Generalized Torsion-Curvature Scale Space Descriptor for 3-Dimensional Curves

Lynda Ayachi, Majdi Jribi and Faouzi Ghorbel
CRISTAL Laboratory, GRIFT Research Group, Ecole Nationale des Sciences de l'Informatique (ENSI), La Manouba University, 2010, La Manouba, Tunisia

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Abstract: In this paper, we propose a new method for representing 3D curves called the Generalized Torsion Curvature Scale Space (GTCSS) descriptor. This method is based on the calculation of curvature and torsion measures at different scales, and it is invariant under rigid transformations. To address the challenges associated with estimating these measures, we employ a multi-scale technique in our approach. We evaluate the effectiveness of our method through experiments, where we extract space curves from 3D objects and apply our method to pose estimation tasks. Our results demonstrate the effectiveness of the GTCSS descriptor for representing 3D curves and its potential for use in a variety of computer vision applications.

1 INTRODUCTION

In the field of computer vision, the ability to accurately describe curves is essential for numerous applications, including object recognition, image segmentation, motion estimation, and tracking. In two-dimensional images, contours can be represented as two-dimensional curves. Over the years, a variety of methods and techniques have been developed for describing these curves in a concise and effective manner. These curve descriptors are essential for many computer vision tasks, as they enable the extraction of useful information from images and enable algorithms to better understand the shape and structure of objects within an image. Two categories of methods have been proposed in the literature: global and local. These terms refer to the scope of the features that are extracted from contours. Global methods typically focus on extracting high-level, overall characteristics of the contour, such as its length or overall shape. Local methods, on the other hand, focus on extracting more detailed, fine-grained features that capture the local structure of the contour, such as the angles between adjacent points or the curvature at different points along the contour. These two types of methods have different strengths and weaknesses, and they are often used in combination to achieve the best performance in contour description and analysis tasks. In the global set of algorithms, there are several methods that have been applied to the task of contour description. One such method is the Fourier descriptor, which has been used in a number of works, including (Persoon and Fu, 1977) and (Ghorbel, 1998). These methods focus on extracting global features of contours, but other methods have been developed that focus on local features instead. For example, the method proposed in (Hoffman and Richards, 1984) partitions the curve into segments at points of negative curvature, which improves the performance of object recognition. In a more recent work, (Yang and Yu, 2018) introduces a multiscale Fourier descriptor that is based on triangular features. This method combines global and local features, addressing the limitations of existing Fourier descriptors in terms of local shape representation. Triangle area representation (TAR) is a multi-scale descriptor that was introduced in (Alajlan et al., 2007). It is based on the signed areas of triangles formed by boundary points at different scales. Another multiscale approach is the Angle Scale Descriptor, which was proposed in (Fotopoulou and Economou, 2011) and is based on computing the angles between points of the contour at different scales. In (Sebastian et al., 2003), a method called Curve Edit was proposed that characterizes the contour using two intrinsic properties: its length and the variations in its curvature. This method has been used for contour registration and matching. Another notable method is the Shape Context algorithm, which was introduced in (Belongie et al., 2002). (Pedrosa et al., 2013) introduced the Shape Saliences Descriptor (SSD), which is based on the identification of points of high curvature on the contour. These points, known as salience points, are represented using the relative angular position and the curvature values at

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multiple scales. Another well-known local descriptor is the Curvature Scale Space (CSS), which was introduced in (Mokhtarian et al., 1997). This descriptor is obtained by extracting the zero-crossing points of the smoothed contour parameterizations using a series of Gaussian functions at different scales. In , a similar approach is used, but the extrema of the Gaussian functions are extracted instead of the zero-crossing points. This type of descriptor has been widely used in tasks such as shape retrieval, classification, and analysis, due to its good performance, robustness, and compactness. In (BenKhlifa and Ghorbel, 2019), this descriptor is extended to the three-dimensional case, and the GCSS (Generalized Curvature Scale Space) descriptor is introduced, which considers an invariant feature on curve points with a given level of curvature. This type of descriptor has been shown to be effective for representing also for 3D curves. Another notable approach for representing 3D curves is the Torsion Scale Space representation, which was introduced by (Yuen et al., 2000). This descriptor is based on the torsion information of the curve, which is a local measure of its non-planarity. Geometrically, space curves that lie on the surface of 3D objects contain valuable information about the shape and structure of those surfaces. (Burdin et al., 1992) proposed a method for extracting 3D primitives, such as long bones, using a set of 2D Fourier descriptors. This set of descriptors is shown to be stable, complete, and endowed with geometrical invariance properties. In a recent work, (Jribi et al., 2021) introduced a novel invariant 3D face description that is invariant under the SE(3) group. This description is based on the construction of level curves of the three-polar geodesic representation, followed by geometric arc-length reparameterization of each level curve. The principal curvature fields are then computed on the sampled points of this three-polar parameterization.

