Feedback in Online Mathematics Tutoring

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Abstract: The goal of this paper is to present issues related to assessment and feedback in the framework of online mathematics tutoring implemented with the help of a chatbot using Artificial intelligence (AI) (Jančařík et al., 2022). The presented project aims to create a teaching course that is intended to help the pupil in independent preparation for the national entrance exams in mathematics for upper secondary schools in the Czech Republic. The course takes the form of a chatbot with which a pupil can communicate in a web browser environment or the Telegram communication application designed for all common operating systems (Windows, macOS, Linux, iOS, Android, ...). The chatbot also includes a communication module using artificial intelligence that can communicate with the pupil beyond the scope of the designed course. The following two questions are addressed in the part of the research that is presented in this paper. The first question is what form of feedback is effective in the given environment and most reflect the nature of tutoring. The second question is how the chosen procedures must be modified for the different areas of mathematics the course focuses on. The paper presents an implementation within the area of algebra and geometry.

1 INTRODUCTION

The use of technology in the teaching of mathematics is a relatively new area of interest for mathematicians and educators, not only in the area of mathematics. Drijvers et al. (2010) summarize the stages of development in the years 1960 to 1990. Already from 1942, there was a significant development in computing technologies, but only in the late 1960s, the focus of mathematicians and mathematics educators turned their attention to the effects of computing on the content of school-level and university-level mathematics (Fey, 1984). One of the main goals of the use of technology is to promote a more active form of student learning.

Technology has also affected the teaching of mathematics, and in the 1980s, theoretical frameworks were developed in which the use of technological tools in education was investigated. Drijvers et al. (2010) draw attention to the Tutor/Tool/Tutees (Taylor, 1980) and White Box/Black Box (Buchberger, 1990) frameworks, among others. In the mode Tutor, the technology presents the materials which the student answers and the technology evaluates their answers. The mode Tool has a similar focus but requires less programming than the mode Tutor. The mode Tutee was described by Taylor as follows: "To use the computer as *tutee* is to tutor the computer; for that, the student or teacher doing the tutoring must learn to program, to talk to the computer in a language it understands" (Taylor, 1980, p. 4).

The use of ICT in education continues to be at the centre of interest of mathematics educators (Verschaffel et al., 2019; Hardman, 2019, Phuong et al., 2022). Lagrange et al. (2001) present a survey of literature about the educational uses of ICT in mathematics education up to 2001. Gissel et al. (2019) published a critical review of various meta-studies about the impact of ICT use on students' learning. Much attention is paid to the use of Artificial Intelligence in education. This is also the focus of this article, which focuses on questions related to evaluation and feedback in the framework of online mathematics tutoring implemented with the

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help of a chatbot with the use of AI. The aim is to develop a system of structurally homogenous courses with a focus on the nationwide entrance examination for Czech secondary schools where the AI would support the learners' experience by answering pupils' questions related or non-related to a given topic.

2 MATERIALS AND METHODS

This section is discussed in two parts. Firstly the definition of *tutoring* and related research is described; secondly, the section talks about *artificial intelligence in education.*

2.1 Tutoring

The term tutoring in this paper refers to tutoring in a school subject which is taught in addition to mainstream schooling. Tutoring is a worldwide phenomenon that has been paid attention to especially in recent decades (Hille et al., 2016; Bray & Silova, 2006). Its main focus is on the "core" subjects, i.e. language and mathematics (Mischo & Haag, 2002). Tutoring in mathematics, albeit in various forms, can be found in the vast majority of countries (Song, 2013). Even though there are several different forms of tutoring, the prevailing form is the form of private supplementary tutoring, i.e. paid tutoring focusing on content from school lessons or preparation for entrance exams (Novotná, 2019). As this is a paid activity, private supplementary tutoring is not available to all students to the same extent. Differences in family socioeconomic status are further exacerbated by a system in which students who are tutored achieve better results (Safarzyńska, 2013). Tutoring used to be typically carried out face-to-face when a pupil or a group of students came to see the tutor, or the tutor came to see the students. Online tutoring expanded during the covid-19 pandemic. The fact that there is no direct contact between the tutor and the student opens up space for the automation of activities. Some studies show the effectiveness of providing online tutoring (e.g. Beal et al. (2007)). Artificial intelligence can contribute to an increase in the frequency of use of online tutoring and can partly or even completely replace the tutor. The tutor can be replaced in the selection of the study trajectory, in the evaluation of results, as well as in communication with the student (Shahbazi & Byun, 2022). These changes aim to improve the quality and accessibility of tutoring forms and thereby reduce the impact of socioeconomic background on a student's performance and achievement (Alhossaini & Aloqeely, 2021). That's why systems like the one described here, which will use the latest technologies, including AI, to deliver free education content to all learners, have a big role to play.

