The Concept of Derivatives Through Eye-Tracker Analysis

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Abstract: In this paper we present a qualitative study on data collected by an eye-tracker tool regarding a Calculus task. One purpose of this research is to highlight the differences and similarities between visual observation of expert and non-expert groups. Analysis of the way of reading a text can provide a lot of information about cognitive processes carried out to solve the task. Moreover, the aim of this study is to analyse, through the eye-tracker tool, the difficulties of students concerning the concept of derivatives and to understand what may trigger a wrong answer to the task.

1 INTRODUCTION

In recent years, there has been an increase in the use of eye-tracking rools for research in the field of Mathematics education. This technique, used also in other fields such as Psychology, Neuroscience or Linguistics (Ferrari, 2004) as well as about cognitive process creativity (Schindler & Lilienthal, 2020), provides information about the way a person looks at a visual stimulus. Thanks to the eye-tracker tool, it is possible to study eye movements while an individual is observing a stimulus. In particular, it is interesting to analyse the eye movements of a person while performing a mathematical task. The way to read a text offers a lot of information about the problem-solving process. Some studies are done in mathematics in high school (Spagnolo et al., 2021) with the eye-tracker tool, while little research is carried out at university level.

In this research study, we focused attention on the problem-solving process of a calculus task involving the concept of derivatives. The task chosen was part of the international survey TIMSS Advanced of 2008. The choice of this task was based on the results of standardised assessment tests (Gambini et al, 2020). In fact, the results show that students have difficulties with the concept of derivatives and the concept of slope of a function. They have difficulties understanding these concepts and explaining the meanings of their cognitive process in a mathematical task (Ferrari, 2017). Therefore, one of the purposes of this research is to understand, using the eye-tracker instrument, students' difficulties regarding these mathematical objects (Almfjord & Hallberg, 2020). In addition, the aim of this analysis is to highlight the difference between the gaze of experts in mathematics (high school teachers, PhD students, academics) and that of non-experts (Andrà et al., 2009; Inglis & Alcock, 2012). The non-expert group is composed of students of scientific faculties who attended a calculus course in the first academic year. Moreover, we wish to observe what has changed since taking the calculus course. Therefore, in this study we wish to make a comparison between standardised assessment results and the responses of the candidates of our sample. Thanks to data collected by the eye-tracker tool and answers to interviews, it is possible to analyse the process carried out by individuals when solving the task. In this way, we can study which elements led candidates to a solution and understand how they did so.

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2 THEORETICAL FRAMEWORK

Eye-tracking allows us to track what a person observes while performing a task. Recently, researchers have used an eye-tracker tool to analyse cognitive processes (Schindler & Lilienthal, 2019). In particular, this tool has been used in research into areas of Mathematics such as Geometry (Schindler & Lilienthal, 2017, Simon et al., 2021), Algebra (Obersteiner & Tumpek, 2016) and interpretation of motion graphs (Ferrara F. & Nemirovsky R., 2005). Moreover, some research studies have been carried out on use of the eye-tracker in high school (Spagnolo et al., 2021). The hypothesis, known as the Eye-Mind Hypothesis, claims that eye movements are linked to cognitive and learning processes. What is observed by the subject offers important information about what is processed by him or her (Just & Carpenter, 1980). Thanks to data collected by eye-tracker instruments, it is possible to analyse the cognitive processes carried out by individuals.

In a mathematical text there are elements belonging to different registers of representation (Duval, 2006). Therefore, when people read a mathematical text, they have to be able to switch from one register of semiotic representation to another (Giberti et al., 2023). Some studies show that the ability to solve a task is related to the ability to read different semiotic representations. Thanks to eye movement analysis, it is possible to study the ability to switch between different representations of mathematical objects (Andrà et al., 2009; Andrà et al., 2015). Moreover, thanks to the eye-tracker tool, it is possible to identify which part of the text attracts more fixations. It enables scholars to study which objects catch students' attention and to obtain more information about their learning process. In recent research by Andrà et al. (2009), a comparative analysis was carried out concerning the approach of experts and non-experts to mathematical representations. Through the eye-tracker tool, they set out to study the pattern of eye movements of the two groups. Following on from this study of experts and non-experts, we wish to carry out a qualitative survey on an analytical concept that seems to present significant difficulties for students. They, in fact, had many problems with the concept of derivatives and slope of a function in a 2008 TIMSS Advanced survey and in an INVALSI task. INVALSI is the institution that provides periodic and systematic testing of Italian students' knowledge and skills; in particular, it manages the National Assessment System (SNV).

