Assessment of Digital and Mathematical Problem-Solving Competences Development

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Abstract: Problem-solving and digital competences assume an essential role in developing students' life-long learning competences. An effective tool to support problem-solving activities is an Advanced Computing Environment (ACE). An ACE is a system that allows to perform numerical and symbolic calculation, make graphical representations, and create mathematical simulations through interactive components. Moreover, it is able to support students in reasoning processes, in the formulation of exit strategies and in the generalization of the solution. The main goal of this paper is to study the development of problem-solving and digital competences of secondary school students solving problems with an ACE in a Digital Learning Environment (DLE). The research question is: "How can we evaluate the evolution of students' problem solving and digital competences during the online training?". To answer the research question, the resolutions of 158 grade 12 students to ten problems carried out during an online training were analyzed. The research methodology was divided into three phases: the analysis of a case study; the analysis of all student evaluations; the analysis of students' answers to a final questionnaire. The results show that solving contextualized problems with the ACE in a DLE enhanced the students' problem-solving and digital competences.

1 INTRODUCTION

Every individual needs to develop skills that can be used throughout their lives: to respond to the challenges of a world in which technologies influence society, teaching and education, to improve as a person and as a worker, and to be an active citizen. In the recommendations relating to the key competences for lifelong learning, the Council of the European Union includes the problem-solving competence and the digital competence (European Parliament and Council, 2018). According to the European Parliament and Council (2018): "Competences, such as problem solving, critical thinking, ability to cooperate, creativity, computational thinking, self-regulation are more essential than ever before in our quickly changing society. They are the tools to make what has been learned work in real time, in order to generate new ideas, new theories, new products, and new knowledge". The Digital competence involves the confident, critical and responsible use of, and engagement with, digital technologies for learning, at work, and for participation in society. These aspects are also mentioned in the Italian National Guidelines (MIUR, 2010), according to which students at the end of upper secondary school should be able to apply mathematical concepts to solve problems, also with the help of technologies. Therefore, proposing problem-solving activities with the use of digital technologies is a teaching methodology that responds to institutional objectives.

This research work has the main goal of evaluating the development of problem-solving and digital competences of secondary school students who carry out problem-solving activities with an Advanced Computing Environment (ACE), during an online training in a Digital Learning Environment (DLE). An ACE, with a special programming language, allows for performing numerical and symbolic computations, plotting two- and three-

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dimensional static or dynamic graphs and programming interactive components in order to generalize a resolution process. An ACE also allows students to approach a problematic situation in the way that best suits their thinking, to use different types of representations according to the chosen strategy and to display the whole reasoning together with verbal explanation in the same page (Barana et al., 2019). All of this makes it an effective tool to support problem solving and mathematics teaching and learning (Brancaccio et al., 2015; Barana et al., 2021). A DLE has been defined as an ecosystem in which teaching, learning, and the development of competence are fostered in classroom-based, online or blended settings. It is made up of a human component, a technological component, and the interrelations between the two (Barana & Marchisio, 2022). According to Suhonen (2005), a DLE is a "technical solution for supporting learning, teaching and studying activities".

The context of our research is the Digital Math Training (DMT) project funded by the Fondazione CRT within the Diderot Project and organized by the Delta Research Group of the University of Turin in Italy. The DMT project every year involves about 3000 upper secondary school Italian students. The main goal of the project is to allow students to develop digital and problem-solving competences by solving contextualized problems with an ACE and collaborating with each other remotely within an integrated DLE (available at the link: https://digitalmatetraining.i-learn.unito.it/) (Barana & Marchisio, 2016; Barana, Boetti & Marchisio, 2022).

This study is guided by the following research question: "How can we evaluate the evolution of students' problem-solving and digital competences during the online training?". To answer the research question, the course of the 12th grade students of the DMT edition of the 2021/2022 school year was analyzed. The submissions and all data relating to the assessments obtained by 158 students during the online training were collected and analyzed. The students' answers to the final questionnaire that they filled out at the end of the online training were also analyzed.

This paper is structured as follows. In the section "Theoretical framework" the methodology of problem solving and problem solving with an ACE are discussed, followed by a brief presentation of the DMT project. In the section "Methodology" the research methodology with which the analysis was carried out is presented. The main results obtained from the analyses are presented in the "Results" section. In the "Conclusions" section some reflections on the results obtained and possible further developments for the research are presented.

2 THEORETICAL FRAMEWORK

2.1 Problem Solving and Problem Solving with an ACE

One of the fundamental skills in Mathematics is the ability to solve problems in everyday situations, which includes the ability to understand the problem, devise a mathematical model, develop the solving process and interpret the obtained solution (Samo et al., 2017). The term "problem solving" refers to mathematical tasks which provide intellectual challenges that improve students' understanding and mathematical development (National Council of Teachers of Mathematics, 2000). Problem solving is a real challenge for students. It involves the use of multiple rules, notions and operations whose choice is a strategic and creative act of the students (D'Amore & Pinilla, 2006). Its value lies not only in being able to find the final solution but also in developing ideas, strategies, skills and attitudes. The focus then shifts from the final solution to the problem-solving process. Solving problems that are contextualized in everyday life activates modeling skills in students and teaches them to recognize how and when to use their knowledge, as well as getting them accustomed to solving problems in real world situations (Baroni & Bonotto, 2015; Samo et al., 2017). Challenging problems should be used, whose content topics have been studied in class or will soon be, with open data in order to offer students a vast range of possibilities to choose from and make decisions about, and that suggest more than one solving strategy (Barana et al., 2022). Through problem solving it is also possible to develop social and civic competences. For example, by solving problems in small groups, students learn to work together, to discuss, to support their own opinions and respect those of others, to discuss and present their ideas. Therefore, by learning problem solving in Mathematics, students acquire ways of thinking, creativity, curiosity, collaborative competences and confidence in unfamiliar situations (Barana et al., 2019).

