Temporal Multidimensional Model for Evolving Graph-Based Data Warehouses

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Abstract: Nowadays, companies are focusing on overhauling their data architecture, consolidating data and discarding legacy systems. Big data has a great impact on businesses since it helps companies to efficiently manage and analyse large volumes of data. In business intelligence and especially decision-making, data warehouses support OLAP technology, and they have been very useful for the efficient analysis of structured data. A data warehouse is built by collecting data from several data sources. However, big data refers to large sets of unstructured, semi-structured or structured data obtained from numerous sources. Many changes in the content and structure of these sources can occur: therefore, these changes have to be reflected in the data warehouse using the bi-temporal approach for the data and versioning for the schema. In this paper, we propose a temporal multidimensional model using a graph formalism for multi-version data warehouses that is able to integrate the changes that occur in the data sources. The approach is based on multi-version evolution for schema changes and the bi-temporal labelling of the entities, as well as the relationships between them, for data evolution. Our proposal provides flexibility to the evolution of a data warehouse by increasing the analysis possibilities for users with the decision support system, and it allows flexible temporal queries to provide consistent results. We will present the overall approach, with a focus on the evolutionary treatment of the data, including dimensional changes. We validate our approach with a case study that illustrates temporal queries, and we carry out runtime performance tests for graph data warehouses.

1 INTRODUCTION

The architectures of data warehouses (DWs) allow the storage of data that are extracted from diverse and heterogeneous data sources in a coherent and integrated way, providing decision-makers with a better understanding of their environments and adequate support for strategic decision-making (Inmon, 1992; Kimball, 1996). Since the appearance of DWs, the warehousing approach has become an important research field in which many problems still need to be solved, particularly problems related to the evolutionary aspect of DWs.

The growth and diversification of data sources through the advent of big data involves changes in the content and structure of DWs. In effect, a schema is designed to meet predefined analysis needs; if these needs change, it can be costly to change the schema. The classical multidimensional model, based on the star schema and its variants, has limited possibilities when it comes to change, and its evolution is complex. These limitations are related to the fixed star model: it is created for analysis needs that are known in advance. We previously proposed a flexible multidimensional model for big data named graph-based agile multidimensional model (GAMM), which is based on an extension of the classical multidimensional model, to support chronological evolution on the conceptual level and the evolution of the graph structure on the logical and physical levels (Benhissen et al., 2023; Benhissen et al., 2022). GAMM allows the evolution of a schema in a data warehouse by creating a new version of the schema at each evolution using evolution functions. Each version corresponds to a data instance extracted from an agile graph data warehouse. A meta-model has been proposed to manage the different schema versions.

In this work, we are interested in the temporal evolution of data in a multi-version DW. In fact, for a real...
representation of the analysis context, periodic and regular data refreshing using heterogeneous sources must be implemented through ETL (extract, transform, load) processes. Indeed, a mismatch between the source data and the data in a DW can alter the consistency of the analyses, which makes the processing of the multidimensional model data extremely important. Conventionally, the refreshment process regularly provides the warehouse with collected and transformed data. These data are integrated into multidimensional structures that are suitable for decision analysis. However, once it is in operation, the data warehouse is not just provided with new facts; there may also be changes in the data of the dimensions, notably in the hierarchical relationships or descriptor attributes. The appropriate management of these changes is a key factor in the consistency of data warehousing systems.

These changes in dimensions necessitate the study of the aspect of temporality; a piece of information/relationship is true during a specific period of time. Temporal data warehouses (TDWs), inspired by the principles and rules of temporal databases, have been adopted to facilitate the management of this notion of temporality, making it possible to offer coherent analyses despite these changes. In this article, we propose a temporal approach for multi-version DWs based on a graph database to keep the evolutionary history of the data, including the changes in the dimensions; this approach also allows flexible temporal queries to provide consistent results.

The rest of the article is organised as follows. First, the notion of changes in the dimensions is presented (Section 2); this is followed by a running motivating example (Section 3). Then, related work on data evolution and temporal approaches in a DW context are presented (Section 4). In Section 5, we present our concept of a temporal multidimensional model for multi-version DWs based on a graph formalism. We then provide in Section 6 examples of temporal queries. In Section 7, we describe a use case based on the Star Schema Benchmark (SSB) dataset to perform functional validation and study the runtime performance. Finally, conclusions and future research topics are presented.

2 CHANGES IN DIMENSIONS

In addition to the insertion of new entities (facts and dimensions) into the data warehouse, existing data could be modified by update operations. These modifications are made by changing existing attributes and/or changing the relationships between hierarchies, which involves updating keys. If they are not handled correctly, these update operations can alter the analyses produced by the DW.

Kimball and Ross studied the problem of changes in dimensions and proposed processing techniques according to the speed of the changes (Kimball and Ross, 2013). For a slowly changing dimension (SCD), the following types of techniques are proposed: (i) the original value of the attribute is maintained, so that the facts are always grouped by this original value (Type 0). (ii) The old value of the attribute is replaced with a new value. The facts will be associated with the current value of the attribute according to the time span of its veracity, which is delimited by a start date and an end date (Type 2). (iv) A new column is added to preserve the current and previous values of the attribute (Type 3). For fast-changing dimensions, the proposed technique involves the addition of a mini-dimension so that frequently analysed or fast-changing attributes are split into a separate dimension. Other so-called hybrid techniques have been proposed that involve combining some or all of the different techniques proposed above to meet the requirements of historical attribute preservation and reporting.