2.2 AI in Education

The Research into the use of AI in education focuses on the following four areas (Zawacki-Richter et al., 2019):

1. Profiling and prediction;
2. Assessment and evaluation;
3. Adaptive systems and personalization;
4. Intelligent tutoring systems.

The here presented research focuses only on one of the forms of using artificial intelligence, namely on its use in the creation of an intelligent tutoring system, i.e. a system in which one-to-one personal tutoring takes place, where the tutor's role is fully or partially taken over by a computer system – artificial intelligence. Despite the significant progress in the development of artificial intelligence in recent years, the difference between AI and a live tutor is still evident. However, research conducted with the first such systems shows that some students may find it much easier to communicate with AI than with a teacher or a tutor (Kim et al., 2020). In another study (E. Park et al., 2011), when educating participants on a certain topic, a robot tutor that provided positive feedback was perceived as attractive and acceptable. As part of Attard's (2021) research, it was found that when using an AI chatbot in the explanation of the mathematics, 73% of the users enjoyed making use of the chatbot, and the same percentage of respondents also expressed a desire to use the chatbot again in the future. In all, these studies support the review of research showing that social robots in educational settings have positive effects on student learning (Belpaeme et al., 2018). In sum, although not always the case, most research on robots in education has shown promising ways that can facilitate effective learning experiences.

AI instruction may provide an effective means for delivering instruction when current events prohibit face-to-face human interaction. Although the first results show the great potential of using AI in tutoring, research in this area is at the very beginning and many questions are still open. One of the most important issues is the design of an appropriate structure for the course and the form of providing feedback.

Tutoring differs from school education in many ways. Thus, assessment and feedback must be

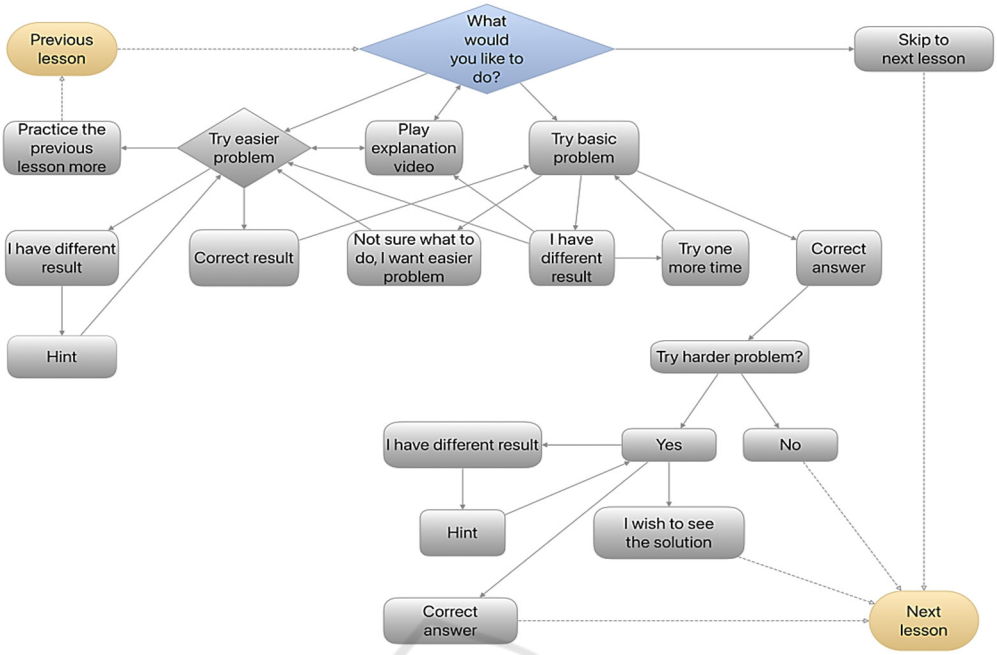

Figure 1: Options the user can choose from at each step.