The resolution of a problem by students can be used to assess progress in problem-solving competences, using an assessment rubric with a score scale (Leong & Janjaruporn, 2015). The score scale describes the reason why a performance was placed in a certain level. The next level guidance provides students with an idea of what should be achieved and what needs to be done to improve. The rubric is one of the best ways to assess problem-solving competence (Jonassen, 2014). It can be used to evaluate problems on different mathematical topics and the evaluations can be compared. Moreover, through rubric assessment, students are provided with relevant feedback on the problem-solving process, since they receive an evaluation on each indicator (Jonassen, 2014). Sharing rubrics with detailed descriptors of the levels is a relevant formative assessment strategy, since it helps students understand the quality criteria (Black & Wiliam, 2009). In fact, through rubric assessment they can understand their actual level, the reference level, and in which area they should work more to reach the goals: these are the three main processes of formative assessment identified by Black and Wiliam (2009) and by Hattie and Timperley (2007). Thus, feedback provided through rubrics can help them bridge the gap between current and desired performance in problem solving (Hattie & Timperley, 2007).

Problem solving is characterized by four fundamental phases described by Polya (1945) in "How to solve it": understanding the problem, devising a plan, carrying out the plan, looking back. The looking back phase consists of reviewing and reconsidering the results obtained and the process that led to them. This allows one to consolidate knowledge, better understand the solution and possibly use the result, or the method, for some other problem. Generalizing is an important process by which the specifics of a solution are examined and questions as to why it worked are investigated (Liljedahl et al., 2016). This process can be compared to the Polya looking back phase, and consists of a verification and elaboration stages of invention and creativity. This makes it possible to move from the single case to all possible cases, to extend and readapt the model developed and to consolidate what has been learned through problem solving (Malara, 2012).

Technologies play a fundamental role in problem solving and make it possible to amplify all phases of the process. An ACE allows to perform numerical and symbolic computations, make graphical representations (static and animated) in 2 and 3 dimensions, create mathematical simulations, write procedures in a simple language, programming, and finally elegantly connect all the different representation registers also with verbal language in a single worksheet (Barana et al., 2020). An important aspect of an ACE for problem solving is the design and programming of interactive components (such as sliders, buttons, checkboxes, text areas, tables and graphics). They enable to visualize how the results change when the input parameters are changed and thus they allow to generalize the solving process of a problem. The use of an ACE for problem solving profoundly affects the entire problem-solving practice and the nature of the problems that can be posed. For example, problems may require difficult pen-and-paper calculations, dynamic explorations, algorithmic solutions to approximate results, and much more. Without having to engage in calculations, students can focus on understanding, exploring and discussing the solving process and the obtained results. The possibility of combining different types of representation in the same worksheet influences the way students approach problems and their strategic choices, favoring high levels of clarity and understanding (Barana et al., 2022). In this way, the ACE is not only a tool, but it becomes an effective methodology that can support problem solving and the learning of Mathematics (Fissore et al., 2019).

Another technology that can enhance the problem-solving methodology is a Digital Learning Environment, i.e., an ecosystem in which teachers and students can share resources and carry out educational activities. In a DLE both the technological component and the human component are important, together with how the activities are designed for the interactions between students, teachers and peers. In a DLE, teachers can propose many different types of activities in a single shared environment; this aspect is essential in an online teaching context, but it can also integrate the teaching experience into ordinary teaching in classroom-based or in a blended mode. In a DLE students can create, share and compare their own works and always be in contact with each other, exchanging opinions and ideas (Barana & Marchisio, 2022).

2.2 The Digital Math Training Project

The DMT Project was funded by the Fondazione CRT within the Diderot Project and was organized by the University of Turin. The DMT was born in 2014 with the aim of developing and strengthening the mathematical, digital and problem-solving competences of secondary school students. The main part of the project consists of an online training in a DLE. The technological component of the DLE is a Moodle platform integrated with the Maple ACE (https://www.maplesoft.com/), developed by the Computer Science Department of the University of Turin. The activities of the DMT project are mainly based on the resolution, with the use of an ACE, of non-routine problems, contextualized in reality and open to different solving strategies. Students solve the problem individually, collaborating asynchronously online with other students. Students are also offered training activities and tools that enable self-learning and collaborative learning to understand how to use an ACE and solve problems. Students from grade 9 to grade 13 participate in the project. The students are divided by grade, then five online training sessions are designed and set up on the platform. During the online training, a problem is proposed to the students every ten days, for a total of 8 problems. The degree of difficulty of the problems gradually increases during the training. Increasing the difficulty of the problems allows students to prepare for a final competition and win a prize. All problems include several requests. The first requests guide students to understand, explore, identify a model and the solution strategy of the problematic situation. The last problem request requires a generalization of the solution through the creation of interactive components.