In this paper, we introduce a new descriptor for 3D curves called the Generalized Torsion Curvature Scale Space (GTCSS). This descriptor is based on the calculation of curvature and torsion measures at different scales. According to the second fundamental theorem of geometry (Friedrich, 2002), this pair of measures is invariant under rigid transformations. This means that two different space curves that have the same values of curvature and torsion should have the same shape. In the next section, we present the mathematical formulation of the GTCSS descriptor, and we demonstrate its effectiveness through empirical results.

2 GENERALIZED CURVATURE TORSION SCALE SPACE

Let $\Gamma(u)$ be a parameterization of the space curve Γ. It is a function of a continuous parameter u defined by:

$$\begin{aligned} \Gamma : [0,1] &\rightarrow \mathbb{R}^3 \\ u &\mapsto [x(u), y(u), z(u)]^T \end{aligned} \tag{1}$$

Where $[x(u), y(u), z(u)]$ are the geometric coordinates of the curve points. It is important to note that the parameterization of the curve is not unique because it depends upon the starting point and the speed we go over the curve. To get rid of this problem, the arc length reparameterization is generally chosen as a solution since it is invariant under Euclidean transformations. For a curve Γ, the arc length parameterization is formulated as follows:

$$\Gamma^*(s) = \left[x\left(\varphi^{-1}(s)\right), y\left(\varphi^{-1}(s)\right), z\left(\varphi^{-1}(s)\right)\right]^T \tag{2}$$

Where $\varphi^{-1}(s)$ represents the inverse of the arc length function defined as:

$$\varphi(u) = s(u) - s(0) = \int_0^u \left\| \frac{d\left(\Gamma(u)\right)}{du} \right\| du \tag{3}$$

Let denote by Γ_σ the smoothed curve for a fixed scale σ and $\kappa(s,\sigma)$ and $\tau(s,\sigma)$ its curvature and torsion :

$$\kappa(s,\sigma) = \frac{\sqrt{(\ddot{z}_\sigma \dot{y}_\sigma - \ddot{y}_\sigma \dot{z}_\sigma)^2 + (\ddot{x}_\sigma \dot{z}_\sigma - \ddot{z}_\sigma \dot{x}_\sigma)^2 + (\ddot{y}_\sigma \dot{x}_\sigma - \ddot{x}_\sigma \dot{z}_\sigma)^2}}{\left(\dot{x}_\sigma^2 + \dot{y}_\sigma^2 + \dot{z}_\sigma^2\right)^{\frac{3}{2}}} \tag{4}$$

$$\tau(s,\sigma) = \frac{\dot{x}_\sigma(\ddot{y}_\sigma \breve{z}_\sigma - \ddot{z}_\sigma \breve{y}_\sigma) - \dot{y}_\sigma(\ddot{x}_\sigma \breve{z}_\sigma - \ddot{z}_\sigma \breve{x}_\sigma) + \dot{z}_\sigma(\ddot{x}_\sigma \breve{y}_\sigma - \ddot{y}_\sigma \ddot{x}_\sigma)}{(\dot{y}_\sigma \ddot{z}_\sigma - \dot{z}_\sigma \ddot{y}_\sigma)^2 + (\dot{z}_\sigma \ddot{x}_\sigma - \dot{x}_\sigma \ddot{z}_\sigma)^2 + (\dot{x}_\sigma \ddot{y}_\sigma - \dot{y}_\sigma \ddot{x}_\sigma)^2} \tag{5}$$

Let a be a substitution of x, y and z. The notation $\dot{a}_\sigma$, $\ddot{a}_\sigma$ and $\breve{a}_\sigma$ stands for respectively the first, second and third derivative of a in scale σ, expressed in equation(6):

$$\begin{cases} \dot{a}_\sigma(s) = a(s) \otimes \dot{g}(s,\sigma), \\ \ddot{a}_\sigma(s) = a(s) \otimes \ddot{g}(s,\sigma), \\ \breve{a}_\sigma(s) = a(s) \otimes \breve{g}(s,\sigma) \end{cases} \tag{6}$$

Where g is a gaussian function.