adapted to these differences. Summative assessment, which is often used in schools, does not seem appropriate in the context of tutoring. Based on an analysis of the effects of summative assessment (Harlen et al., 2002), two reasons can be given why it is advisable not to rely merely on summative assessment in tutoring. The first reason is that summative assessment motivates only some students and increases the gap between higher and lower-achieving students. By tutoring, however, we want to reduce the differences between higher and lower-achieving students (without reducing the performance of high achievers). The second reason is that summative assessment motivates students towards performance goals rather than towards learning goals, as required for continuing learning (Harlen et al., 2002). The goal of tutoring should not only be to achieve short-term results but to prepare for continuous learning. The aim of assessment should therefore not be an evaluation of achieved goals, but rather the level of mastery of the needed knowledge, procedures and skills. Fiori et al. (2004) work with the term process-oriented assessment and state that by assessing students' problem-solving processes rather than products alone, we may provide them with more formative feedback as compared with the other techniques. We consider the provision of this kind of feedback to be essential for effective tutoring. Roa (2006) states that when using ICT in education it is important to utilize both formative and summative evaluation. On the one hand, it is important to determine not only the tools that allow for the learning of a particular subject area but those that allow for the correct feedback. Sadler (Sadler, 1998) points out that the quality of feedback is a crucial issue.

In the paper, we focus on the use of AI for two purposes. One of them is its independent use by students as a tool for self-checking the correctness of the solution or as an aid to finding a possible way to the right results. We classify this as used in the Tutee role (Taylor, 1980). The system can also be used by a teacher who wants to introduce students to some procedures that they do not know yet or do not know how to use. This is the mode of the Tutor (Taylor, 1980).

Petty (2002) states the following motivational reasons from a survey among students:

- The things I am learning are useful to me;
- The qualification I will get is good for me;
- I usually have good results in my studies and this success boosts my self-confidence;
- If I study well, it will be appreciated by my teacher or my classmates;
- If I don't study, there will be unpleasant (and quite immediate) consequences;
- The things I am learning are interesting and make me curious to learn more;
- The teaching is fun.

3 RESULTS

The goal of our research is to design an online course that will help students to prepare on their own for the national entrance exams from mathematics for upper secondary schools in the Czech Republic. The course covers the following four areas*: Number and Arithmetic Operations*, *Dependencies, Relations and Work with Data*, *Geometry in Plane and Space*, and *Non-Standard Application Problems*.

The course takes the form of a chatbot and can be run in a web browser or the Telegram communication application designed for all common operating systems. Due to the need to implement the course into the Telegram environment, the user interface consists exclusively of elements suitable for touch phones and tablets. Thus, the user communicates with the chatbot mainly using selection fields or text fields. The chatbot's response is typically verbal, with an image, a gif, an URL link, or another decision-making level with a selection from the pre-offered response fields. Thanks to the integration of artificial intelligence, the student can even drop the discussed topic and ask questions to which the chatbot responds. This AI system can also supply relevant information pupils might need to solve the task. For example AI responds pupil when he or she asks about reconnecting to the course and starting over. Questions and answers of a pupil are continuously reflected upon and new functions are added (such as searching for relevant information on wikipedia or other databases of educational resources). AI is being trained on questions and reactions of pupils'.