3 RUNNING MOTIVATING EXAMPLE

![Figure 1: Multidimensional model schema for retail sales using DFM formalism.](image)
tual schemas of our example (Figure 1). Our example is composed of a fact ‘Sales’ and three dimensions, the ‘Product’, ‘Customer’ and ‘Date’ dimensions. For the sake of consistency, we will use this schema configuration as a running example during the presentation of all parts of our proposed approach. The ‘Product’ dimension is described by the attributes ‘Product_Name’ and ‘Unit_Price’ and a hierarchical level ‘Category’. This dimension therefore has a single hierarchy on the analysis axis (‘Product’, ‘Category’). The ‘Customer’ dimension is described by the attributes ‘Customer_Name’, ‘Address’ and ‘Phone’, and a hierarchical level ‘City’. The dimension ‘Date’ has two hierarchical levels, i.e. ‘Month’ and ‘Year’. The fact ‘Sales’ is described by the measure ‘Sales_Amount’.

In the following, we present an example of the temporal evolution of data in a DW to illustrate a case involving changes in dimensions. Note that the representation of instances in this example illustrates the instances of a temporal DW based on the principle of temporal databases. Indeed, this representation allows the data to have an advanced chronological precision, a piece of information is true during a specific lifespan characterised by a From_Date (FD) and a To_Date (TD), and entities are characterised by a Valid_Time (VT) and/or a Transaction_Time (TT) (mono- or bi-temporal approach) (Golfarelli and Rizzi, 2011). This notion of temporality offers a level of analysis that better represents the real world, particularly when it comes to managing the temporality of aggregation links.

The instances of the ‘Customer’ dimension shown in Table 1 indicate that the customer Mary Saveley initially lived in the city of Paris from 01/01/2020 to 31/07/2020 and then moved to Lyon, where she has lived since 01/08/2020. Similarly, the instances of the ‘Product’ dimension shown in Table 2 indicate that the product Mozzarella was classified into the category ‘Fresh’ from 01/01/2020 to 31/03/2020 and was then reclassified into the category ‘Dairy’ on 01/04/2020.

Table 4 shows some instances of the fact ‘Sales’ that describe the ‘Sales_Amount’ for the customer Mary Saveley and the product Mozzarella. These sales amounts represent the customer’s purchases when she lived in Paris and when she lived in Lyon. Additionally, these same amounts represent the sales of the product Mozzarella when it belonged to the category ‘Fresh’ and when it belonged to the category ‘Dairy’. These changes in the dimensions require a temporal treatment of the queries in order to avoid any mismatch in the results. In general, the Type 2 technique of adding a new row with the new value while taking into account the time interval of the veracity of the data is the most representative of the real world, offering consistent analyses. Indeed, the Type 0 technique of maintaining the original value and the Type 1 technique of replacing it with the new value lead to information losses, as does the Type 3 technique of adding a new column; this technique cannot be applied continuously due to the modification of the schema with each addition. As for the technique of adding a mini-dimension, this technique is meant to be used in a particular case; it is not a general solution to the problem of changes in dimensions.

4 RELATED WORK

There are several research works in the literature that have addressed many facets of temporal DWs (Faisal et al., 2017; Golfarelli and Rizzi, 2018), including the type of temporality, conceptual and logical level, data evolution, changes in dimensions, delayed measurements, implementation of approaches, temporal queries and aggregation in temporal relationships. We can cite the work of (Blujipte et al., 1998), who presented a temporal approach based on the suppression of the time dimension that makes it possible to manage the chronological aspect of the data and to replace the time dimension with temporal labels at the level of each of the instances; in particular, the VT label or the TT label (or both for a bi-temporal approach) can be used. (Mendelzon and Vaisman, 2000) presented a temporal multidimensional model that allows temporal OLAP queries. (Golfarelli and Rizzi, 2007) presented alternative design solutions, which can be adopted in the presence of late measurements, to support different types of queries that allow a meaningful historical analysis in the presence of late measurements. (Faisal and Sarwar, 2014) presented a classification of queries based on the input and output attributes of the query and studied the performance of Type 2 and hybrid SCDs. (Saroha and Gosain, 2015) presented an approach that makes it possible to manage dimension data and track retroactive and proactive updates in a bi-temporal DW using both the VT and TT. (Garani et al., 2016) presented an approach for the logical modelling of TDWs based on the temporal star schema in which time is treated not as another dimension but as time attributes in every temporal dimension. (Phungtua-Eng and Chittayasothorn, 2019) utilised temporal database features, including the concept of VT state tables, to solve the SCD problem of data warehouses notably SQL. (Ahmed et al., 2015; Ahmed et al., 2020) presented a logical model and querying technique for temporal
Table 1: Dimension CUSTOMER.

<table>
<thead>
<tr>
<th>Surrogate_Key</th>
<th>Customer_ID</th>
<th>Customer_Name</th>
<th>City_ID</th>
<th>From_Date</th>
<th>To_Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>SkCust001</td>
<td>Cust001</td>
<td>Mary Saveley</td>
<td>City001</td>
<td>01/01/2020</td>
<td>31/07/2020</td>
</tr>
<tr>
<td>SkCust002</td>
<td>Cust001</td>
<td>Mary Saveley</td>
<td>City002</td>
<td>01/08/2020</td>
<td>31/12/9999</td>
</tr>
</tbody>
</table>

Table 2: Dimension PRODUCT.