Once the curvature and torsion functions of the smoothed curve are obtained, the next step corresponds to the extrema extraction and the thresholding. Extrema, that have significant curvature and torsion variations in the sense of shape information, are extracted. The number of extrema decreases after each gaussian convolution and the space curve becomes smoother as illustrated in Figure 1.

Therefore, a thresholding is performed in order to eliminate local extrema that have low absolute curvature and torsion variations. Curvature extrema that are

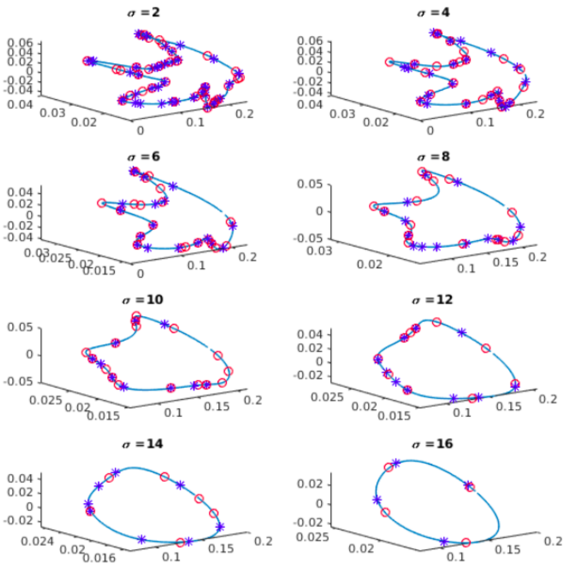

Figure 1: An example of a space curve in different levels of smoothing, from $\sigma = 2$ to $\sigma = 16$.

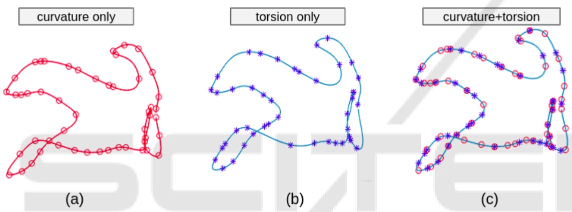

Figure 2: (a) The curvature keypoints (b) The torsion keypoints (c) The curvature and torsion keypoints.

higher than the threshold ε_κ are kept in the set $\Omega_\sigma(\varepsilon_\kappa)$ which can be formulated as follows:

$$\Omega_\sigma(\varepsilon_\kappa) = \{\kappa \in \Omega_\sigma \quad ; |\kappa| > \varepsilon_\kappa\} \tag{7}$$

The torsion extrema that are higher than the threshold ε_τ are kept in $\theta_\sigma(\varepsilon_\tau)$

$$\theta_\sigma(\varepsilon_\tau) = \{\tau \in \theta_\sigma \quad ; |\tau| > \varepsilon_\tau\} \tag{8}$$

The next step is the generalization part. It consists on seeking points of the curve having the same curvature values as the set Ω_σ and the same torsion values as the set θ_σ. The objective of this step is the enrichment of the point set. It allows reaching areas that are not selected in the previous steps but having the same level of interest (same curvature and torsion) as the extracted points of interest. Reciprocal images are described as follows:

$$\kappa_\sigma^{-1}(\Omega_\sigma(\varepsilon_\kappa)) = \{s \in [0,1] \quad / \kappa_\sigma(s) \in \Omega_\sigma(\tau)\} \tag{9}$$

$$\tau_\sigma^{-1}(\theta_\sigma(\varepsilon_\tau)) = \{s \in [0,1] \quad / \tau_\sigma(s) \in \theta_\sigma(\tau)\} \tag{10}$$

The previous steps are repeated a number of times chosen empirically. The selected points at each scale σ are stored, respectively, in $F_\kappa(\sigma)$ and $F_\tau(\sigma)$. They can be described as follows:

$$F_\kappa(\sigma) = \left\{\Gamma(s,\sigma) \quad / s \in \kappa_\sigma^{-1}\left\{\Omega_\sigma(\varepsilon_\kappa)\right\}\right\} \tag{11}$$

$$F_\tau(\sigma) = \left\{\Gamma(s,\sigma) \quad / s \in \tau_\sigma^{-1}\left\{\theta_\sigma(\varepsilon_\theta)\right\}\right\} \tag{12}$$

3 EXPERIMENTATIONS AND RESULTS

In this section, we evaluate the performance of our proposed method on 3D object datasets. As there are currently no datasets available for space curves, we generate them from widely used 3D object datasets in order to validate our approach. The method we use for extracting space curves from 3D objects allows us to evaluate the performance of the GTCSS descriptor in terms of pose retrieval. We demonstrate the effectiveness of our method on simple pose estimation tasks, and compare its performance to existing state-of-the-art methods.