The annual tests involve all Italian students of grades 2, 5, 8, 10 and 13. In recent research, the authors have analysed why students encounter difficulties in some tasks. They argue that in cases where students have to apply only one procedure, they are able to give the correct answer more easily. When they have to interpret the meaning of the concept, they are in great difficulty (Gambini et al. 2020). To improve understanding of a concept, Tall (Tall, 2003) suggests working on its meaning in the graph. In particular, Tall talks about three mathematical worlds in which the mathematical concept takes shape: the embodied world, symbolic world and axiomatic world. In the embodied world, in fact, an individual learns through perception. In this case, it is useful to work on the graph to show the meaning of the derivative, and then to delve into symbolic and axiomatic meaning. The INVALSI' results, in fact, show that in tasks where the concept of the derivative was linked to the concept of velocity, students were able to answer more correctly than in the task presented in this paper (Gambini et al., 2020).

The aim of this research study is to investigate the problem-solving process as performed by experts and non-experts, and to analyse their eye movements. Thanks to the responses to the interviews, it was possible to understand candidate awareness about what they looked at while performing the task. In addition, it is possible to examine which elements caught the attention of the candidates and what changed after attending the Analysis I course.

The research questions are:

- Is there a difference between which elements caught the attention of experts and non-experts?
- How do these elements influence the problem-solving process of the candidates?
- Is there a difference between the results of standardised assessment and the solution proposed by the non-expert sample?

We predict that the observational approach of the experts and non-experts is different. We think the experts' viewpoint focused most on the angular coefficient of the line equation. Instead, we believe that non-experts looked mostly at the area of the graph where lines meet the curve and the ordinate of the tangency point.

3 METHODOLOGY

This paper presents a qualitative analysis based on data obtained using the eye-tracker instrument. The subjects of this research were students of scientific courses, for example Chemistry, Engineering, Physics or first year of Mathematics, and Mathematics graduates, PhD students or high school teachers of Mathematics. The group composed of scientific faculty students who took Calculus in the first year is called the "non-experts" group, while the term "experts" is used to refer to the group composed of PhD students, high school teachers or Mathematics graduates. The two types of candidates were compared in order to investigate tracked cognitive processes and highlight similarities and differences between the two groups.

In this study a screen-based eye-tracker instrument was used, which can collect gaze data at 60 Hz. This tool is designed for fixation-based analysis, and it consisted of a binocular camera with a precision of 0.10° RMS and an accuracy of 0.3° under optimal conditions. These values of precision and accuracy were necessary to obtain the heat map as in the following figures (for example, see Figure 3).The method is based on a collection of images of both eyes by camera; in this way, it is possible to have a better position of the gaze in the space and the diameter of the pupil. The eye-tracker tool was linked to a computer to analyse data collected with software. This software provides tools of analysis like creation of heatmaps, gaze plots and video recording of eye movements. In this way it was possible to perform a comparative analysis between the data of candidates. The heatmap is a graph in which the most interesting areas are represented with a warm colour. These areas were observed for many times or for a long time; therefore, these areas captured most attention from the candidates. The gaze plot provides information about the trajectory of the eye movements on the screen. Fixation durations are used to represent time spent watching the visual stimulus. The eye-tracker instrument was calibrated for each subject. In fact, before being shown the stimulus, the candidate had to follow with his/her eyes the cursor to calibrate the tool; the test started after this phase.

Candidates were given a Calculus task on the concept of derivatives with no time limit to solve it. Candidates read the text of the task on a monitor where an eye-tracker camera was placed. The eyes of the candidates in this research, while performing the task, were monitored by the eye-tracker instrument. Therefore, candidates knew that they had to keep their eyes on the screen throughout the test. The eye-tracker detected and recorded eye movements while the subjects were performing the task. After candidates had solved the task, a blank screen was shown to them, so that the recording of eye movement data was stopped, while keeping the candidates' eyes on the screen at all times. Afterward, candidates were subjected to an interview to understand the problem-solving process chosen. In addition, during the interview of the subjects, the task was shown to them again to detect and record their eye movements during this phase. Data collected by an eye-tracker is useful to understand the cognitive processes of candidates and motivation of the problem-solving strategy chosen. During the interview, the following questions were asked:

- What did you look at the most - the graph or the text of the task?
- Which elements in the text most caught your attention?
- Which elements in the graph most caught your attention?
- Which element did you start from when looking for the solution?
- What element enabled you to find the solution?
- Did you first read the text of the task and then look at the graph, or vice versa? Why did you do this?

