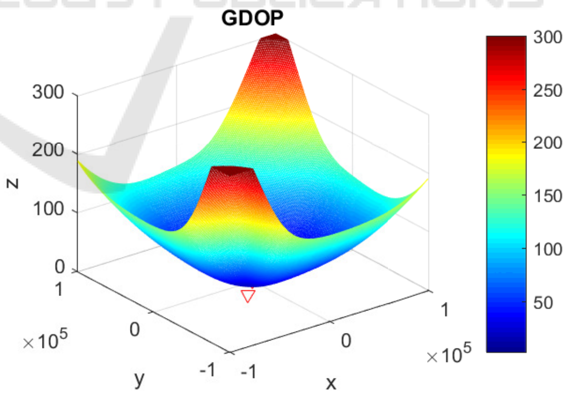

Figure 4: 3D GDOP calculation for star geometry.

To ensure meaningful calculations, in cases where the GDOP value at a specific location is excessively large or cannot be calculated, a value of 300 is assigned.

These simulation results indicate that the positions of the sensors has a direct impact on the performance of localization, since changing their positions leads to varying GDOP values. Thus, the deployment geometry of the sensors directly influences the performance of localization. From the former, it can be seen clairly that the star geometry outperforms the square geometry in terms of performance.

These results reveal some intriguing observations. Firstly, it is evident that the GDOP is notably higher (worse) at the center of the square configuration. This finding aligns with the report of (Li et al., 2011), which highlights that deploying the base stations (sensors) exclusively along the perimeter of an area leads to poor DOP at the center of the polygon. Secondly, in the square geometry, a notable degradation in GDOP occurs when moving along the direction of the square's medians beyond the receiver's area (as depicted in Figure 3). Consequently, this configuration is unsuitable for target localization due to the substantial ambiguity zone it presents.

Finally, in the scenario of the star geometry, the localization accuracy exhibits a consistent pattern as one moves farther from the sensors. Additionally, the GDOP remains good within the inner region encompassing the receiver positions and the surrounding area. For instance, when aiming for a tolerated localization error of less than 100 m, the location system covers a contiguous area exceeding $[100 \times 100]\ Km^2$.

The star geometry can be achieved by moving one of the sensors to the center of the square geometry. Consequently, placing a receiver at the center of a polygon enhances the GDOP within the sensor's area.

Therefore, this geometry will be utilized in subsequent analyses to study the impact of various factors on localization performance. These factors include the baseline angle, heights of the four sensors, and baseline length.

5.1 Effect of Baseline Angle

Let us consider the sensor deployment geometry depicted in Figure 5. The coordinates of the four sensors are as follows

$$
\begin{aligned}
(X_1, Y_1, Z_1) &= (0, 0, 0)(m) \\
(X_2, Y_2, Z_2) &= (R \times \sin(\theta_1), -R \times \cos(\theta_1), 0)(m) \\
(X_3, Y_3, Z_3) &= (-R \times \sin(\theta_2), -R \times \cos(\theta_2), 0)(m) \\
(X_4, Y_4, Z_4) &= (0, -R, 0)(m)
\end{aligned}
$$

The RMS ranging error and the baseline length are set to fixed values: $\sigma_r = 0.2\ m,\ R = 20 \times 10^3\ m$.

To investigate the impact of baseline angles, θ_1 and θ_2, we conducted a comprehensive analysis by varying their values within the range of 15° to 165°. For each configuration, the average GDOP was computed over the designated area of interest. The results obtained from the simulations are presented in Figure 6.

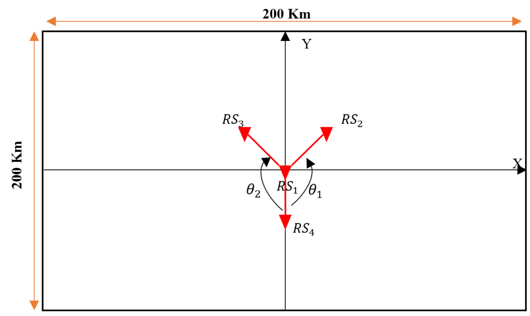

Figure 5: The selected geometry to study the effect of baseline angle.

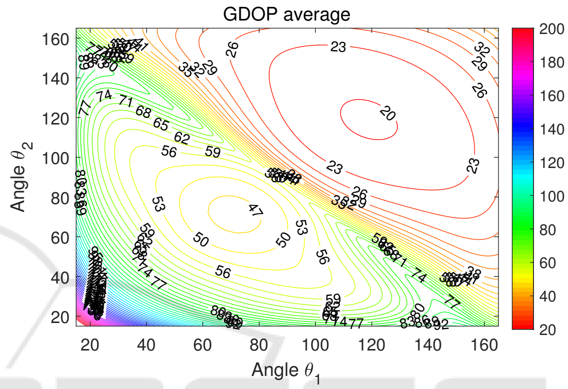

Figure 6: Effect of baseline angles variation on location performances. Angles are provided in degrees.

