

Optimization Analysis for an Uncovered Wagon Transportation with an Interactive Animated Simulation-Based Platform for Multidisciplinary Learning

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Keywords: Interactive Simulation-Based Learning, Engineering and Non Engineering Education, Computer Supported Education, Virtual Tools and Augmented Learning Environments.


Abstract: At an earlier stage of European funding for projects on technology-enhanced learning, the main thrust was to develop e-learning technologies and on projects that sought to promote the take-up of platforms and services. This contribution is prepared by students after attending lectures of a multidisciplinary course in the context of a complementary studies frame. The students of this course summarized, through a case study, concepts and methods in a straightforward, but structured way. Thanks to the help of a software tool based on Python, an original and open learning platform is realized for students and lecturers and it represents a part of this contribution. Concerning the specific lecture, cargo loads and transportation are important logistical topics in many industries. They dictate the profit of the final product, expenditure, time consumption and labor force utilization. The optimal transportation of any type of cargo is crucial for businesses. More in general, it is possible to say that the proposed problem can be generalized and applied in other main economic problems in which optimization problems are involved. In this work we focused on the optimization of fluid transportation under specified conditions, or constraints, in other words. The aim of the project is to determine the optimal parameters of the system to control the transportation in an optimal way. This material which includes an open software to test the developed concepts through the lecture can be used by students and lecturers. An open link is accessible to the users.

1 INTRODUCTION

At an earlier stage of European funding for projects on technology-enhanced learning, the main thrust was to develop e-learning technologies and on projects that sought to promote the take-up of platforms and services. At the initial stage, the need-based use of web for knowledge acquisition was seldom considered as 'e-learning'. At a later phase, major European projects were funded to support the development of knowledge process methodologies with web tools and specific software solutions (such as the EU-funded Mature project inside the FP7 European Project). The aim there was to analyse the knowledge-intensive work processes and practices of 'knowledge workers'. Using the platform will allow participating companies to integrate and transfer knowledge in a more natural and direct way. In this sense, the crucial concept of the transferability is automatically guaranteed

via knowledge integration of industry and academic world platform. In the past, without upcoming Internet of Things, several "remote laboratories" have been set up for teaching and research purposes in the field of automatic control and robotics. In a remote laboratory, physical systems settled at specific locations in the world are made available through the internet to remote users for performing experiments and validation tests in location where plant models are not available or are difficult to be reproduced (Gomes and Bogosyan, 2009), (Casini and Garulli, 2016), (Casini et al., 2004), (Leva and Donida, 2008), (Balestrino et al., 2009), (Tzafestas, 2009). All these structures, although having permitted tangible progresses in the field to be achieved, have the drawback of requiring complex hardware and demanding communication equipment. Instead, fully virtual laboratories have been recently developed by several companies, also in the framework of collaborations among higher education institutions or under the patronage of governmental institutions – see, e.g., Labster and Virtual Labs. These solutions are generally oriented to sci-

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ence and engineering. Also, it is noticeable that EU-funded projects have been conceived to devise virtual laboratories. This is the case, for instance, of the projects Next-Lab and ELLIOT. All these projects are milestones either as general purpose tools in science education (like the H2020 project Next-Lab) or in their own fields of interest (IOT like in the FP7 project ELLIOTT or physical systems). The proposed problem is modelled and solved using a python environment which fits the recommendation of the 2013 EU Communication on Opening Up Education. To sum up, in this context, the main features that showcase the novelty and innovative aspects of this contribution, with the direct participation of the students, can be stated as follows:

- introducing new and paradigmatic topics in the teaching and learning contents, which reflect the latest EU research funding programmes in the broad field of control engineering;
- providing a technical and cultural resource among different universities and companies to share available expertise and knowledge promoting in this sense a knowledge platform which structurally guarantees a direct transferability;
- providing a flexible learning and teaching structure in which the user can actively interact to test the proposed solution and to modify the virtual models.