3.1 Dataset

The experiments in this paper were conducted on the 3DBodyTex dataset (Saint et al., 2018), which contains 400 high-resolution 3D scans of 200 different subjects. The subjects are captured in at least two poses: the "U" pose and another random pose belonging to a fixed set of 35 poses. The dataset provides a useful benchmark for evaluating the performance of our proposed method on human pose estimation tasks. (Saint et al., 2018). The Figure 3 illustrates some examples of the dataset.

3.1.1 Space Curves Extraction

Our idea involves generating space curves from three-dimensional objects, on which we then apply our GTCSS descriptor. In this work, we focus on acquisitions of the human body. To create a space curve for a given object, we take the following steps:

- Identify a set of landmarks on the object, located on the head, shoulders, elbow, wrist, hips, knees, and ankles.
- Connect each pair of successive landmarks with a geodesic line, as shown in Figure 5.
- Concatenate all of the geodesic lines to create the final space curve for the object.

The obtained 3D curve accurately represents the shape of M and captures its global morphological variations. As a result, it can be used for pose recognition, as it lies on mobile areas of the body.

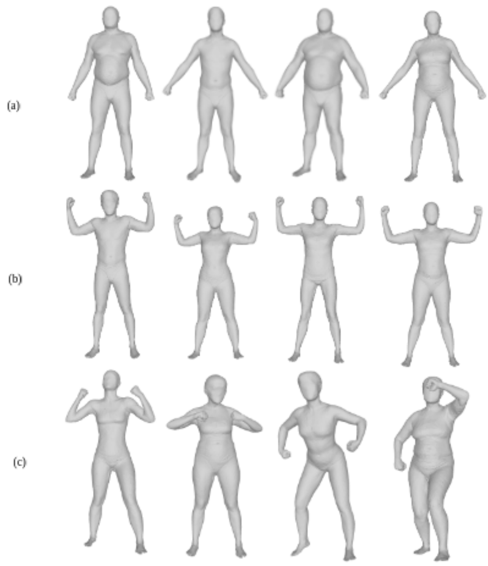

Figure 3: Samples from the 3DBodyTex dataset: (a) ”A” pose, (b) ”U” pose, (c) random pose.

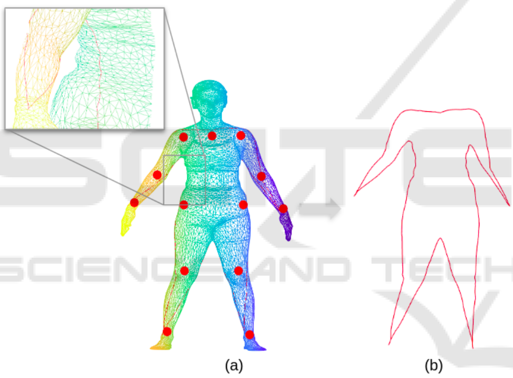

Figure 4: (a) Spots in red, the chosen landmarks (b) Space curve generation.

3.2 Similarity Metric

Let $\pi(\sigma)$ be the set of points, represented by their arclength parametrization, and obtained by concatenating $F_\kappa(\sigma)$ and $F_\kappa(\sigma)$, as illustrated in Figure 2 and expressed in the following equation:

$$\pi(\sigma) = F_\kappa(\sigma) \cup F_\tau(\sigma) \tag{13}$$

As a similarity metric, we employ the Dynamic Time Warping (DTW) distance developed by Sankoff aat al. (Sankoff and Kruskal, 1983) to compare 3D curve descriptors as illustrated in Figure 5. As stated in (Ratanamahatana and Keogh, 2004), the proposed representation generates a pseudo time series, and the DTW guarantees an invariance relatively to the initial point. Let A and B be two space curves of length N_A and N_B respectively, and represented by two signatures $S(A) = \{S(a_1), S(a_2) \ldots S(a_{N_A})\}$ and $S(B) = \{S(b_1), S(b_2) \ldots S(b_{N_B})\}$. The path that minimizes the cumulative distance between these two series represents the distance between them and denoted by $D(S(a_i), S(b_i))$:

$$D(S(a_i),S(b_j)) = \min \begin{cases} D(S(a_i),S(b_{j-1})) \\ D(S(a_{i-1}),S(b_j)) \\ D(S(a_{i-1}),S(b_{j-1})) \end{cases} + D(S(a_i),S(b_j)) \tag{14}$$