<table>
<thead>
<tr>
<th>Surrogate_Key</th>
<th>Product_ID</th>
<th>Product_Name</th>
<th>Category_ID</th>
<th>From_Date</th>
<th>To_Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>SkProd001</td>
<td>Prod001</td>
<td>Mozzarella</td>
<td>Categ001</td>
<td>01/01/2020</td>
<td>31/03/2020</td>
</tr>
<tr>
<td>SkProd002</td>
<td>Prod001</td>
<td>Mozzarella</td>
<td>Categ002</td>
<td>01/04/2020</td>
<td>31/12/9999</td>
</tr>
</tbody>
</table>

Table 3: Levels CITY and CATEGORY.

<table>
<thead>
<tr>
<th>City_ID</th>
<th>City_Name</th>
<th>Category_ID</th>
<th>Category_Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>City001</td>
<td>Paris</td>
<td>Categ001</td>
<td>Fresh</td>
</tr>
<tr>
<td>City002</td>
<td>Lyon</td>
<td>Categ002</td>
<td>Dairy</td>
</tr>
</tbody>
</table>

Table 4: Fact SALES.

<table>
<thead>
<tr>
<th>Sale_ID</th>
<th>Customer_ID</th>
<th>Product_ID</th>
<th>Order_Date</th>
<th>Sales_Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sale001</td>
<td>Cust001</td>
<td>Prod001</td>
<td>01/03/2020</td>
<td>1500</td>
</tr>
<tr>
<td>Sale002</td>
<td>Cust001</td>
<td>Prod001</td>
<td>01/05/2020</td>
<td>2200</td>
</tr>
<tr>
<td>Sale003</td>
<td>Cust001</td>
<td>Prod001</td>
<td>01/09/2020</td>
<td>1800</td>
</tr>
<tr>
<td>Sale004</td>
<td>Cust001</td>
<td>Prod001</td>
<td>01/11/2020</td>
<td>2000</td>
</tr>
</tbody>
</table>

These studies have provided many solutions to the management of data evolution in data warehouses and to the problems related to SCDs, in particular due to the incorporation of temporal support in SQL:2011 (Kulkarni and Michels, 2012) and its implementation in some DBMSs, e.g. SQL Server 2016, Oracle 12c, IBM DB2 and Teradata (Poscic et al., 2018). However, the work in the literature is based on the entity-relationship (ER) model, which limits the evolution of these multidimensional models, particularly in terms of the schema (Benhissen et al., 2023; Benhissen et al., 2022). This led us, in this paper, to propose an alternative approach for temporal data management in DWs by using graph-oriented NoSql databases, especially after the promising results obtained by the models proposed by (Campos et al., 2016; Debrouvier et al., 2021). Our proposal offers a global solution to the SCD problem and allows temporal queries with concordant results.

5 TEMPORAL MULTIDIMENSIONAL MODEL BASED ON A GRAPH FORMALISM

To present our approach, we will provide a brief summary of our previously proposed model, called the graph-based agile multidimensional model (GAMM) (Benhissen et al., 2023); we consider the evolution of the schema in this model. In this paper, we deal with the temporal evolution of the data and we enrich the formalism of the model to take into account this evolution, particularly the changes in dimensions, as well as the evolution of the schema.

We define agility in a multidimensional model as its organisational capacity to build data warehouses in a scalable way while prioritising the goals of business teams. Agility involves responding to new business objectives and the integration of new data sources in a flexible and incremental way while maintaining all previous builds. This flexibility delivers more business value to decision-makers and allows them to better manage their environment.

GAMM is a flexible approach to schema and data evolution in data warehouses. The model allows designers to integrate new data sources and accommodate new user requirements to enrich the analytical capabilities of the data warehouse. It is an approach based on a multi-version scalable schema model and a data warehouse stored in a unique global graph database Figure 2.

Each schema version (SV) is valid for a period of time (T) characterised by a Starting_Time (ST) and an Ending_Time (ET), and it corresponds to a data instance (DInst) extracted from the graph-based temporal data warehouse (GTDW). A meta-model is implemented for the management of schema versions whose validity periods are in sequential order. We will develop in the following subsections the formal-
isation of our approach (Subsection 5.1), the concept of the GTDW (Subsection 5.2) and the transformation rules (Subsection 5.3).

5.1 Formalisation of the Approach

On the conceptual level, the approach represents an extension of classical multidimensional modelling based on the fact, measure, dimension, level and hierarchy concepts. Indeed, due to the evolution of the schema and data over time and for historical purposes, the temporal concept has been introduced according to the following definitions.