This work can be seen as a paradigmatic example of a framework for an integrated transnational approach to academic teaching and learning, that contributes to the development of the engineering, and non-engineering, communities, meeting their needs through an innovative cloud-based virtual platform shared by external organizations in the first instance and spread among stakeholders in the long-term.

1.1 Starting with a Virtual Lecture

This contribution proposes a structure of the lecture in the context of a complementary course which should be open for Master's students also with no background. This material is prepared by students after attending some lectures in this multidisciplinary course which is in this case dedicated to optimization techniques in the context of a complementary study frame. In this study frame, engineers and non-engineers can be admitted. The students of these courses summarized, through a case study, concepts and methods in a straightforward but structured form. Thanks to the help of a software tool, an original and open-learning platform is realized for students and lecturers. This learning platform can be used both by students and

lecturers. Students can interactively test the effect of physical variables directly on the developed platform. Teachers can use this material for their lectures and they can be inspired by this material for further developments. The animated structure of the simulation realizes a friendly platform to be used intuitively. This lecture connects various disciplines. This allows the students to set additional priorities in parallel to their special studies and it gives the opportunity to sharpen their individual competence profile - subject-related, non-specialist or interdisciplinary. The shipment loading problem is not only an engineering one. In fact, the problem formulated in the context of an optimization problem can be seen as an economical one in which possible cost functions can be defined together with possible constraints. The shipment loading problem is highly overspread and has variations (Aksentijevic et al., 2020). The cargo can be relocated by different means of transport between cities or countries, or can be carried from one department of the plant to another one, depending on the industrial targets. This project elaborates on the issue of fluid transportation in the uncovered carriage.

1.2 Structure of the Paper

This paper contains the problem description, see Section 2. In this section possible issues and the main aim are explored together with the system explanation, which helps to understand its parameters, objects, conditions and states. This part is a fundamental one and includes knowledge related to the background. In Section 3, methods, which have been used to solve the problem formulated in Section 2, are shown. In Section 4, constraints are explicitly considered and the Karush-Kuhn-Tucker criteria are used to find the optima. Section 5 explains the obtained results in order to give the possibility to the readers to have access to the interpretation of the visualized results. Conclusion and future work session closes the paper. The access to the virtual lecture can be found via the following link: [link](#).

2 PROBLEM DESCRIPTION

The project assumes a wagon that transports some amount of fluid. The wagon has a mass M_W and an interior of cuboid shape, open at the top. The interior has height r , length d and depth b . The wagon can be filled with the fluid. Therefore, the volume of the fluid is:

$$V = h \cdot d \cdot b, \quad (1)$$

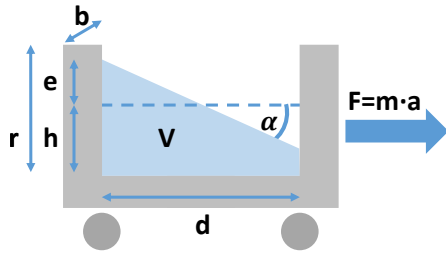


Figure 1: Wagon: when accelerated, the fluid moves and the excess e gets bigger. The angle α can be defined as: $\tan(\alpha) = 2e/d$.

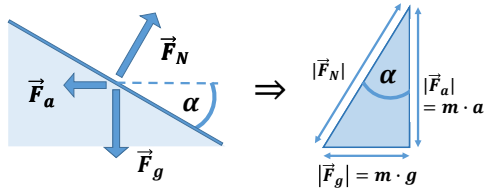


Figure 2: Forces: The different forces acting on the fluid result in a tilted surface with angle α , where $\tan(\alpha) = g/a$ with the acceleration of the cart a and the gravitational acceleration g . Since the wagon accelerates in the forward direction, a fluid particle of mass m in the reference system of the wagon feels a force $F_a = a * m$ in the opposite direction of the same magnitude, as well as a normal force F_N and the gravitational force $F_g = mg$.