The problems' solutions worked out by the students are assessed by tutors according to a rubric designed to evaluate the competences in problem-solving while using an ACE. The rubric is an adaptation of the one proposed by the Italian Ministry of Education to assess the national written exam in Mathematics at the end of Scientific Lyceum, developed by experts in pedagogy and assessment. The rubric has 5 indicators, each of which can be graded with a level from 1 to 4. The first four indicators have been drawn from Polya's model and refer to the four phases of problem solving; they are the same included in the ministerial rubric. The project's adaptation mainly involves the fifth indicator, and entails the use of the ACE, which we chose to separate from the other indicators in order to have and be able to provide students with precise information about how the ACE was used to solve the problem. Since the objective of the project is developing problem solving with technologies, it has been considered appropriate to evaluate the improvements also in the use of the ACE in relation to the problem to solve (Barana et al., 2022). The five indicators are the following:

- Comprehension: Analyze the problematic situation, represent, and interpret the data and then turn them into mathematical language (score between 0 and 18);
- Identification of a solving strategy: Employ solving strategies by modeling the problem and by using the most suitable strategy (score between 0 and 21);
- Development of the solving process: Solve the problematic situation consistently, completely, and correctly by applying mathematical rules and by performing the necessary calculations (score between 0 and 21);
- Argumentation: Explain and comment on the chosen strategy, the key steps of the building process and the consistency of the results (score between 0 and 15);
- Use of an ACE: Use the ACE commands appropriately and effectively in order to solve the problem (score between 0 and 25).

A total score (maximum of 100) is given to each resolution. Finally, each evaluation is integrated with personalized feedback from the tutors, relating to the evaluation obtained and containing advice on how and what to improve. At the end of the training, all participants are asked to fill out a satisfaction questionnaire.

3 METHODOLOGY

The research question is: "How can we evaluate the evolution of students' problem-solving and digital competences during the online training?". To answer the research question, the course of the 12th grade students of the DMT edition of the 2021/2022 school year was analyzed. The submissions and all data relating to the assessments obtained by 158 students during the online training were collected and analysed. The analysis was divided into three phases: the analysis of how digital and problem-solving competences vary in an exemplary case study; the analysis of all student evaluations from the beginning to the end of the online training; the analysis of the students' answers to the final questionnaire. The average number of submissions was 92 in the first half of the training (first four problems) and 49 in the second half (last four problems). The data collected were organized in a table containing, for each student and for each of the 8 problems, the evaluations relating to the five indicators of the assessment rubric and the total score. The table was important both for the analysis of the case study and for the analysis of student assessments (the trend of total assessments and the trend of assessments of individual indicators).

A significant and exemplary case study was selected which showed an overall improvement in the scores of the five indicators and whose competences showed significant changes during the training. For the analysis of the case study, all the solutions to the problems made by the student were analysed, and some explanatory examples were reported. The assessment rubric was used to analyze the submissions, paying particular attention to the level descriptors of each indicator. A correspondence was sought between the assessments given by the tutors and the competences achieved by the student, to analyze in detail how they changed over time. Furthermore, the educational value of personalized feedback from tutors was examined. All of these investigations made it possible to make a global assessment of the progress of problem-solving and digital competences in the selected case study.

A second level of analysis concerned all the evaluations of the students in the sample, in order to obtain a global vision and a more complete study of the evolution of the students' competences. In order to be able to effectively evaluate any improvement between an initial and a final phase of the training, it was necessary to examine the students who had actively participated. For this reason, the analysis sample was restricted to students who had solved at least five problems (66 students in total), regardless of what they were. After that, it was necessary to identify a submission that represented the initial level of competences and a final submission that represented the level of competences achieved by participating in the training. As the initial submission, the one relating to the second problem was chosen, because the first problem had fewer requests as it did not ask for the generalization of the solution. This further narrowed the sample to students who had solved the second problem (61 students in total). The last assignment completed by each student was chosen as the final submission. For this part of the analysis, we will call the second problem the "initial problem" and the last submission problem the "final problem". We will call the total scores relating to the initial problem "initial assessments" and those relating to the final problem "final assessments". The Wilcoxon signed rank test for paired samples was then carried out with RStudio, to compare the initial and final evaluations and measure any increases or decreases. The Wilcoxon test was chosen because the data did not represent a normal distribution. The test made it possible to verify whether the difference between the median of the initial evaluations and that of the final evaluations was zero. Finally, to confirm the results obtained from the test, the box-plots relating to the initial evaluations and final evaluations were created, which made it possible to deepen the study.

To continue and further the analysis, we moved on to study the development of the competences of the 158 students of the initial sample during the entire training. To carry out this analysis, the trend of the arithmetic averages of the evaluations in the individual indicators and in the total was studied. Since the sample of the population on which the averages were carried out varied over time and since the outliers had a great influence on the arithmetic mean, the box-plots relating to the evaluations were created and analyzed, comparing the results of the two analyses.