Candidates were vocally recorded for later analysis of their answers to the interview questions. In this way, the data collected by the eye-tracker were reconnected to subjects' answers, making it possible to analyse the cognitive process triggered by candidates.

4 ANALYSIS OF RESPONSES TO THE TASK

This task was included in the TIMSS Advanced survey of 2008. Moreover, a similar version of it was used in the pre-test of grade 13 in the INVALSI survey.

In this experimentation, thanks to data collected by the eye-tracker tool, it is possible to carry out a qualitative analysis organised in levels.

The text of the task is as follows:

"The line of equation $y = \frac{3}{2}x - 2$ is tangent at point P with abscissa equal to 2 to the graph f in the image. What is the value of f ' (2)?"

In the first macro-level of analysis, it is possible to divide candidates into three categories:

- in the first category, there are candidates who prefer to focus their attention on the text of the question. They almost completely ignored the graph of the task. We call this category "type T";

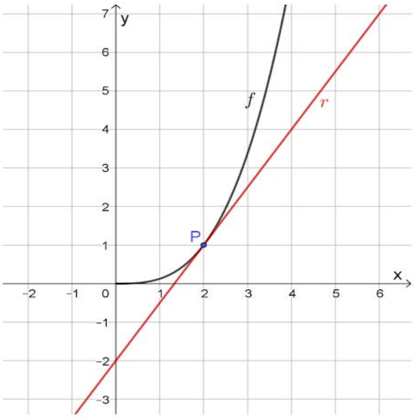

Figure 1: Graph of the task.

- in the second category, there are candidates who read the text quickly and after spending more time on the graph, try to solve the task through the graph information. We call this category “type G”;
- in the third category, there are candidates who favour neither the graph or the text. Eye movements of individuals move between text and graph with quick saccades. We call this category “type TG”.

In the micro-level of analysis of these categories, we tried to highlight the distribution of expert and non-expert students. It is possible to observe that the expert candidates belong to the first category (we call these candidates T-E). In contrast, non-expert candidates are subdivided into category type T (T-NE), category type TG (TG-NE) and there is one individual who belongs to the category type G (G-NE).

Apart from the clear division in these categories between expert and non-expert candidates, it is important to point out that the task is an open-ended question, and, moreover, the answer is in the stimulus. Analysis of the graph is not crucial to find the correct solution to the problem. In fact, to solve the task correctly, it is necessary to connect the concept of derivative in a point, expressed by *f ‘(2)* in the text, with the angular coefficient of the tangent line, expressed by the equation $y = \frac{3}{2}x - 2$.

The first concept, belonging to a purely analytical representation, is linked to the second one (belonging to a purely geometric representation) through the concept of angular coefficient (algebraic/analytical representation) of the tangent line (geometric representation), thus expressed through a "mixed" representation, according to the following scheme (Figure 2):

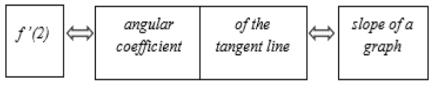

Figure 2: Scheme of representations involved.

4.1 T-E Candidates

The heatmap in Figure 3 (below) shows that the fixations of the expert candidates focused on those parts of the text connected with the angular coefficient. In fact, these parts are sufficient to solve the task.

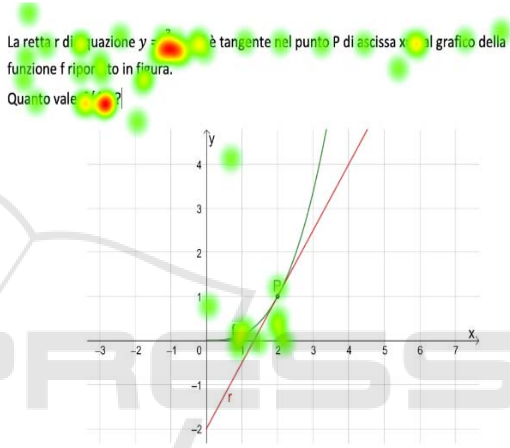

Figure 3: Heatmap of the T-E candidates.