Definition 1: GAMM is represented as follows:

\[
\text{GAMM}(t) = \{F, D, \text{FAssoc}[F, D](t)\}.
\]

\(\text{GAMM}(t)\) represents the schema version at a time \(t\). \(t \in T = [\text{ST}, \text{ET}]\) represents the period of validity of the schema version, where \(\text{ST}\) is the Starting Time of the version and \(\text{ET}\) is the Ending Time of the version. \(F = \{f_i(t), i \in \{1, *, \}\}\), represents the set of facts at the moment \(t\). \(f_i(t)\) represents the fact \(f_i\) at a time \(t\). \(D = \{d_j(t), j \in \{1, *, \}\}\), represents the set of dimensions according to which \(f_i(t)\) is analysable at a time \(t\). \(d_j(t)\) represents the dimension \(d_j\) at a time \(t\). \(\text{FAssoc}[F, D]|(t) : f_i(t) \mapsto \{d_j(t), \text{ST}, \text{ET}\}\), where \(j \in \{1, *, \}\), represents the association function of the set of dimensions \(\{d_j(t)\}\) and the fact \(f_i(t)\) at a time \(t\).

Example:

\[
\text{GAMM}(t_0) = \{\{\text{Sales}\}, \{\text{Customer}, \text{Product}\}\},
\]

\(\text{Sales} \mapsto \text{Customer}, \text{Product}\}\}

and \(\text{ST}_0 = l_0 < \text{ET}_0\).

Definition 2: A measure is an indicator allowing the analysis of the business subject represented by the fact; it is numerical and aggregable and is defined as follows:

\[
M(t) = \{M\_\text{label}, l^m_0\}.
\]

\(M\_\text{label}\) represents the measure identifier. \(l^m_0, k \in \{1, \,*\}\), represents the set of instances of measure \(M\).

Example:

\[
M(t_0) = \{\text{Sales\_Amount}, l^m_0 : \{1500\}\}.
\]

Definition 3: A fact represents a subject analysed by GAMM. It is defined as follows:

\[
F(t) = \{F\_\text{label}, [M], \text{MAssoc}[F, M](t), [\text{VT}]\}.
\]

\(F\_\text{label}\) represents the fact name. \([M]\) = \{\(m_k(t)\), \(k \in \{1, *, \}\)\}, represents the set of measures associated with the fact at a time \(t\). \(m_k(t)\) represents a measure \(m_k\) at an instant \(t\). \(\text{MAssoc}[F, M]|(t) : f_i(t) \mapsto \{m_k(t), \text{ST}, \text{ET}\}\) represents an association function of the set of measures \(\{m_k(t)\}\) and the fact \(f_i(t)\) at a time \(t\), where \(\text{ST}\) is the Starting Time and \(\text{ET}\) is the Ending Time. \([\text{VT}] = \{l'^i_1\}, i \in \{1, *, \}\)\}, represents the set of instances of fact \(F\). Each instance \(l'^i\) = \{[l'^m_0], \text{VT}, \text{TT}\}\). \(\text{VT} = \text{Valid Time}\) represents the time point at which a fact instance is stored in the model. \(\text{VT} = \text{Valid Time}\) represents the time point at which a fact instance is true in relation to reality.

Example:

\[
F(t_0) = \{\text{Sales}, M_o, \text{MAssoc}[F, M](t_0), l^m_0\}\.
\]

\(M_o : \{\text{Sales\_Amount}\}\).

\(\text{MAssoc}[F, M]|(t_0) : \{\text{Sales} \mapsto \text{Sales\_Amount}\}\)\).

\(l^m_0 : \{l^m_0 : \{1500\}, 05/01/2020, 15/06/2020\}\).

Definition 4: An attribute is an element of description of a dimension or a hierarchical level to which it is associated. It is defined as follows:

\[
A(t) = \{A\_\text{label}, \text{Attribute}, [P]\}.
\]

\(A\_\text{label}\) represents the attribute name. \(\text{Attribute}\) represents the value of the description attribute. \([P]\) = \{\(l^a_0\), \(k \in \{1, *, \}\)\}, represents the set of instances of attribute \(A\). Each instance \(l^a_0\) = \{\text{value}, \text{VT}, \text{TT}\\). \(\text{VT} = \text{Valid Time}\) represents the time point at which an attribute is true with respect to reality. \(\text{TT} = \text{Transaction Time}\) represents the time point at which an attribute is stored in the model.

Example:

\[
A(t_0) = \{\text{Product Name}, \text{name}, \}
\]

\(l^a_0 : \{\text{Mozzarella}, 05/01/2020, 15/06/2020\}\).

[A figure of the architecture of GAMM is shown.]

Figure 2: Architecture of GAMM.

Example:

\[
A(t_0) = \{\text{Product Name}, \text{name}, \}
\]

\(l^a_0 : \{\text{Mozzarella}, 05/01/2020, 15/06/2020\}\).
Definition 5: A level represents the degree of detail of an analysis perspective according to a given hierarchy. It is defined as follows:

\[ L(t) = \{L\text{Label}, L\text{ID}, A, [l], L\text{Assoc}[L, A](t), \text{Rel}([l^1], [l^2]) \} \]

- \( L\text{Label} \) represents the level name.
- \( L\text{ID} \) represents the level identifier.
- \( A = \{a(t), i \in [1, \ast] \} \), represents the set of attributes associated with the level at a time \( t \).
- \( [l^1] = \{l^1_k \}, k \in [1, \ast] \) represents the set of instances of level \( L \). Each instance \( l^1_k \) is \( \{\text{id}, \text{VT}, TT\} \).
- \( \text{VT} = \text{Valid}\text{Time} \) represents the time point at which a level is true compared to reality.
- \( TT = \text{Transaction}\text{Time} \) represents the time point at which a dimension is stored in the model.
- \( L\text{Assoc}[L, A](t) : L(t) \rightarrow \{a(t), ST, ET\} \) represents the association function of the set of attributes \( \{a(t)\} \) and the level \( L(t) \) at a time \( t \), where \( ST \) is the Starting Time, \( ET \) is the Ending Time and \( j \in [0, \ast] \).
- \( \text{Rel}([l^1], [l^2]) \) represents the relationships between instances of levels \( \{L \} \) and attributes \( \{A\} \), where \( \text{Rel}([i^1_m], [i^2_n]) = \{\text{Relation}\text{Label}, \text{From}\text{Date}, \text{To}\text{Date}\} \).
- \( \text{Relation}\text{Label} \) represents the relationship name.
- \( \text{From}\text{Date} \) represents the starting date of the relationship.
- \( \text{To}\text{Date} \) represents the ending date of the relationship.