The students' answers to the final questionnaire were analyzed to also consider the students' point of view and to draw the final conclusions. 69 students replied to the questionnaire. The questionnaire contained questions relating to various aspects of the project, such as: the degree of appreciation of the use of ACE for problem solving; the usefulness of having learned to use it; the difficulties encountered in solving problems; the usefulness of developing digital and problem-solving competences in the world of work; a self-assessment of mathematical, problem-solving and digital competences at the end of the training. The typology of the questions is mainly Likert scale questions, where students can select an answer from 1 = "not at all" to 5 = "very much". All analyses were performed using Excel software and RStudio statistical software.

4 RESULTS

4.1 Analysis of the Case Study

Through the analysis of the case study, it was possible to observe a general improvement in problem-solving and digital competences in the case of a student who obtained a low score (below 50) in the first problem and a high score in the last. The scores of the case study (see Figure 1) start with a low initial evaluation of 33/100, in the first half of the training it remains approximately constant while, subsequently, there is a significant improvement which sees an evaluation of 93/100 in its last submission. The increase in the degree of difficulty of the problems made it possible to consider these results as particularly significant and to select the student as a case study.

From the graph of the evaluation trends relating to the individual indicators (see Figure 2) it is possible to observe that all the indicators show a significant improvement. They reflect the trend of the total

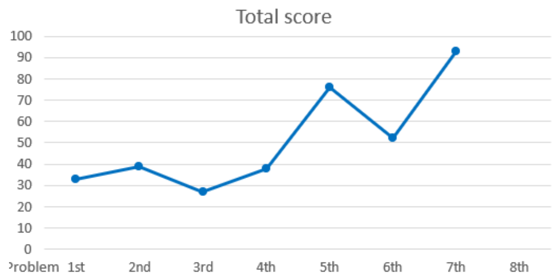

Figure 1: Trend of the overall assessments of the case study.

scores: starting from low evaluations, they show a notable improvement in the second half of the training, with the exception of the "use of Maple" indicator which shows a progressive even if not linear improvement during the entire training. As well as the overall evaluations, also the evaluations of the single indicators do not show a continuous and linear improvement. Many factors influence this aspect: the non-compulsory nature of the extracurricular project, the progressive increase in the complexity of the problems, school and personal commitments, the mathematical knowledge possessed by the student.

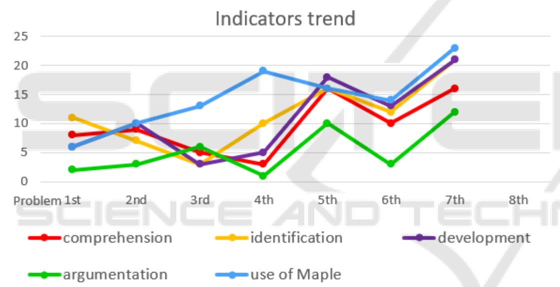

Figure 2: Trend of the evaluations of the case study divided by indicators.

The graph in Figure 2 shows an initial lowering of the scores in the indicators "understanding the problematic situation", "identifying a solution strategy" and "developing the resolution process". This may be due to the fact that the student is initially not completely accustomed solving contextualized problems and finds it difficult to solve problems with increasing difficulty. At the same time, the improvement of the indicators in the last submissions, when the problems have a higher degree of complexity, is particularly relevant. In the submission of the first problem, the case study does not fully develop the solution process. The student carries out some calculations without arguing the steps taken and the strategies chosen and provides a short final answer to one of the questions in the problem. The student uses the worksheet as a simple writing sheet and is not familiar with Maple commands yet. In the submission of the second problem the student still does not develop and does not fully discuss the proposed resolution. In this case, however, the student tries to use the ACE to create an array of point coordinates and to open packages with more advanced commands (see Figure 3). Feedback from tutors has been effective for student improvement. The feedback for the resolution of the first problem was: “The solution is only partially correct. The use of Maple and the argument are poor but don't give up, for the next problems it will be better! I advise you to comment more on both the results found and the individual steps". In the resolution of the second problem, the student begins to comment on the chosen strategies, such as: "I include "plots" to be able to use the "pointplot" command"; "Imagining that we have an exponential curve, the value that we will have on the tenth day will be around 1600 new cases"; "If you draw a line between the three points, the new cases on the 19th day will be around 2500." From the student's resolution and these last comments, it is possible to notice how the student is still unable to identify a solution strategy to model the problem and to develop the resolution. The student shows that they confuses exponential trend and linear trend and demonstrates that he does not know how to make the best use of the ACE. In fact, the student does not obtain a mathematical expression that models the problematic situation, and is not able to use the commands to show a graph that adequately describes it and for this reason he/she tries to "imagine" it.