It is worth noting that the deployment geometry of the receivers significantly affects the performance of localization. Based on the obtained results, the optimal configuration is characterized by angles of 120° between the different baselines $[RS_1 - RS_2]$, $[RS_1 - RS_3]$, and $[RS_1 - RS_4]$. This particular configuration exhibits the lowest average GDOP.

5.2 Effect of Sensor Height

In this scenario, we focus on the optimal sensors' deployment geometry obtained in the previous subsection 5.1, where the baseline angles were set at θ=120°. We now proceed to vary the height of the main sensor while keeping heights of the other sensors (auxiliary) at a fixed level. Similarly to previous simulations, we calculate the average GDOP for each configuration. The results are presented in Figure 7.

We observe that the larger the difference between the height of the reference sensor and that of the other sensors, the more the localization performance is degraded (Figure 7). Therefore, the height of the main sensor in a TDOA-based localization system should be close to that of the other sensors to achieve better localization.

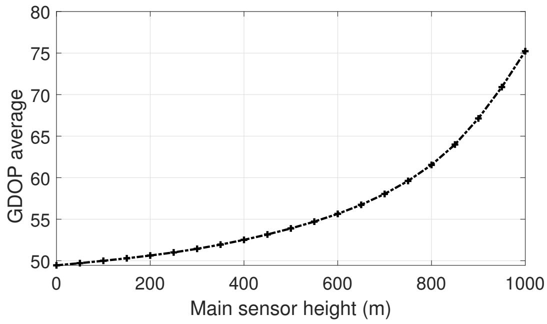

Figure 7: Effect of the main sensor height on location performances.

Now, let us explore the effect of changing the height of an auxliary sensor on the localization performance.

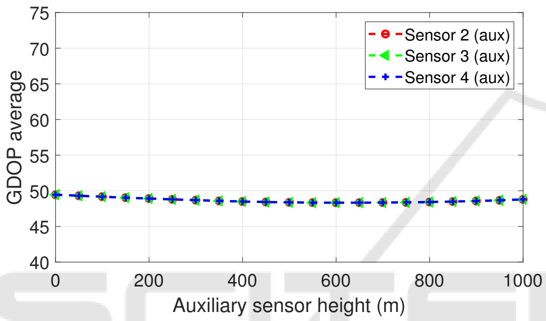

Figure 8: Effect of auxiliary sensor height on location performances.

When we keep the height of the main sensor fixed and vary the heights of the remaining sensors one by one, we observe that it has minimal impact on the localization performance (Figure 8). The values of GDOP remain practically stable throughout these variations. This indicates that the height of individual auxiliary sensors does not significantly impact the overall localization system performance.

5.3 Effect of Baseline Length

The baseline lenght is another factor to consider in evaluating the performance of TDOA-based localization. Its impact is investigated by variying the length of the three baselines in the same way, examining its effect on the GDOP values. It is important to note that the deployment geometry remains the same (star geometry with baseline angle θ=120°) throughout this analysis. The simulation results are presented in Figure 9. The values are presented in meters.

It is observed that lengthening baseline length leads to improved localization accuracy. This improvement can be attributed to the larger TDOA values obtained with greater distances, leading to en-

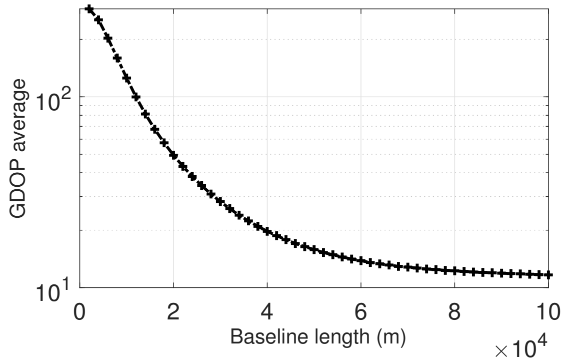

Figure 9: Influence of baseline length variation on location performances.

hanced accuracy in the estimation of the source's location. However, it is important to note that there is a limit to increasing the baseline length as it can lead to a potential risk of signal detection failure by the sensors, where the emitted signal from the source may not be detected by a sensor. In such cases, with fewer available equations, it becomes challenging to accurately locate the source. Therefore, careful consideration must be given to strike a balance between maximizing baseline length for improved accuracy while ensuring reliable signal detection and localization.

6 CONCLUSION

This paper focuses on the optimization of sensor deployment in a 3D TDOA-based location system. GDOP is used as a performance metric to evaluate localization performance. The GDOP for a TDOA location system with four sensors is derived and analyzed. It is shown that the deployment geometry of the sensors directly impacts localization performance. A comparison was first made between a square geometry and a star geometry. The latter was efficient and its localization accuracy exhibits a consistent pattern as one moves farther from the sensors. Thus, the star geometry was selected to investigate the influence of other factors on localization performance, namely : the baseline angle, sensor height, and baseline length. It is found that a sensor deployment configuration with baseline angles of 120° gives optimal results. As for the impact of sensor height, only changes in the main sensor height significantly affect localization errors, as it serves as the reference in TDOA methods. Finally, increasing the baseline length enhances location accuracy. Future research should focus on formulating an optimization problem for sensor deployment and proposing algorithms to design an optimal strategy. The effectiveness of the strategy should also be evaluated.