where h is the height up to which the wagon is filled. The total Mass of the wagon filled by a fluid with density ρ is:

$$M = M_W + \rho \cdot V. \quad (2)$$

The wagon can be accelerated (and decelerated) with a maximal force of F_{max} . Figure 1 shows the level change of the fluid in the wagon, caused by the movement with acceleration. Since the wagon gets accelerated, the fluid moves. The forces that result from the acceleration can be seen at Fig. 2. The position $x(t)$ of the wagon after some time with initial values of velocity and acceleration follows the next relation:

$$x(t) = x_0 + v_0 \cdot t + \frac{1}{2} a_0 \cdot t^2 + \frac{1}{6} j \cdot t^3. \quad (3)$$

The goal of our work is to determine optimal values for the parameters in (3), as well as an optimal height h of the fluid, such that the volume V that is moved over a distance $L = x(T)$ per time T is maximised. The boundary conditions under which the optimization takes place are those that the maximal achieved acceleration times the wagons mass is below the maximal force the wagon can produce and that the fluid does not swap over, which puts a constraint on the maximal angle of α (see Figs. 1 and 2). To sum up the observations mentioned above, we will determine the problem with objective function, constraints and optimization method in Section 3.

3 CONSTRAINED OBJECTIVE FUNCTION

This part considers the calculations which lead to the specification of objective function and constraints. The initial position and velocity of the system are zero:

$$x(t = 0) = 0 \Rightarrow x_0 = 0, \quad (4)$$

$$\dot{x}(t = 0) = 0 \Rightarrow v_0 = 0, \quad (5)$$

$$\Rightarrow x(t) = \frac{1}{2} a_0 \cdot t^2 + \frac{1}{6} j \cdot t^3, \quad (6)$$

$$= \frac{1}{2} t^2 \cdot (a_0 + \frac{1}{3} j \cdot t). \quad (7)$$

The distance, travelled by the wagon with the fluid for the defined time, submits to the following constraint:

$$x(t = T) = L, \quad (8)$$

$$\Rightarrow L = x(T) = \frac{1}{2} T^2 \cdot (a_0 + \frac{1}{3} j \cdot T). \quad (9)$$

The velocity of the wagon at its destination should be zero:

$$\dot{x}(t = T) = 0, \quad (10)$$

$$\Rightarrow 0 = \dot{x}(T) = a_0 \cdot T + \frac{1}{2} j \cdot T^2. \quad (11)$$

This leads to the following equation for the jerk coefficient j which determines the acceleration's change over time:

$$\Rightarrow j = -2 \frac{a_0}{T}. \quad (12)$$

We substitute the found parameters into the distance defined by (9) and express the time variable:

$$\Rightarrow L = \frac{1}{6} T^2 \cdot a_0, \quad (13)$$

$$\Rightarrow T = \sqrt{\frac{6L}{a_0}}. \quad (14)$$

The objective function can therefore be written as:

$$\Rightarrow \frac{V}{T} = h \cdot d \cdot b \cdot \sqrt{\frac{a_0}{6L}}. \quad (15)$$

Now, we can rewrite the kinematic equations of the system in the following way:

$$x(t) = a_0 t^2 \left(\frac{1}{2} - \frac{1}{3} \frac{t}{T} \right), \quad (16)$$

$$v(t) = \dot{x}(t) = a_0 t \left(1 - \frac{t}{T} \right), \quad (17)$$

$$a(t) = \ddot{x}(t) = a_0 \left(1 - 2 \frac{t}{T} \right). \quad (18)$$

The force $F(t)$ that has to act on the wagon at the time t to guarantee the desired dynamics has the following form:

$$F(t) = M \cdot a(t) = (M_W + \rho \cdot V) \cdot (a_0 + t \cdot j), \quad (19)$$

$$|F(t)| \leq F_{max}, \quad (20)$$

and is constrained by the maximal force F_{max} that the wagon can generate. Since the distance L that is travelled by the cart has to be greater than zero, as well as the time that it took to travel it, we know that:

$$\frac{6L}{T^2} > 0, \quad (21)$$

$$\Rightarrow a_0 > 0, \quad (22)$$

$$\Rightarrow \max_t a(t) = \max_t a_0 \left(1 - 2\frac{t}{T}\right), \quad (23)$$