These fixations are linked to the cognitive process of the expert candidate, who does not perform a mental calculation. This result is confirmed by the subsequent interviews. More than one expert candidate states: “the elements I looked at the most (in the text, NdA) are: angular coefficient and the question of the task”. The occasional gazes at the graph are connected to the ease of the task. In fact, this represents a disorienting element for experts, so they check more closely the request of the task and make control evaluations. These could be due to the subject’s anxiety, which is inversely proportional to the perceived difficulty of the task. In fact, an expert candidate says: “I saw that the task was very easy and that is why I thought that there was a trap […] I checked that the abscissa of P was 2”.

The candidate's heatmap, recorded during the interview (Figure 4), is the one drawn while the subject is solving the task. However, there is a fixation (the only noteworthy example) of the candidate on the point of tangency. The rest of the graph was almost ignored.

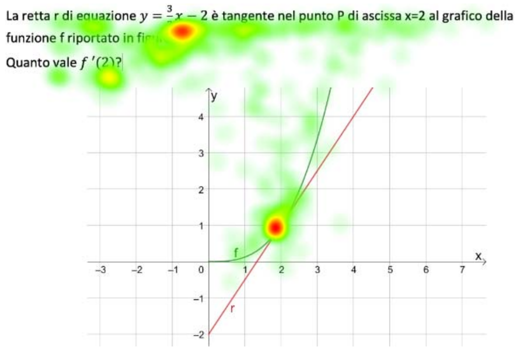

Figure 4: Heatmap of the T-E candidates during the interview.

This action can be justified because the expert candidate gives the local value of the derivative of the function at a point. It is possible to say that expert candidates see the task's graphic register as a confirmational element. In fact, they are able to highlight essential elements of algebraic or analytical nature useful in providing an answer based on simple definitions, decreasing the phase of calculation or graphic/geometrical analysis. This process of reduction of useful information is clear by the textual part "*at point P with abscissa equal to 2*" was observed less by the candidates, because this information belonged to "*f ' (2)*".

4.2 T-NE Candidates

Non-expert candidates who looked mostly at the textual area of the task have a heatmap which varies little from that of T-E candidates. However, their conclusions are different; this means that the use in the cognitive process of the visual elements, obtained by eye exploration phase, is different, interpreting incorrectly acquired information.

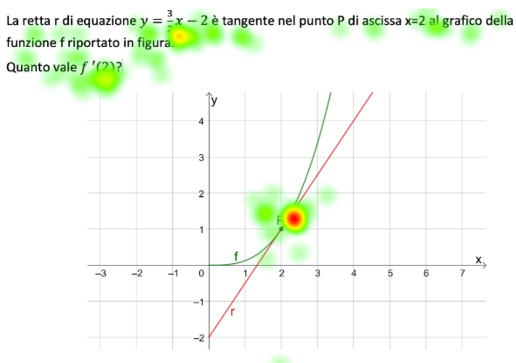

Figure 5: Heatmap of the T-NE candidates.

Although the candidate's attention is focused on the same textual elements, it is possible to observe that it is more uniformly distributed across the text (Figure 5). This marks a lower ability to select the elements useful in solving the task. Confirming this weakness, non-expert saccades are shorter than those of expert candidates and their fixations have a shorter duration (as can be seen from the gaze plot, which we have not reported here due to limited space). Although the abscissa of the tangency point was ignored, it is the only part of the graph which may be considered essential. In contrast with the expert candidates, observation of the graph belongs to the exploratory phase, and it was soon abandoned, because no useful elements were identified to solve the task. This is evidenced by the saccades between the textual part and graphical part, which were almost absent. This indicates the absence of any cognitive process of providing links between textual and graphical data.

By analysing the textual register, it is possible to observe many saccades between the line equation and the demand of the task. This can be justified by the way the T-NE candidates perform the problem: they compute the line's derivative and determine the solution by equating the function's derivative at line's derivative in the same point. Therefore, the candidate knows the link between derivative and tangent line, but not between derivative and angular coefficient. Therefore, he needs to carry out explicit calculations to solve the task, indicating a need for formal justification to determine the solution of a mathematical problem. This is also confirmed by a discomfort, expressed in the following interviews, about inexplicit knowledge of the function *f(x)*. One candidate states: "I could not explicitly compute the derivative of the function in 2, I calculated the derivative of the tangent line in 2 and I thought the two values were equal". Therefore, candidates T-NE prefer an analytical/algebraic method of solving the problem and this leads them to do analytical calculations to determine solution. The graph is an irrelevant element for them.