The aim of the course is not to test the student, but to improve their abilities in and understanding of the given areas. That is why we decided to implement the course in a form where students do not get the usually presented choice of answers of which only one is correct and the others are wrong. Instead, we decided to use a form where only one answer is given and this answer is correct. The other options allow the student to ask for advice or to give up on the solution. It is thus up to the student to solve the task and then answer whether they have reached the desired result. We expect a higher level of motivation from this form (Petty, 2002) when the student does not feel that he is being tested and is not penalized for choosing the wrong answer or for asking for advice. If the student selects that they need advice, the chatbot will show them a detailed solution to the given task, or a procedure that can be used for the solution of the task after some minor modification. For each topic, questions are graded according to difficulty, allowing the student to skip an easy task if they feel confident in the area. Moreover, after solving a standard task, it is up to the student to decide whether they want to try to solve a difficult task. While working in the course, the student is also allowed to play a video with an explanation that will link them to YouTube or another video server. Since our goal is to find out whether a student has improved in the area after completing the course and how they worked with the multiple-choice format without distractors, their progress is recorded anonymously. Among other things, the time between displaying a question and selecting an answer or the performance on a pre-test and post-test is monitored. The student has the opportunity to generate topics for the course based on the results of the pre-test. They do not have to go through all the areas if they for example only have problems in algebra.

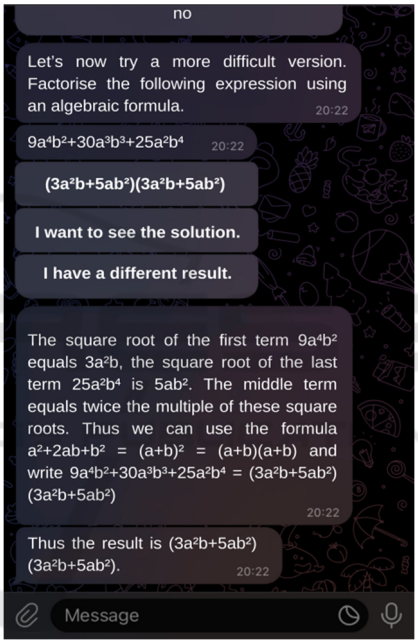

Figure 2: Chatbot environment for Algebraic expressions topic.

A more detailed functioning of the chatbot is illustrated in Figure 1, which shows what options the student chooses from at each step. Specific examples of the use of algebraic identities and unit conversions are shown in Figures 2 and 3. In Figure 2, the student's task is to factorise an algebraic expression using the formula for the square of the sum. As the student did not know what to do, they selected the option "I want to see the solution". Through this, the student got access to an in-detail explained solution to the problem. In Figure 3, the student's task is to convert units of length. The student chose the correct answer, and in response, the chatbot commended the

student as well as gave them useful tips on what to look out for when converting.

The fundamental question addressed in our research was whether this principle can be applied in all areas of mathematics that the course focuses on. Our original intention was to apply a uniform scheme across all topics so that the student could get used to the homogeneity of the environment and could work in it effectively. However, this turned out to be impossible, as different areas of mathematics require different ways of presentation. While, for example, in algebra, it is possible to offer an answer in the form of an algebraic expression, there is no such possibility in the field of geometry. Not only is it not possible to express the solution with a one-line verbal answer, but also a solution in the form of a picture may not be sufficient for the student to understand. The nonlinearity of the answer in the form of a picture combined with the fact that the construction can usually be done in many ways means that we have to approach it differently than to offer one correct answer. While displaying the correct solution in arithmetic and algebra allows the student to check their understanding of the procedure and helps to eliminate numerical errors, in geometry showing one solution may confuse the student, especially if the student proceeded differently or found a different solution.

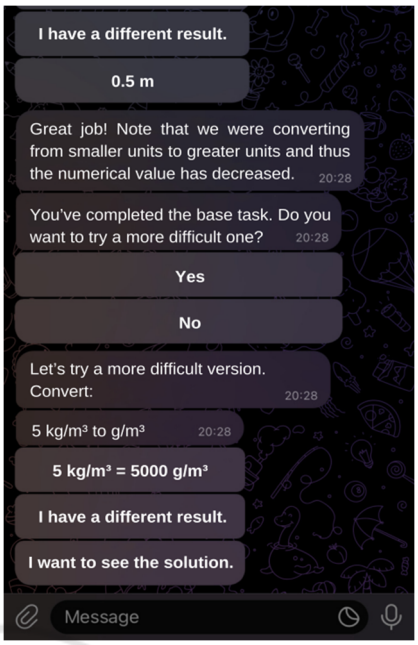

Figure 3: Chatbot environment for Unit conversions.