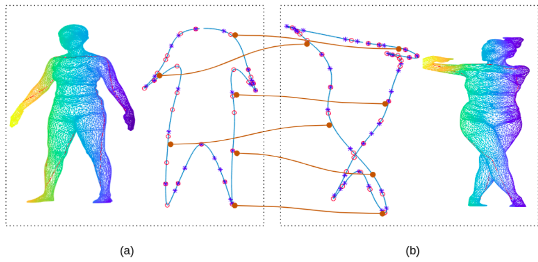

Figure 5: The comparaison process between two models with different poses.

3.3 Results

We apply our method to each model in the dataset, using the GTCSS description of each object. To determine the optimal scale, we perform a study to compute the retrieval rate using a gaussian kernel, as shown in Figure 6. We experimentally set the thresholds for the curvature and torsion measures to be $\varepsilon_\kappa = 10^{-2}$ and $\varepsilon_\tau = 10^{-3}$, respectively, in order to eliminate points with very low curvature and torsion. The curvature and torsion of the obtained curves vary significantly. For example, a human body in a running pose will have varying curvature and torsion values in the arms and legs, and almost zero values elsewhere. In contrast, a human body in a standing position (or A pose) will have mostly monotonous or zero curvature and torsion values. Figure 7 demonstrates the utility of the combination of curvature and torsion measures for pose estimation.

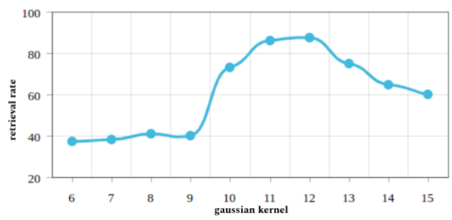

Figure 6: Dependency of the retrieval rates and the gaussian kernel.

Our approach is evaluated in terms of pose retrieval. We use the k-nearest neighbor (kNN) algorithm with k=1 to obtain the scores of the pairwise shape matching. For each model, we compute the dynamic time warping (DTW) distance using a super-

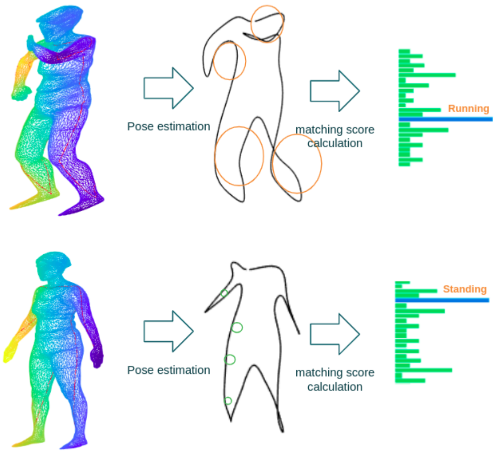

Figure 7: The decision process based on the descriminative measures of curvature and torsion in two space curves representing two different poses. In red, zones with high variations. In green, zones with low variations.

vised list of poses consisting of 36 descriptors: one model descriptor for each pose. An example of a good and bad matching is illustrated in Figure 8.

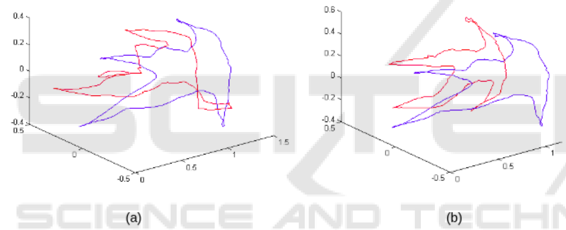

Figure 8: (a) An example of bad matching (b) An example of a good matching.

Table 1: The obtained retrieval rate for different category of poses.