In the submission of the third problem there is an improvement in the use of Maple and in identifying and implementing solution strategies for modeling the problem. The student is still unable to identify the correct strategies for modeling the problematic situation, however he demonstrates originality and creativity in developing the entire resolution through an interactive component (see Figure 4). The interactive component consists of an interactive table, a slider and a text area. As the values of the slider vary, the values of the last column of the table and the result of the problem in the text area change. The programming code to create the interactive component (top right in Figure 4), shows the correct use of the commands to take input data (for example: parameter:=Do(%Slider0)) and return output values (for example: Do(%TextArea0=parameter)). The code also includes a nested loop. This represents an improvement in programming proficiency. Despite the originality of the problem-solving idea, it is not an effective strategy due to a poor understanding of the problem situation and the lack of identification of a correct modeling.

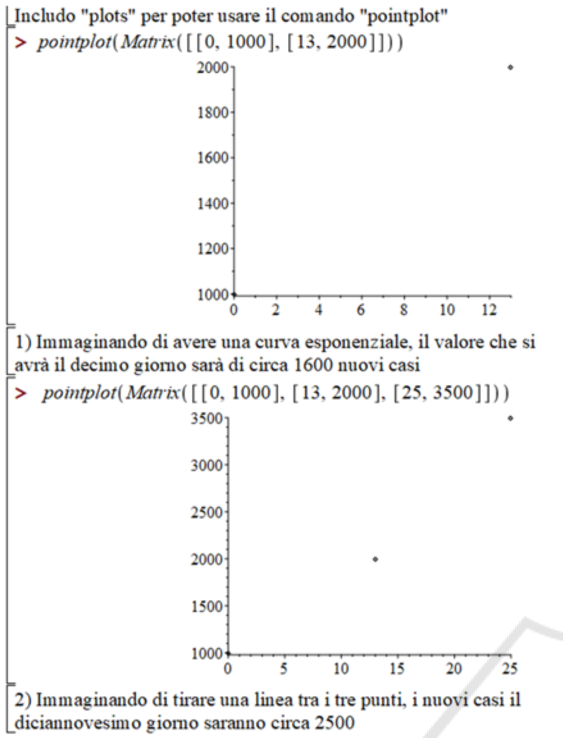

Figure 3: Solution of the second problem submitted by the case study.

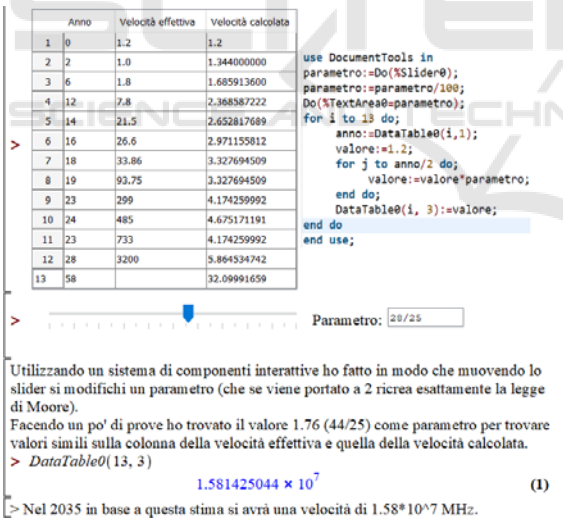

Figure 4: Resolution of the third problem submitted by the case study.

During the training, the student's digital competences gradually improve and it is possible to observe an improvement also in the indicators "understanding the problematic situation", "identifying a solution strategy" and "developing the resolution process" starting from the fifth problem. In the last submission, despite a small error of understanding, the student fully develops the resolution and implements effective strategies, through modeling consistent with the interpretation of the problem. The proposed problem is contextualized in the advertising field, and speaks of a disco that advertises its sound system. Given the formula that describes the sound intensity expressed in decibels (dB) as a function of the sound intensity expressed in W/m2, the question was:

- to calculate the total sound intensity in dB of four 100 dB loudspeakers, the one in dB of each single loudspeaker knowing the total loudness of 400 dB and to state whether the declaration of the disco regarding its sound system is capable of diffusing music at 400 dB having four speakers of 100 dB each, correct or tendentious;
- to construct an example that would show how, given several speakers to which a different sound intensity is associated, the total sound intensity in dB could be approximated with that of the speaker to which the greater intensity is associated;
- to create a system of interactive components which, given two loudspeakers to which two sound intensities in dB are associated, would return the total sound intensity in dB and at least two graphs;

The student incorrectly understands the intensity formula provided by the text, replacing the sound intensity expressed in W/m2 with that expressed in dB. Despite this, the development of the resolution of the problem is interesting. For example, to find the intensity of each single loudspeaker, the student constructs a while loop which increases the total intensity at each cycle, checking that the decibels obtained with it do not exceed the maximum threshold indicated by the problem. When the latter is exceeded, the cycle returns the total intensity that caused the maximum value to be exceeded. The total intensity is then divided by 4 to obtain the intensity of each individual speaker. This strategy allows to solve the problem in a clear, schematic and effective way by exploiting a piece of code that automatically controls the steps to be carried out in order to obtain the desired result under the conditions required by the problem. For this reason, the student obtained the highest marks for the indicators "identifying a solution strategy" and "developing the solution process" but not in "understanding the problematic situation". The indicators in fact, as components of problem solving, are closely related but, at the same time, each of them has its own "identity" which characterizes and distinguishes it from the others.