REFERENCES

Al Harbi, F. S. and Helgert, H. J. (2010). An improved chan-ho location algorithm for tdoa subscriber position estimation. *International journal of computer science and network security*, 10(9):101–105.

Álvarez, R., Díez-González, J., Alonso, E., Fernández-Robles, L., Castejón-Limas, M., and Perez, H. (2019). Accuracy analysis in sensor networks for asynchronous positioning methods. *Sensors*, 19(13):3024.

Chan, Y. and Ho, K. (1994). A simple and efficient estimator for hyperbolic location. *IEEE Transactions on Signal Processing*, 42:1905 – 1915.

Deng, Z., Wang, H., Zheng, X., Fu, X., Yin, L., Tang, S., and Yang, F. (2019). A closed-form localization algorithm and gdop analysis for multiple tdoas and single toa based hybrid positioning. *Applied Sciences*, 9(22):4935.

Diez-Gonzalez, J., Alvarez, R., Prieto-Fernandez, N., and Perez, H. (2020). Local wireless sensor networks positioning reliability under sensor failure. *Sensors*, 20(5):1426.

Díez-González, J., Álvarez, R., Verde, P., Ferrero-Guillén, R., and Perez, H. (2022). Analysis of reliable deployment of tdoa local positioning architectures. *Neurocomputing*, 484:149–160.

Elgamoudi, A., Benzerrouk, H., Elango, G. A., and Landry Jr, R. (2021). A survey for recent techniques and algorithms of geolocation and target tracking in wireless and satellite systems. *Applied Sciences*, 11(13):6079.

Foy, W. H. (1976). Position-location solutions by taylor-series estimation. *IEEE transactions on aerospace and electronic systems*, (2):187–194.

Li, B., Dempster, A., and Wang, J. (2011). 3D DOPs for positioning applications using range measurements. *Wireless Sensor Network*, 3:343–349.

Li, W., Yuan, T., Wang, B., Tang, Q., Li, Y., and Liao, H. (2017). Gdop and the crb for positioning systems. *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, 100(2):733–737.

Qin, Z., Wang, J., and Wei, S. (2016). A study of 3d sensor array geometry for tdoa based localization. In *2016 CIE International Conference on Radar (RADAR)*, pages 1–5. IEEE.

Shehu, Y. (2018). Position estimation performance evaluation of a linear lateration algorithm with an SNR-based reference station selection technique. *Nigerian Journal of Technology*, 34.

Shehu, Y. and Sha'ameri, A. (2018a). Closed-form 3-D position estimation lateration algorithm reference pair selection technique for a multilateration system. *Journal of Telecommunication*, 10.

Shehu, Y. and Sha'ameri, A. (2018b). Effect of path loss propagation model on the position estimation accuracy of a 3-dimensional minimum configuration multilateration system. *International Journal of Integrated Engineering*, 10:35–42.

Sun, S., Wang, Z., and Wang, Z. (2016). Study on optimal station distribution based on tdoa measurements. In *2016 International Conference on Computer Engineering, Information Science & Application Technology (ICCIA 2016)*, pages 283–288. Atlantis Press.

Thompson, R., Balaei, A., and Dempster, A. (2019). Dilution of precision for GNSS interference localisation systems.

Vankayalapati, Naresh; Kay, S. Q. D. (2014). TDOA based direct positioning maximum likelihood estimator and the cramer-rao bound. *IEEE Transactions on Aerospace and Electronic Systems*, 50.

Wan, P., Ni, Y., Hao, B., Li, Z., and Zhao, Y. (2018). Passive localization of signal source based on wireless sensor network in the air. *International Journal of Distributed Sensor Networks*, 14(3):1550147718767371.

Wang, D., Yin, J., Chen, X., Jia, C., and Wei, F. (2019). On the use of calibration emitters for tdoa source localization in the presence of synchronization clock bias and sensor location errors. *EURASIP Journal on Advances in Signal Processing*, 2019(1):1–34.

Wang, Y., Zhou, T., Yi, W., and Kong, L. (2022). A gdop-based performance description of toa localization with uncertain measurements. *Remote Sensing*, 14(4):910.

Wong, S., Jassemi-Zargani, R., Brookes, D., and Kim, B. (2017). A geometric approach to passive target localization. *S&T Organization*.

Zhang, J. and Lu, J. (2020). Analytical evaluation of geometric dilution of precision for three-dimensional angle-of-arrival target localization in wireless sensor networks. *International Journal of Distributed Sensor Networks*, 16(5):1550147720920471.

Zhang, L. and TAN, Z. (2008). *A new TDOA algorithm based on Taylor series expansion in cellular networks*. Frontiers of Electrical and Electronic Engineering in China, SP Higher Education Press.