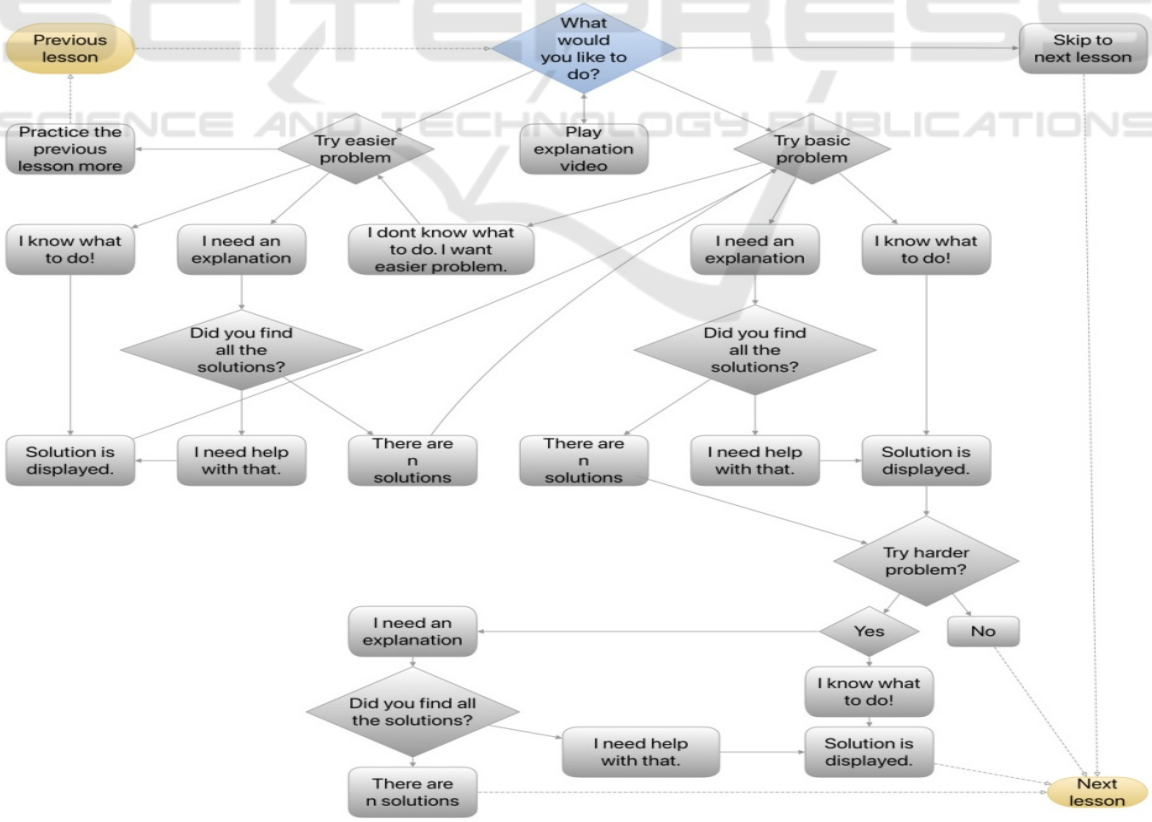

Figure 4: Options the user can choose from at each step in the field of construction tasks.

In the case of algebraic problems, the chosen procedure was implemented in such a way that the student was shown the correct answer together with the assignment. The implementation of the entire procedure can be found in Figure 1. In the case of construction tasks, the correct answer cannot be given immediately for the above reasons. This was solved by giving the student the choice between the options "I need an explanation" and "I know how to do it". This means that the second option is different from how it works in the previous areas as there is no visible solution or advice on how to proceed directly on the button.

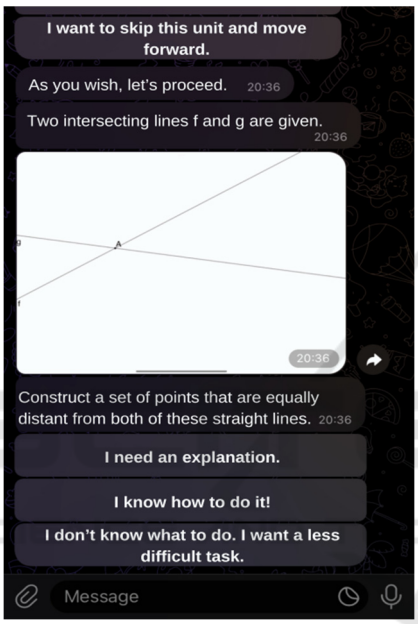

Figure 5: Chatbot environment for geometry.