Example:

\[ L(t_0) = \{\text{Category}, \text{Category}\text{Id}, A_0, i_0, \text{LAssoc}[L, A_0](t_0), \text{Rel}([l^1_0], [l^2_0])\} \]

\( A_0 \): \{\text{Category}\text{Name}\}.

\( i_0 \): \{\text{Category001, 05/01/2020, 15/06/2020}\}

\( \text{LAssoc}[L, A_0](t_0) \): \{\text{Category} \rightarrow \text{Category}\text{Name}\}.

\( \text{Rel}([l^1_0], [l^2_0]) \): \{\text{Category}\text{To}\text{Name}, 05/01/2020, 31/12/19999\}

\( i^2_0 \): \{\text{Fresh, 05/01/2020, 15/06/2020}\}.

Definition 6: A dimension is an axis of analysis according to which the business subject is analysed. It determines the level of detail of the measures and is defined as follows:

\[ D(t) = \{D\text{Label}, D\text{ID}, [A], [l^d], D\text{Assoc}[D, A](t), \text{DAssoc}[D, L](t), \text{Rel}([l^d], [l^2]), \text{Rel}([l^3], [l^4]), \text{Rel}([l^5], [l^6]) \} \]

- \( D\text{Label} \) represents the dimension name.
- \( D\text{ID} \) represents the dimension identifier.
- \( A = \{a(t), i \in [1, \ast] \} \), represents the set of attributes associated with the dimension at a time \( t \).
- \( [l^d] = \{l^d_k \}, k \in [1, \ast] \) represents the set of dimensions of dimension \( D \). Each instance \( l^d_k \) is \{\text{id}, \text{VT}, \text{TT}\}.
- \( \text{VT} = \text{Valid}\text{Time} \) represents the time point at which a dimension is true in relation to reality.
- \( TT = \text{Transaction}\text{Time} \) represents the time point at which a dimension is stored in the model.
- \( D\text{Assoc}[D, A](t) : d_j(t) \rightarrow \{a(t), ST, ET\} \) represents the association function of the set of attributes \( \{a(t)\} \) and the dimension \( d_j(t) \) at a time \( t \), where \( ST \) is the Starting Time and \( ET \) is the Ending Time.
- \( \text{Rel}([l^d], [l^2]) \) represents the relationships between instances of dimension \( D \) and attributes \( A \), where \( \text{Rel}([l^d], [l^2]) = \{\text{Relation}\text{Label}, \text{From}\text{Date}, \text{To}\text{Date}\} \).
- \( \text{Relation}\text{Label} \) represents the relationship name.
- \( \text{From}\text{Date} \) represents the starting date of the relationship.
- \( \text{To}\text{Date} \) represents the ending date of the relationship.

Example:

\[ D(t_0) = \{\text{Product}, \text{Product}\text{Id}, A_0, l^d_0, \text{DAssoc}[D, A](t_0), \text{DAssoc}[D, L](t_0), \text{Rel}([l^d_0], [l^2_0]), \text{Rel}([l^3_0], [l^4_0]), \text{Rel}([l^5_0], [l^6_0])\} \]

\( A_0 \): \{\text{Product}\text{Name}\}.

\( l^d_0 \): \{\text{Product001, 05/01/2020, 15/06/2020}\}

\( \text{DAssoc}[D, A](t_0) \): \{\text{Product} \rightarrow \text{Product}\text{Name}\}.

\( \text{Rel}([l^d_0], [l^2_0]) \): \{\text{Product}\text{To}\text{Name}, 05/01/2020, 31/12/19999\}

\( i^2_0 \): \{\text{Mozzarella, 05/01/2020, 15/06/2020}\}

\( \text{DAssoc}[D, L](t_0) \): \{\text{Product} \rightarrow \text{Category}\}.

\( \text{Rel}([l^d_0], [l^3_0]) \): \{\text{Product}\text{To}\text{Category}, 05/01/2020, 31/12/19999\}

\( i^3_0 \): \{\text{Product001, 05/01/2020, 15/06/2020}\}. 

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Definition 7: A hierarchy is a projection of analysis by level along the axis defined by the dimension. It is organised from the finest to the coarsest granularity level, thus offering analysis possibilities for ascending groupings through roll-up and descending groupings through drill-down. The hierarchy is defined as follows:

\[ H(t) = \{L,R_0|L_j,L_k\}[t]\].

\(L = \{l_i(t)\}, i \in [1,*]\), represents the set of aggregation levels constituting a hierarchy \(H(t)\) at a time \(t\).