$$= a(0) = a_0, \quad (24)$$

similarly we can show, that for $t < T$ the minimal acceleration is: $\min a(t) = a(T) = -a_0$. Now, we can reformulate the constraint (19):

$$a_0 \leq \frac{F_{max}}{M}. \quad (25)$$

The second constraint deals with the problem of containing the fluid inside the wagon. The maximal rising of the fluid's level e should be smaller than the difference between the fluid's height without acceleration and the height of the carts rim:

$$e \leq r - h. \quad (26)$$

The maximal excess e is in relation to the width of the cart d and the angle α by which the fluid's surface is tilted according to the following equation:

$$e = \tan(\alpha) \frac{d}{2}, \quad (27)$$

(see Fig. 1). According to the force diagram at Fig. 2, we can define the angle α by the forces acting on the fluid. At the maximal acceleration a_0 , at which the maximal rising of the fluid happens, the forces are the following:

$$\vec{F}_g = M_f l \cdot g, \quad (28)$$

$$\vec{F}_a = M_f l \cdot a_0, \quad (29)$$

$$\Rightarrow \tan(\alpha) = \frac{|\vec{F}_a|}{|\vec{F}_g|} = \frac{a_0}{g}. \quad (30)$$

Now, we can rewrite the constraint 26:

$$\frac{d}{2} \frac{a_0}{g} \leq r - h. \quad (31)$$

We will keep the minimization of the objective function:

$$Loss = -\frac{V}{T} = -h \cdot d \cdot b \cdot \sqrt{\frac{a_0}{6L}}. \quad (32)$$

4 Karush-Kuhn-Tucker

We can use the Karush-Kuhn-Tucker (KKT) criteria (Brezhneva et al., 2009) to do the optimisation with constraints. We want to minimise the function $f(h, a_0)$ under the constraints $g_i(h, a_0) \leq 0$:

$$f(h, a_0) = -dhb\sqrt{\frac{a_0}{6L}}, \quad (33)$$

$$g_1(h, a_0) = \frac{da_0}{2g} + h - r \leq 0, \quad (34)$$

$$g_2(h, a_0) = a_0(M_W + d h b \rho) - F_{max} \leq 0. \quad (35)$$

From this result, the following KKT conditions are derived:

$$\nabla f(h^*, a_0^*) + \sum_{i=1}^2 \mu_i^* \nabla g_i(x^*) = 0, \quad (36)$$

$$g_1(h^*, a_0^*) \leq 0, g_2(h^*, a_0^*) \leq 0, \quad (37)$$

$$\mu_1^* \geq 0, \mu_2^* \geq 0, \quad (38)$$

$$\mu_1^* g_1(h^*, a_0^*) = \mu_2^* g_2(h^*, a_0^*) = 0, \quad (39)$$

from equation (36), we get two separate equations:

$$-db\sqrt{\frac{a_0}{6L}} + \mu_1 + \mu_2 a_0 d b \rho = 0 \quad (40)$$

$$-\frac{1}{2} d h b \sqrt{\frac{1}{a_0 \cdot 6L}} + \mu_1 \frac{d}{2g} + \mu_2 (M_W + d h b \rho) = 0. \quad (41)$$

Equation (39) also leads to two equations:

$$\mu_1 \left(\frac{da_0}{2g} + h - r \right) = 0, \quad (42)$$

$$\mu_2 (a_0 [M_W + d h b \rho] - F_{max}) = 0. \quad (43)$$

Now, we can solve these equations to get the optimal a_0 , h and μ_1 and μ_2 . First, we will do a distinction of different cases.