During the training, the student's argumentative ability also improves: in the last submission the student discusses the steps taken and the strategies chosen, leading the reader to follow the reasoning made. The results show that solving contextualized problems with the ACE enhanced the student's problem-solving and digital competences.

4.2 Analysis of the Assessments of all Students

For the analysis of the evaluations of all the students, the Wilcoxon test was carried out to evaluate any improvement between an initial phase and a final phase of the training. The p-value of 0.89>0.05 did not allow us to reject the null hypothesis according to which the medians of the initial evaluations and of the final evaluations were equal. This result is satisfactory for the purpose of this research. In fact, since the final evaluations relate to problems of greater difficulty, the equality or a non-significant difference in the evaluations shows that the students have developed competences to solve problems of greater difficulty, suggesting an improvement in these competences. This result was confirmed by the box-plots relating to the initial and final evaluations (see Figure 5).

Indeed, they show that the medians are the same, with a value of 84/100, indicating a high starting level (above 70) which becomes more significant when related to the final submission, reflecting more developed problem-solving and digital competences. In the initial problem, the median is very close to the third quartile indicating a high number of evaluations between 84 and 89, while in the final problem 50% of the evaluations are distributed symmetrically with respect to the median with evaluations between 74 and 95. These results satisfy expectations: 25% of the evaluations with a value greater than 84, which initially was between 84 and 89, in the final problem are distributed between 84 and 95, indicating that a greater number of students took evaluations greater than 89. At the same time, the first quartile passes from corresponding to an evaluation of 72 to an evaluation of 74, indicating that a greater number of students have obtained an evaluation higher than 74. The greater dispersion found in the final problem compared to the initial problem can be justified by the increase in the difficulty of the problems, which therefore led to a greater variability of the evaluations. At the same time, however, the dispersion to the right of the median and the increase in the value of the first quartile show a general improvement in students' competences.

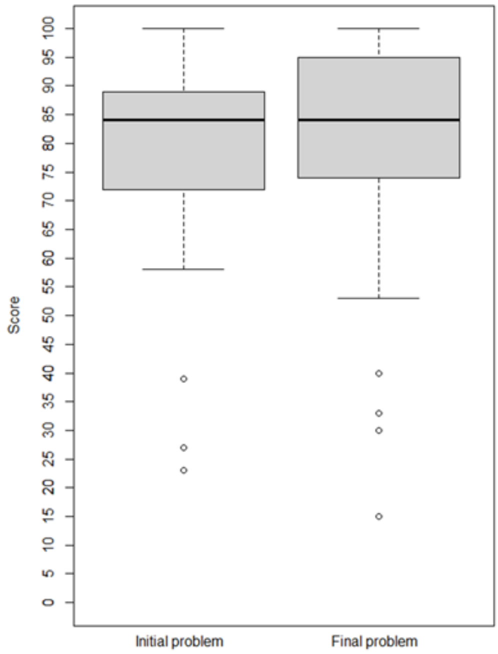

Figure 5: Box-plot of the overall evaluations related to the initial and final problem.

By studying the trend of the average evaluations during the entire training, it was possible to expand the analysis and obtain a more complete vision of the development of the students' problem-solving and digital competences. The investigation of average ratings showed an overall improvement in problem-solving and digital competences. In fact, from the graphs of the trend of the average evaluations of the individual indicators and of the total ones (see Figure 6), a slight improvement can be observed for all the indicators, even though not continuous or linear, with a general decrease in the sixth problem. In particular, the "argumentation" indicator shows a more evident improvement, with a more regular trend and a progressive improvement. This aspect indicates that, although it was complex to understand, identify and develop a solution strategy, the students were able to explain and justify their resolution in an even more precise, complete and pertinent way. In problem six there is a drop in scores on all indicators. The students' difficulties can also be seen in the discussions in the forum on the platform: "Hi, I didn't quite understand the third request"; "I wanted to ask what was the exp function in the formula that I didn't understand"; "Hi, regarding point 2 of the problem, how did you do it (in a very general way)? Did you use a more algebraic or graphical approach? Because graphically it seems to me very complex to visualize, while algebraically I find it more it difficult to find the right commands". A slight worsening of all indicators in the second problem can also be observed. This aspect is not surprising since the first problem, being at the beginning of the training, had a lower difficulty as it did not require the generalization phase of the resolution, thus leading to generally higher scores.

Since the arithmetic mean is influenced by outliers, the latter may not effectively represent the assessment of students' results during training. We therefore decided to also analyze the median of the total scores obtained during the training.

From the box-plots of the evaluations relating to the total score (see Figure 7) it can be observed, for each problem, a distribution of half of the evaluations approximately symmetrical with respect to the median and a generally reduced width of the interquartile ranges, indicating a concentration of the evaluations around the median. This indicates that the median gives a good representation of the evaluation obtained by the students.

In the sixth problem, the interquartile range is instead wider, indicating a wider distribution of evaluations. This implies that the median, in this case, is less representative of the evaluations obtained by all the students for that problem. This may be due to the considerations made previously on the difficulty encountered by the students in solving that problem.