4.3 G-NE Candidates

Non-expert candidates, who focus their attention mainly on the graphical part of the task, have an opposite approach from T-E candidates. The analysed data show some saccades between the line equation in the textual part (with particular attention to angular coefficient) and tangency point in the graph part. Candidates almost completely ignored the rest of the text, including the request of the task. The candidates know the importance of the angular coefficient of the tangent as a solving element, but they are not able to determine a direct link between the coefficient and the derivative of the function *f*. Therefore, they are unable to transfer the information obtained about the tangent line from an analytical point of view to the graph of the function represented. They try to use the knowledge of the angular coefficient obtained in the textual part to find a connection with the graphical element in order to enact a cognitive process to solve the problem.

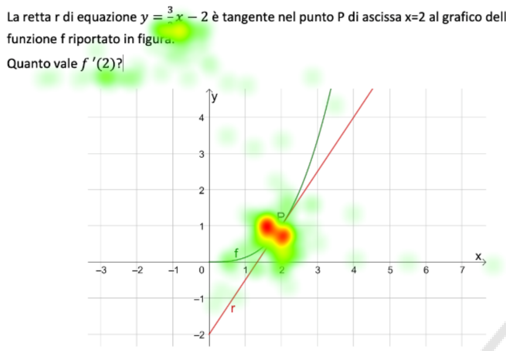

Figure 6: Heatmap of the G-NE candidates.

The heatmap (Figure 6) shows that fixations of G-NE candidates are focused on graphical properties of the point of tangency, which is observed through eye movements along the tangent line and the behaviour of the function. This is the only category in which candidates try to determine graphically the analytical behaviour of the function, looking for distinctive visual elements, such as intersection with abscissas axis or transition at points near the tangency point (helped also by the presence of the numbers in the graph). The heatmap highlights many fixations and saccades in a large (global) area that follow the behaviour of the green curve. One candidate states: "I tried to understand what the parabola equation was …". From this perspective, we can point out that, sometimes, a non-expert candidate associates increasing nonlinear behaviour with a parabola graph. This probably occurs because a parabola is the most familiar nonlinear behaviour for high school students. Therefore, the cognitive process of a candidate G-NE follows an opposite process to that of T candidates: they behave as if the graphical register were essential to obtain all the information needed to solve the task, and afterwards to translate it into the analytical register.

4.4 TG-NE Candidates

Candidates of this category display many saccades between the text of the task and the graph. Their approach to the execution of the problem is based on a continuous comparison between the textual part, with fixations focused on the angular coefficient of tangency line and question of the task *f '(2)* (similarly to candidates of category T), and the graphical part, with fixations focused in particular on the point of tangency, but with considerable saccades and short fixations following the behaviour of the function up to the axis origin (Figure 7).

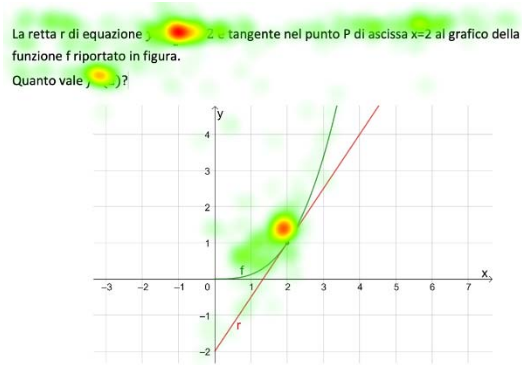

Figure 7: Heatmap of the TG-NE candidates.

A comparative approach of this type requires a continuous change of the semiotic register, from algebraic to graphical. This transition occurs through a mental process that requires the transformation of two registers using analytical knowledge that one should acquire after a Calculus course, which makes this approach the most complicated. In fact, from this method it is possible to posit a typology of analytical mistakes presented by TG-NE candidates, connected to some misconceptions:

1. confusion between the value of the function at the point and the value of the correspondent derivative in the same point: the TG-NE candidates check the ordinate of the tangency point and often answer the task question with that value. One of them states: "I checked that tangency point was at 2 and as tangent line and function have the same value in that point, that value is the solution…";
2. a wrong attribution of globality to the local problem: visual attention (and cognitive) to the behaviour of the function even at the points far from the tangency point, are a feature. A TG-NE candidate states: "…I tried to observe only the tangency point, but not knowing the behaviour of the function, I was not able to understand the derivative";
3. (linked to previous point) the lack of distinction between the value of the derivative of the function in one point

$f'(x_0)$ and derivative of function $f'(x)$. This is highlighted also in the language used: one candidate states that “the first derivative of the function at that point is a tangent line to the function at that point, identifying a number (derivative in one point) with a curve (tangent line). From this perspective the behaviour of the supposed parabolic (mentioned earlier) and the supposed analytic quadratic behaviour for the function, can explain why the derivative is a line (tangent line). This excludes the fact that if a behaviour was exponential or logarithmic, the tangent curve could not be a straight line.”