4.1 Case1: $\mu_2 = 0$

If μ_2 is 0, the four equations above are reduced to the following three equations:

$$-db\sqrt{\frac{a_0}{6L}} + \mu_1 = 0, \quad (44)$$

$$-\frac{1}{2} d h b \sqrt{\frac{1}{a_0 \cdot 6L}} + \mu_1 \frac{d}{2g} = 0, \quad (45)$$

$$\mu_1 \left(\frac{da_0}{2g} + h - r \right) = 0. \quad (46)$$

From (44) we get

$$\mu_1 = db\sqrt{\frac{a_0}{6L}}, \quad (47)$$

which can be set into (45) to get a relation between the optimal h and a_0 :

$$h = \frac{da_0}{g}. \quad (48)$$

Using these two equations in (46), we get:

$$db\sqrt{\frac{a_0}{6L}} \left(\frac{3da_0}{2g} - r \right) = 0. \quad (49)$$

Since d, b, a_0 and L are all unequal 0, we get the optimal solutions in this case as:

$$a_0 = \frac{2rg}{3d}, \quad (50)$$

$$h = \frac{2r}{3}. \quad (51)$$

4.2 Case2: $\mu_2 > 0$

4.2.1 Case 2.1: $\mu_1 = 0$

From $\mu_1 = 0$ follows for (44):

$$db\sqrt{\frac{a_0}{6L}} = 0. \quad (52)$$

Since $d, b,$ and a_0 cannot be equal to zero, this case cannot be optimal.

4.2.2 Case 2.2: $\mu_1 > 0$

If $\mu_1 > 0$ and $\mu_2 > 0$, (42) and (43) are simplified in the following way:

$$h = r - \frac{da_0}{2g}, \quad (53)$$

$$a_0(M_W + d h b \rho) - F_{max} = 0. \quad (54)$$

Now, we can solve these two equations to get the two unknowns h and a_0 :

$$a_0 \left(M_W + db\rho \left[r - \frac{da_0}{2g} \right] \right) - F_{max} = 0, \quad (55)$$

$$\Rightarrow a_0^2 - a_0 \cdot 2g \frac{M_W + db\rho r}{d^2 b \rho} + \frac{2g F_{max}}{d^2 b \rho} = 0, \quad (56)$$

$$\Rightarrow a_0 = g \frac{M_W + db\rho r}{d^2 b \rho} \pm \sqrt{g^2 \left(\frac{M_W + db\rho r}{d^2 b \rho} \right)^2 - \frac{2g F_{max}}{d^2 b \rho}}. \quad (57)$$

Using this solution for a_0 , we get the optimal h using (53).

5 RESULTS

In this work the team accomplished the project via programming language Python. It has necessary modules and libraries for optimization problems solving, (see *scipy*), tools for the results visualization and graphs' construction. The Sequential Least Squares Programming (SLSQP) method has been applied. This method is suitable for this issue, because it is non-linearly constrained gradient-based optimization for a scalar function of one or more variables (Fu et al., 2019). As it was noticed at the Section *KKT* we minimize the objective function (33) under constraints (34)-(35). Using the constant variables:

$d = 20.0$ # wagon length
 $b = 2.5$ # wagon width
 $r = 3.0$ # wagon height
 $g = 9.8$ # free fall acceleration
 $L = 120$ # distance of transportation
 $\rho = 1$ # fluid density
 $M_w = 50$ # wagon mass
 $F_{max} = 200$ # maximal pulling force,

the optimal values of the height and acceleration have been found automatically:

Optimal height: 2.0
 Optimal acceleration: 0.98.

We implied the received values into the movement law in order to see the alterations of position, velocity and acceleration due time. The graphs are presented in Figs. 3-5, accordingly.

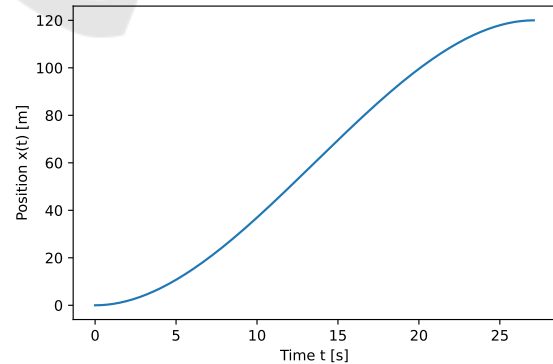


Figure 3: Optimal position of the cart over time.