In the seventh problem, however, the median corresponds to an evaluation of 95/100 and is close to the third quartile, indicating a large number of submissions with a very high evaluation (above 95). This justifies the peak that is also found in the graph of average ratings. The trend of the medians is very similar to that of the average evaluations of the various indicators and of the total. For this reason, it was possible to confirm what emerged from our the analysis of the average ratings.

4.3 Analysis of Students' Answers to the Final Questionnaire

The last part of the analysis concerned the students' answers to the satisfaction questionnaire, to understand their point of view on some aspects of online training and on the development of their competences. The first question examined was: "In solving problems, which of the following aspects gave you difficulty?". The answers (see Table 1) show that the students found approximately the same degree of difficulty in developing all the competences related to the five indicators of the evaluation grid. The average values of the answers are all between 3.13 and 3.20. Only the "argumentation" indicator has an average response of 2.70, so students had less

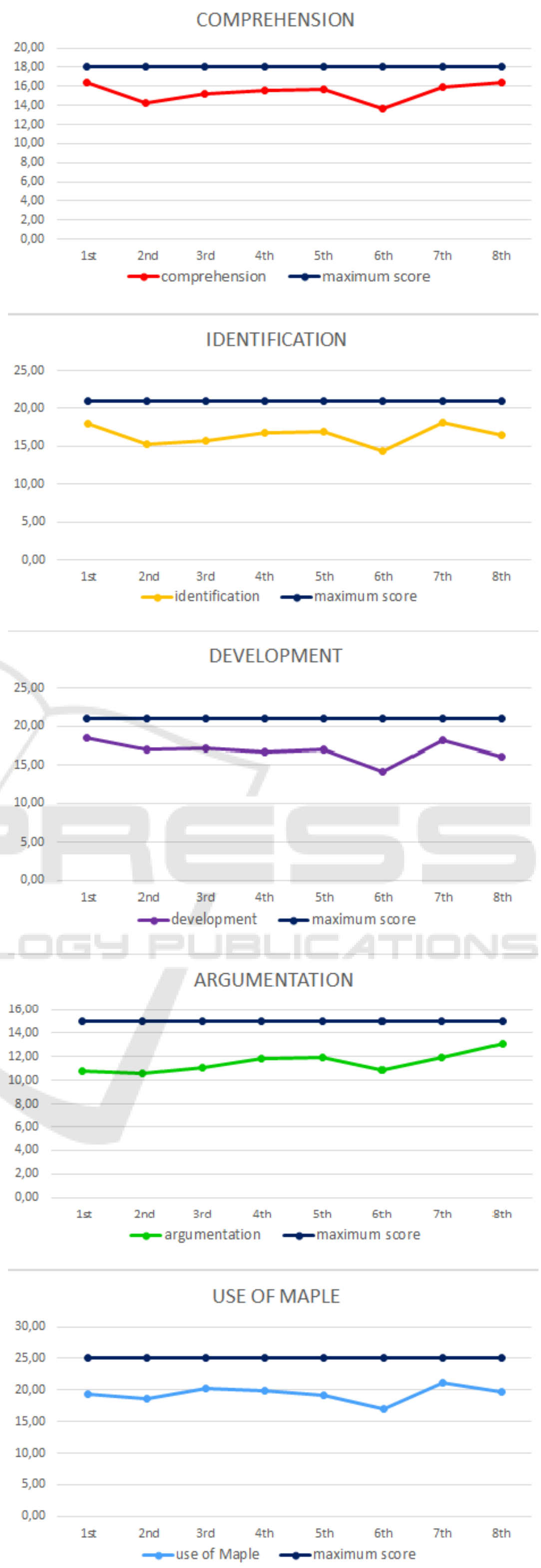

Figure 6: Graphs of the trends of the average evaluations of the single indicators.

difficulty developing this skill. This is also confirmed by the constant increase in ratings for this indicator.

These results reflect what was observed in the analysis of the assessments of all students. In particular, students experienced a slightly greater difficulty in generalizing the problem, indicated by an average of 3.48 out of 5.

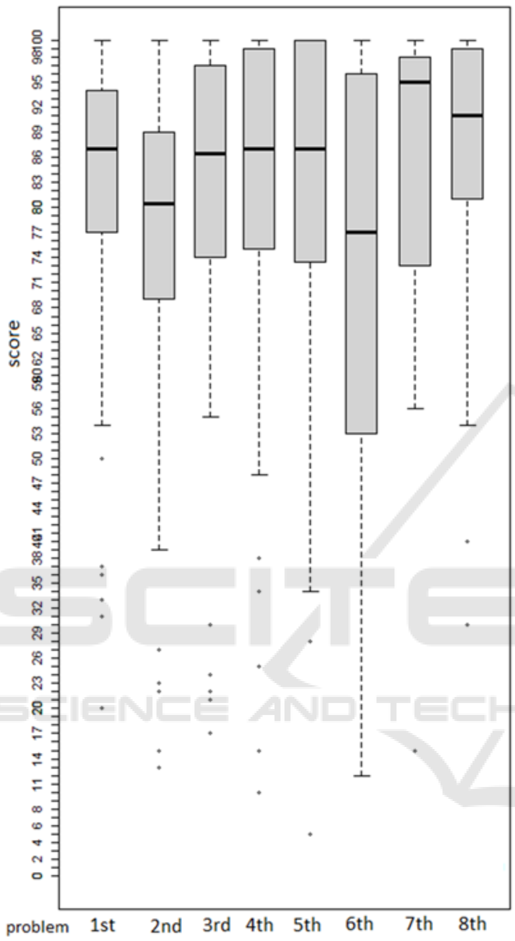

Figure 7: Boxplot of the overall evaluations of all the students.