\(R_0|L_j,L_k\)[t] represents the aggregation function between the different levels \(\{l_i(t)\}\) constituting a hierarchy \(H(t)\) at a time \(t\), with \(j \in [1,*]\), \(k \in [1,*]\) and \(j \neq k\).

Assume that \(D_i(t) \in D\) with \(i \in [1,*]\), \(\forall H_j(t) \in H\) with \(j \in [1,*]\), and \(L_j^h(t)\) is directly related to \(D_i(t)\) so that

\[ D_i(t) \prec L_j^h(t) \prec L_{2j}^h(t) \prec \ldots \prec L_{kj}^h(t) \prec All.\]

\(D_i(t)\) represents the finest level of aggregation of the analysis axis.

\(L_j^h(t)\) represents the coarsest level of aggregation in the hierarchy \(h_j\).

\(All\) represents the global aggregation level of the dimension.

Example: Analysis projection by COUNTRY and CITY for the CUSTOMER dimension can be performed as follows:

\[ D(t) = (Customer) \prec L_j^h(t) = (City) \prec L_{2j}^h(t) = (Country) \prec All.\]

This formalism allows us to provide flexibility in the evolution of the model both at the schema level and at the level of the data instances while preserving the history of these evolutions. To study the evolution of the data and for a clearer explanation, we assume that the schema does not change. Indeed, the data instances in the GTDW evolve incrementally and without redundancy. All data instances of previous schemas will be available for consultation. When the relationship between the entities of a dimension/level and a lower level changes, a new link to the new corresponding instance in this level will be created, with an FD label that is set to the creation date and a TD label that is set to 31/12/9999. Additionally, the TD of the old relationship is set to the same creation date. Thus, each relationship will be valid during a specific time interval. The same principle is applied in the case of changing descriptor attributes since these are also represented by external nodes in relation to the business concept. The queries can be parameterised with these time labels to obtain results that match the real state of the DW and take into account any changes in the dimensions.

5.2 Graph Temporal Data Warehouse

The GTDW is an extension of the classical DW distinguished by the introduction of the temporal concept in the aggregation relationships due to the changes in the data. Indeed, it is characterised by the separation of business concepts from their descriptors (attributes), allowing each entity to have an independent evolution. The graph formalisation and the graph database implementation were adopted to overcome the constraints generated by the use of an ER model. Indeed, the graph formalisation offers more flexibility for the model, particularly in terms of evolution (Akoka et al., 2021). In addition to the representative quality of the interconnected data and the use of information-carrying links, particularly for the notion of temporality in aggregation links, the use of graphs was preferred due to the absence of integrity constraints, the absence of a pre-established schema and the possibility of representing each value of a tuple (or all of the tuple) using a node of the graph. This graph implementation is unlike relational tables, which are order schemas composed of horizontal lines and vertical columns in which the addition/removal of a column affects the whole structure. A (NoSQL) graph database (GDB) was used to represent the GTDW based on our formalisation. It presents data in the form of a graph (vertex/edge) using physical pointers between nodes, thus avoiding joins in queries; this is advantageous, particularly in a big data context. GDBs are also characterised by the absence of a data type and the possibility of integrating information into the relationships between data. Additionally, the concept of graphs has been adopted to allow us to carry out an advanced analysis of the data representation; this makes it possible to perform an online analysis to produce explicative and predictive models.

5.3 Transformation Rules

According to the formalisation presented in subsection 5.1 and the characteristics of the graph databases, the business concepts, as well as the descriptors, will be represented by nodes, and the relationships between these concepts will be represented by edges (Figure 3). According to the formalisation of our approach, we have established the following rules for moving from a classical multidimensional model to a graph multidimensional model:

1. Each tuple of a fact is represented by its own node.
2. Each (business/descriptor) value of a dimension, level or attribute is represented by its own node.
3. All relationships are represented by edges.
4. The fact nodes contain the measures.
5. The fact nodes are directly related to the dimension nodes.
6. The level nodes are related to a dimension / another level node according to their depth.
7. The attribute nodes are directly related to the dimension/level nodes.
8. The levels constitute hierarchies according to axes of analysis organised from the finest to the coarsest level of aggregation.
9. All edges between the (business/descriptor) values of a dimension, level or attribute have a chronological FD label and TD label to determine the lifespan of the relationship.
10. All nodes have a chronological VT label and TT label in accordance with the principles of bi-temporal databases.

By applying these rules to the schema of our running example, we obtain the schema represented in Figure 3. We consider the three dimensions PRODUCT, CUSTOMER and DATE, the fact SALES, the measure Sales Amount and the hierarchical levels CATEGORY (for PRODUCT) and CITY (for CUSTOMER). The dimensions and levels are described by nodes representing the descriptors.

![Figure 3: Logical schema for the GTDW.](image1)

This model separates the business concepts from the descriptors (attributes) to allow the independent evolution of each entity. Indeed, facts, dimensions and hierarchy levels are represented by nodes corresponding to the basic multidimensional concepts to which other nodes representing descriptors are linked. A temporal label consisting of a VT and a TT has been assigned to all entities to allow the identification of the different instances. Additionally, the FD and TD parameters determine the lifespans of the relationships in the dimensions, allowing the GTDW to consistently process the queries and avoid any mismatch in the results due to changes in the dimensions.