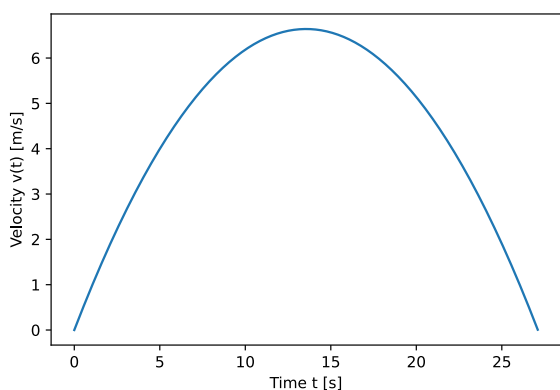


Figure 4: Optimal velocity of the cart over time.

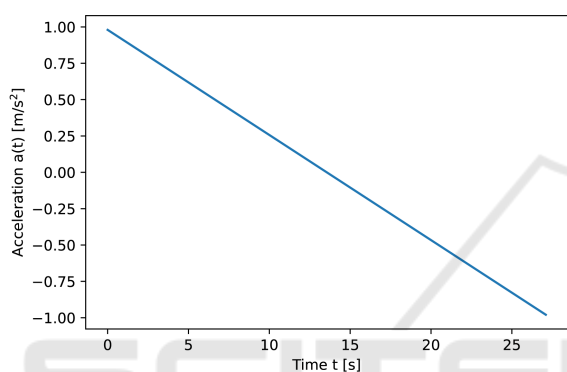


Figure 5: Optimal acceleration over time of the cart.

We replicated the level of the fluid in the wagon as a blue line at a rectangle in a 2D space. Figure 6 shows the changes of fluid’s level at three moments of time during the movement.

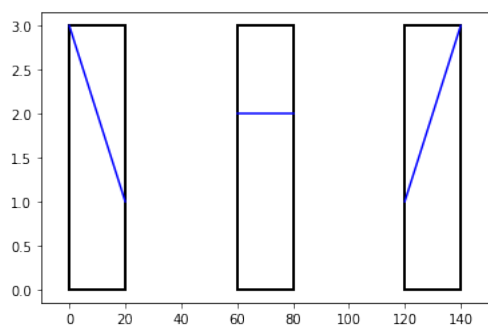


Figure 6: Wagon with fluid over time.

The project provides the solution for the technical side of the fluid transportation problem in the uncovered wagon. After the issue’s exploration the parameters of the system, which include the characteristics of the cart and fluid, have been defined. The analysis led the team to the optimization of the fluid transportation via the determination of optimal values for the fluid height and the acceleration of the wagon. The

received results could be applied to real world situation, for example to achieve the maximal profit for a specific transportation task. The output affects the volume of the fluid and the time of the wagon relocation. This may decrease shipment or energy costs and increase the utility of the transportation.

6 CONCLUSION AND FUTURE WORK

This contribution deals with an interactive software platform dedicated to realization of lectures in the context of optimization problems. The platform, together with the lecture structure were realized by students after attending lectures of a multidisciplinary course in the context of a complementary study. The proposed lecture considers an optimization problem in the context of transportation. The contribution is addressed to students and lecturers who can find inspiration and idea for further development or they can directly use this material. This contribution is on track with EU-funded projects which have been conceived to devise virtual laboratories, as, for instance, the projects Next-Lab and ELLIOT. As the contribution is on track with the main guideline and research directions of EU in the field of didactic, future work can include also building a set of different examples with even more possibilities to change parameters and conditions in the initialization phase of the simulations. The platform is fully accessible and the software in use is Python. An open link is accessible to the users.

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This work was inspired by the lecture ”Optimization Techniques” held by Paolo Mercorelli within the scope of the Complementary Studies Programme at Leuphana University of Lueneburg during the summer semester 2022. In this framework, students can explore different disciplinary and methodological approaches from the second semester onwards, focussing on additional aspects in parallel with their subjects and giving them the opportunity to sharpen skills across disciplines.

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