Definitely, in the approach used by TG-NE candidates, it is possible to highlight a marked distance between information acquired from observation of the text and information obtained by visual analysis about behaviour of the function. Therefore, we can say that the graphical register is a distracting element for these candidates.

5 CONCLUSIONS

This work is part of a more general project, which sets out to analyse (by means of the eye-tracker tool) the data obtained from the administration of questions based on concepts learned in a standard Calculus course. Two kinds of candidates were involved: the experts, including university professors, high school professors, doctoral students and master's students, and the non-experts, i.e., students enrolled in the first years of an undergraduate degree course of a scientific faculty. The basic idea is that, by comparing the data obtained, it is possible to "reconstruct" the different approach and cognitive path used to tackle a mathematical problem. The purpose is twofold: on the one hand, it is possible to take a "snapshot" of the delicate transition that a student faces in moving from secondary school to university; on the other hand, it is possible to try to derive useful indications to improve the teaching of mathematics in a first-year university course. In this paper, a qualitative analysis was presented of a question based on a quantitative analysis about an INVALSI task. It concerns the link between the concept of the derivative, the angular coefficient of the tangent line and the slope of the graph of a "smooth" function at a point. The results of the Italian INVALSI assessment referring to the same task were as follows: Correct 13%, Incorrect 57%, Missing 30%. As we mentioned above, the purpose of the eye-tracking analysis is to figure out the student behaviour.

The use and interpretation of the different theoretical concepts used in the test allowed us to hypothesise the cognitive processes implemented by the different types of participants. The nature of the representations involved in the scheme in Figure 2 are represented by the two different and distinct areas of interest distributed over the question: the textual part expressed in analytical/algebraic register and the figurative part expressed in geometric register. What was possible to observe is a very clear characterisation of the four expert candidates (T-E candidates), who preferred a purely analytical approach. For them, therefore, the main visual (and cognitive) area of interest was textual, with particular attention paid to the question request ($f'(2)$) and the angular coefficient of the tangent line, while the figure assumed only the role of confirmation or control. For such candidates, the previous scheme is strongly shifted to the left and the answer to the question was unanimous and correct. The division of the nine non-expert candidates was more complex. Three of these candidates (T-NE candidates) also followed a purely analytical approach, but the link between the first derivative and the angular coefficient of the tangent line was less decisive: they preferred to calculate the derivative of the equation of the tangent line and to identify the concept of the derivative of the function with that of the tangent line. These candidates answered the question correctly and the figural register was essentially irrelevant. Four of them chose an "intermediate" approach (TG-NE candidates), with areas of interest evenly distributed between text and graphics. For these candidates, the diagram was the main focus: they tried to relate the equation of the tangent line to its graph, losing sight of the (local) concept of the first derivative at a point and the (global) graph of the tangent line. Two of them answered incorrectly, confusing the slope of the graph of a function with the value of the function at that point. Finally, one non-expert candidate (G-NE candidate) approached the question from the opposite side to that of the experts. His area of interest containing the figure was clearly predominant: he tried to determine the slope of the graph from the slope of the tangent line, calculated using the grid (although his answer was not correct). The graph takes priority in his approach and his diagram is strongly shifted to the right. It was not possible to deduce from this purely qualitative analysis that there are statistically significant correlations between the different methods of approaching the question, the candidates' prior knowledge and the outcome of the question itself. To this end, we intend to acquire a large amount of data in the coming months so that we will be able to carry out a more quantitative analysis. However, the information acquired about the cognitive processes were important to underline the observation of a mathematical problem articulated in different registers, such as the one we experimented, and the theoretical information that should be acquired as the primary objective of a basic course in Calculus, in order to gain useful information on the best teaching methods that can be used and possible technologies suited to support such methods.

We think this can also be helpful from a teacher professional development perspective (Spagnolo et al., 2022).

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