6 QUERIES IN THE GTDW

We represent the instances of our running example (shown in Tables 1, 2, 3 and 4) with our temporal approach based on a graph, as shown in Figure 4. The FD and TD parameters were used to identify the lifespans of the relationships between dimensions and levels. The labels of nodes and the names of the relationships are not included in the graph to keep the schema from being cluttered.

![Figure 4: Example of a data instance from the GTDW.](image2)

The schema shows that the relationship between the customer Mary Saveley, who has the identifier Cust001, and the level Paris, which has the identifier City001, is valid during the time interval from 01/01/2020 to 01/08/2020; meanwhile, the relationship between this same customer and the level Lyon, which has the identifier City002, is valid during the time interval from 01/08/2020 to the present. In the same way, the relationship between the product Mozzarella, which has the identifier Prod001, and the level Fresh, which has the identifier Categ001, is valid during the time interval from 01/01/2020 to 01/04/2020; meanwhile, the relationship between this same product and the level Dairy, which has the identifier Categ002, is valid during the time interval from 01/04/2020 to the present. The use of a graph database makes it easier to implement this temporality in the relationships between the instances since the links are also information carriers, just like the nodes. In our case, the links between the CUSTOMER nodes and the CITY nodes contain the attributes FD and TD, making it possible to determine the lifespans of the relationships between these instances. This is also true for the links between the PRODUCT nodes and the CATEGORY nodes.

Figure 5 illustrates the lifespans of relationships involving the levels CITY and CATEGORY, where the intersections of the different lifetimes create three time intervals denoted by $T_1$, $T_2$ and $T_3$. These tem-
poral changes in relationships require the use of temporal queries to obtain consistent results. We illustrate through some examples (Table 5) how graph databases could be used to extend the principle of relational algebra (Vaisman and Zimányi, 2022; Ahmed et al., 2015) to handle time-varying information in DW.

Below, we propose three examples of illustrative queries using temporal aggregation for, respectively, (i) the level CITY, (ii) the level CATEGORY and (iii) the levels CITY and CATEGORY jointly. The queries are written in the Cypher request language (CRL), which is specific to the Neo4j graph database.

**Query 1:**

MATCH (cn:city_name)<-[r:customer_city]-(c:customer)<-[s:sales] WHERE r.From_Date<=s.valid_date<r.To_Date RETURN cn.city_name, c.customer_id, SUM(s.sales_amount)

Query 1 shows the sales amounts for each city for the customer Cust001. Results (Table 6) are obtained for each of the cities that the client lived in. Note that in the CRL, the clause `RETURN value1,..., value_n,AGGREGATE_FUNCTION(ATTRIBUTE)` makes it possible to group the aggregation by `value1,..., value_n`.

**Query 2:**

MATCH (cn:category_name)<-[r:product_category]-(p:product)<-[s:sales] WHERE r.From_Date<=s.valid_date<r.To_Date RETURN p.product_id, cn.category_name, SUM(s.sales_amount)

Query 2 shows the sales amounts for each category for the product Prod001. Results (Table 7) are obtained for each of the categories that the product was assigned to.

**Query 3:**

MATCH (ctn:city_name)<-[r:customer_city]-(c:customer)<-[s:sales]-(p:product)<-[r:product_category]-(c:category) WHERE r1.FD<=s.valid_date<r1.TD AND r2.FD<=s.valid_date<r2.TD RETURN ctn.city_name, cgn.category_name, SUM(s.sales_amount)

Query 3 shows the sales amounts for different cities and categories. Results (Table 8) are obtained for each of the cities that the client lived in and for each of the categories that the product was assigned to.

Using the temporal parameters From_Date and To_Date, accurate results are obtained regardless of the changes made in the dimension instances. The same principle is applied to changes in the attributes of the dimensions due to the separation of business concepts from their descriptors (attributes), allowing each entity to have an independent evolution, and the temporal parameters FD and TD are also implemented in the relationships between these attributes. This feature, which is available due to the ability of graph databases to use information about the relationships between entities, offers advantages when it comes to formulating temporal queries.

7 VALIDATION

For validation purposes, we carried out two case studies based on the Star Schema Benchmark (SSB): the first study was performed for the functional validation of our approach, and the second study was used to perform runtime tests for a graph DW.

As part of the functional validation study, we chose to perform the instantiation process on Neo4j using the SSB data while generating several temporal changes in the aggregation relationships of some hierarchical levels and attributes; temporal queries generated from all 13 SSB queries were then applied to this temporal DW. Note that in the initial SSB schema, there were no hierarchical levels; we generated them from the attributes, as shown in Figure 6.

As the SSB data span from 1992 to 1998, the following changes in relationships were made:

1. For the dimension CUSTOMER, the assignments of one hundred randomly selected customers were changed on the hierarchical levels C_CITY, C_NATION and C_REGION. Thus, these clients were assigned to two different levels according to the two time intervals $T_1 = [FD_1 = 01/01/1992, TD_1 = 01/01/1994]$ and $T_1' = [FD_1 = 01/01/1994, TD_1 = 31/12/9999]$.

---

3https://github.com/Kyligence/ssb-kylin
Table 5: Temporal operators.