Figure 4 shows how the chatbot works in the field of construction tasks. Having asked for an explanation, the student is offered a detailed verbal answer, an animation (gif) showing the step-by-step construction in a graphical software, a stationary image of the solution, or a link to a video explaining the phenomenon/construction/validity of a critical step. The student's task is then to determine whether the problem has more than one solution. Having clicked on the option "I know how to do it", the student is shown only a stationary image of one of the solutions. The student is then asked if they have found other solutions. They can choose from the options "Yes, the task has *n* solutions (congruent solutions are taken as 1)", where *n* is dynamic, depending on the task, and the option "I need advice on the number of solutions". Selecting the latter option means the student is shown a detailed solution as if they selected the option "I need an explanation" in the very beginning. Examples of construction problems are in Figures 5 and 6.

Figure 5 shows the assignment that the student gets having chosen the difficulty of the task they want to solve. This is described in words, sometimes it is supplemented with an illustrative picture. Figure 6 shows an explanation of another problem is displayed, where the student first sees the described animation and then a picture of the solution. A set of other solutions to the problem are also discussed here.

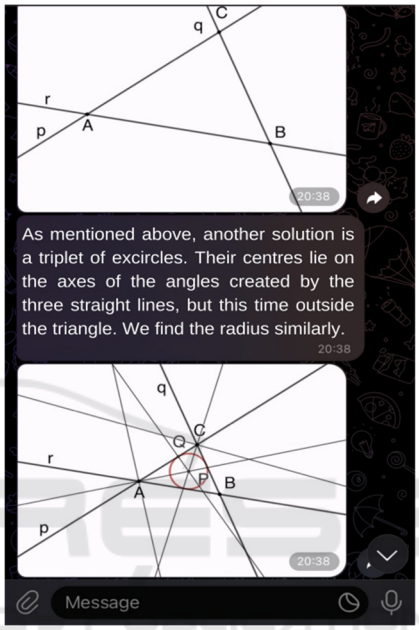

Figure 6: Offered solution of geometry task.

For each of the areas, at the end of the unit, the student is offered links to other tasks or units of a similar type created by teachers and uploaded to the Ema.cz server. The tasks that are recommended to students have been evaluated by the authors of the paper in terms of quality and only the most suitable ones have been selected. The course thus primarily helps the student identify which areas they still need to practice, it offers tasks and explanations but also other resources where the student can improve or practice their knowledge.

Before making the course available to students, we plan to add links to interactive applets created in GeoGebra (https://www.geogebra.org/, accesed on 15th March 2023) that will allow the student to construct in a graphical software environment with a limited palette of tools (circle, line, line segment, compass, intersection, ...) and the software will automatically evaluate the correctness of the solution, similar to the Euclidea application (https://www.euclidea.xyz/, accesed on 15th March 2023).

4 DISCUSSION & CONCLUSIONS

Our research confirms that it is necessary to proceed differently in different areas of mathematics. In the paper, this is documented in two different areas of school mathematics – algebraic problems and construction problems in geometry.

Petty (2002) states that anything that surprises, arouses curiosity or anticipation or provokes thought helps to motivate students. Pupils can be encouraged to take an active approach to learning, among other things, by presenting them with activities in which they will correct and check their work (either on their own or with each other), by making them study at least some topics on their own from books and by using inquiry-based approach and by allowing them to experiment actively.

As we have shown in the paper, the use of a chatbot meets these recommendations and can be considered a suitable tool for fostering understanding in problem-solving in mathematics. In further research, we plan to focus on the use of the chatbot in other areas of school mathematics, and on examining the relationship between teachers and pupils to it. Attention should also be paid to the introduction of AI tools in teacher education.

The tutoring system is about to be tested with pupils during the spring of 2023. Their feedback as well as data from their answers (the time it took for them to click the right answer) will be collected and analyzed to further improve the system.

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