Table 1: Students' answers to the question about the difficulties encountered in solving problems.

	Mean	St.Dev.
Interpret the text	3.20	1.1
Identify a solution strategy	3.13	0.87
Complete the resolution process	3.16	1.04
Discuss the solution	2.70	1.06
Generalize the problem	3.48	1.02
Use Maple	3.17	0.79

The second question examined was: "Please indicate to what extent you think you have acquired the following competences in online training". The responses (see Table 2) indicate that, from the students' point of view, participating in an online training in a DLE and using an ACE for problem solving fostered the development of their math, digital and problem solving (with an average of 3.12, 3.46, 3.52 respectively). The third question examined was: "Please indicate to what extent you think these competences will be useful in the world of work". Table 3 shows the results. It is interesting to observe how students find mathematical competences (with an average of 3.60) and problem-solving and digital competences (with an average of 4.19 and 4.25 respectively) useful in the world of work, revealing the strong awareness of importance of these competences for their future, even outside the school context.

Table 2: Students' answers to the question on the development of their competences.

Acquired competences	Mean	St.Dev.
Mathematical competences	3.12	0.72
Digital competences	3.46	0.70
Problem-solving competences	3.52	0.85

Table 3: Students' answers to the question on the usefulness of the competences in the world of work.

Utility in the world of work	Mean	St.Dev.
Mathematical competences	3.60	0.89
Digital competences	4.25	0.72
Problem-solving competences	4.19	0.81

The last question analyzed concerned the school average of the students in mathematics (expressed by the students in a grade from 1 to 10) at the beginning and at the end of the online training. For 30% of the students the average improved, for 67% of the students the average remained unchanged and for 3% of the students the average worsened. In particular, the average decreased only in two students who went respectively from 10 to 9.5 and from 8 to 6. Furthermore, 55% of the students whose average remained unchanged had a high starting average (above 8) . For this reason, the results obtained are satisfactory and show a general improvement in the mathematical competence of the students participating in the online training.

5 CONCLUSIONS

This research work had the main objective of evaluating the development of problem-solving and digital competences of secondary school students who carry out problem-solving activities with an

ACE during an online training in a DLE. To answer the research question, the online training of grade 12 students from the 2021/2022 school year edition of the DMT project was examined. The analysis was developed following three phases: the analysis of an exemplary case study; the analysis of all student evaluations; the analysis of the students' answers to the final questionnaire submitted at the end of the training. The results show that the problem-solving activities with an ACE carried out during the online training allowed the development of all problem-solving competences (in particular argumentation) and digital competences. In fact, the use of an ACE in problem solving has made it possible to support all phases of problem solving, allowing to focus on the resolution process, on exploration and on the results obtained, and to exploit different types of representation in the same environment. Furthermore, the ACE, with the creation of interactive components, has favored the process of generalization of the problem, an important phase of problem solving which, from what emerged from the questionnaire, is considered difficult to tackle by students. In the generalization phase students have to design and program the interactive components in such a way that they take data as input, process a result and return an output of the results of the problem. In this way, it is possible to generalize the initial situation and see how the solution of the problem changes as the initial data vary. This is not easy but it allows them to develop abstraction and programming competences using a specific language. The growing difficulty of the problems has also helped to foster the development of problem-solving and digital competences, stimulating the commitment, participation and training of the students, who in this way have developed and consolidated their competences.

The analysis of the case study submissions showed that the evaluation system had a positive impact on the development of students' competences. The personalized feedback from the tutors and the comparison of the evaluations obtained with the shared assessment rubric have allowed the students to establish their own level of competence and to understand what and how to improve, which are the three important processes of formative assessment (Black & Wiliam, 2009).

Since the development of problem-solving and digital competences, key competences for lifelong learning and problem-solving activities with an ACE are also part of the institutional objectives, it is desirable to promote these activities within the school context, entrusting the competences of problem-solving and digital skills a central role in teaching.

A limitation of this study is the variation in the number of students who turned in problem resolutions over the course of training. Future research could propose problem-solving activities with an ACE during lessons at school, in order to carry out the analysis on a sample of students that does not vary over time. It would be interesting to compare the development of problem-solving and digital competences using a control sample of the same education level, made up of students who do not participate in the activities. In this way it would be possible to further evaluate the effectiveness of problem-solving activities with an ACE for the development of these competences. However, this is not easy because some problem requests would be difficult to implement without the use of technologies. This type of project shows how technology can be used naturally in ordinary teaching. It allows the teacher to rethink the teaching methods, and at the same allows the student to develop mathematical, digital and problem-solving competences.

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