<table>
<thead>
<tr>
<th>Temporal operation</th>
<th>Case</th>
<th>Condition in query</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporal union</td>
<td></td>
<td>WHERE MIN(R1.From_Date, R2.From_Date) &lt;= s.valid_date &lt; MAX(R1.To_Date, R2.To_Date)</td>
</tr>
<tr>
<td>Temporal join</td>
<td></td>
<td>WITH CASE WHEN R1.From_Date &gt;= R2.From_Date THEN R1.From_Date ELSE R2.From_Date END as min_date, CASE WHEN R1.To_Date &lt;= R2.To_Date THEN R1.To_Date ELSE R2.To_Date END as max_date WHERE min_date &lt;= s.valid_date &lt; max_date</td>
</tr>
<tr>
<td>Temporal difference</td>
<td></td>
<td>WITH CASE WHEN R1.From_Date &gt;= R2.To_Date THEN R1.From_Date ELSE R2.To_Date END as min_date, CASE WHEN R1.To_Date &lt;= R3.From_Date THEN R1.To_Date ELSE R3.From_Date END as max_date WHERE min_date &lt;= s.valid_date &lt; max_date</td>
</tr>
<tr>
<td>Temporal aggregation</td>
<td></td>
<td>WHERE R1.From_Date &lt;= s.valid_date &lt; R1.To_Date AND R2.From_Date &lt;= s.valid_date &lt; R2.To_Date</td>
</tr>
</tbody>
</table>

Table 6: Results of Query 1.

<table>
<thead>
<tr>
<th>Customer_ID</th>
<th>City_Name</th>
<th>Sales_Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cust001</td>
<td>Paris</td>
<td>3700</td>
</tr>
<tr>
<td>Cust001</td>
<td>Lyon</td>
<td>3800</td>
</tr>
</tbody>
</table>

Table 7: Results of Query 2.

<table>
<thead>
<tr>
<th>Product_ID</th>
<th>Category_Name</th>
<th>Sales_Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prod001</td>
<td>Fresh</td>
<td>1500</td>
</tr>
<tr>
<td>Prod001</td>
<td>Dairy</td>
<td>6000</td>
</tr>
</tbody>
</table>

Table 8: Results of Query 3.

<table>
<thead>
<tr>
<th>City_Name</th>
<th>Category_Name</th>
<th>Sales_Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paris</td>
<td>Fresh</td>
<td>1500</td>
</tr>
<tr>
<td>Paris</td>
<td>Dairy</td>
<td>2200</td>
</tr>
<tr>
<td>Lyon</td>
<td>Dairy</td>
<td>3800</td>
</tr>
</tbody>
</table>

2. For the dimension SUPPLIER, the assignments of one hundred randomly selected suppliers were changed on the hierarchical levels C_CITY, C_NATION and C_REGION. Thus, these suppliers were assigned to two different levels according to the two time intervals $T_2 = [FD_2 = 01/01/1992, TD_2 = 01/01/1996]$ and $T_2' = [FD_2 = 01/01/1996, TD_2 = 31/12/9999]$.

3. For the dimension PART, the SIZE attributes of one hundred randomly selected products were changed. Thus, these products had two different sizes; they had one size for the time interval $T_3 = [FD_3 = 01/01/1992, TD_3 = 01/01/1995]$ and another size for the time interval $T_3' = [FD_3 = 01/01/1995, TD_3 = 31/12/9999]$.

We use the query Q3.3 from the SSB queries as an illustrative example:

```sql
OPTIONAL MATCH (c:cc_city) <-[:c_c_name]-(c_city)<-[r1:customer_city]-(c:customer) <-[:order_customer]-(l:lineorder)-[:order_date]->(d:date),(s:sc_city)<-[:s_c_name]-(s_city)<-[:s_c_name]-(s:sc_region)-[:s_r_name]-(r:region)
```
We found through the results obtained that the graph approach performed better for the queries Q1.2, Q1.3, Q2.2, Q2.3, Q3.3 and Q3.4 (from 2x to 11x better). The two approaches were quite close, with the relational approach being slightly faster, for the queries Q3.2 and Q3.3, and the relational approach performed better for the queries Q1.1, Q2.1, Q3.1, Q4.1 and Q4.2 (2x to 8x better). These differences in the execution times of the response graph approach depend on the filter factors (FFs), the number of dimensions and the number of edges to be covered. Indeed, Neo4j can be very efficient when the FF is low, just as it can become less efficient as the FF and the number of dimensions increase, which implies that many relations need to be browsed to carry out an aggregation function; on the other hand, Oracle is quite homogeneous in terms of the execution time of the queries. However, in the context of temporal approaches, a performance study must be carried out on the entire integration and storage process.

8 CONCLUSION

In this paper, we proposed a temporal multidimensional model for multi-version data warehouses based on a graph formalism. Temporal labelling has been used for entities and the relationships between them to allow the optimal management of data instances and to preserve the evolutionary history of these data, especially for changes in dimensions, in addition to providing the capacity for schema evolution offered by the concept of multi-versioning in DWs. We have established rules for moving from a classical multidimensional model to the temporal graph multidimensional model, and an example of an instance and examples of temporal queries have been provided. We have validated our approach with two use cases by carrying out functional validation and runtime performance experiments. We plan to carry out a performance study of the entire integration process and further study OLAP queries and graph cubes. In addition, we plan to carry out a study on advanced analyses in graph models in order to utilise online analysis to produce explanatory and predictive models.

REFERENCES


Ahmed, W., Zimányi, E., and Wrembel, R. (2015). Tempo...