	Rate GTCSS (%)	Rate TSS (%)
A pose	88.2	67.08
U pose	89.2	65.3
Run pose	81.56	63.91

Our proposed descriptor GTCSS is compared to TSS (Yuen et al., 2000) with σ ranges from 0.5 to 12 with the step =0.2. Table.1 shows the retrieval results of the GTCSS on 3DBodyTex dataset using KNN algorithm, k=1 for some poses: A pose, U pose and Run pose.

4 CONCLUSION

The problem of 3D curve description remains a significant challenge in the field of computer vision, with the need for robust and efficient methods that can accurately represent the shape of 3D curves in a manner that is invariant to various transformations. In this paper, we introduce a new descriptor for 3D curves called the Generalized Torsion Curvature Scale Space (GTCSS) that is based on the calculation of curvature and torsion measures at different scales. This descriptor is invariant under rigid transformations, making it well-suited for representing the shape of 3D curves. To address the challenges associated with estimating these measures, we employ a multi-scale technique in our approach. By estimating torsion and curvature at multiple scales, we are able to mitigate the cumulative errors that can arise from the computation of multiple derivatives in the torsion equation. We demonstrate the effectiveness of our approach through experiments, where we extract space curves from 3D objects and apply our method to pose estimation tasks. In future research, we plan to refine our method and conduct further studies on the GTCSS parameters to evaluate their effectiveness for 3D object recognition.

REFERENCES

Alajlan, N., El Rube, I., Kamel, M. S., and Freeman, G. (2007). Shape retrieval using triangle-area representation and dynamic space warping. *Pattern Recognition*, 40(7):1911–1920.

Belongie, S., Malik, J., and Puzicha, J. (2002). Shape matching and object recognition using shape contexts. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 24(4):509–522.

BenKhlifa, A. and Ghorbel, F. (2019). An almost complete curvature scale space representation: Euclidean case. *Signal Processing: Image Communication*, 75:32–43.

Burdin, V., Ghorbel, F., Tocnaye, J.-L., and Roux, C. (1992). A three-dimensional primitive extraction of long bones obtained from fourier descriptors. *Pattern Recognition Letters*, 13:213–217.

Fotopoulou, F. and Economou, G. (2011). Multivariate angle scale descriptor of shape retrieval. *Proceedings of SPAMEC*, 2011.

Friedrich, T. (2002). *Global Analysis: Differential Forms in Analysis, Geometry, and Physics*, volume 52. American Mathematical Soc.

Ghorbel, F. (1998). Towards a unitary formulation for invariant image description: Application to image coding. *Annales des Telecommunications/Annals of Telecommunications*, 53:242–260.

Hoffman, D. and Richards, W. (1984). Parts of recognition. *Cognition*, 18(1):65–96.

Jribi, M., Mathlouthi, S., and Ghorbel, F. (2021). A geodesic multipolar parameterization-based representation for 3d face recognition. *Signal Processing: Image Communication*, 99:116464.

Mokhtarian, F., Abbasi, S., and Kittler, J. (1997). Efficient and robust retrieval by shape content through curvature scale space. In *Image Databases and Multi-Media Search*, pages 51–58.

Pedrosa, G. V., Batista, M. A., and Barcelos, C. A. (2013). Image feature descriptor based on shape salience points. *Neurocomputing*, 120:156–163. Image Feature Detection and Description.

Persoon, E. and Fu, K.-S. (1977). Shape discrimination using fourier descriptors. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-8:388–397.

Ratanamahatana, C. A. and Keogh, E. (2004). Everything you know about dynamic time warping is wrong. In *Third Workshop on Mining Temporal and Sequential Data*, pages 22–25.

Saint, A., Ahmed, E., Cherenkova, K., Gusev, G., Aouada, D., and Ottersten, B. (2018). 3dbodytex: Textured 3d body dataset. In *2018 International Conference on 3D Vision (3DV)*, pages 495–504. IEEE.

Sankoff, D. and Kruskal, J. B. (1983). Time warps, string edits, and macromolecules: The theory and practice of sequence comparison. In Sankoff, D. and Kruskal, J. B., editors, *Time Warps, String Edits, and Macromolecules: The Theory and Practice of Sequence Comparison*. Addison-Wesley Publication.

Sebastian, T., Klein, P., and Kimia, B. (2003). On aligning curves. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 25:116–125.

Yang, C. and Yu, Q. (2018). Multiscale fourier descriptor based on triangular features for shape retrieval. *Signal Processing: Image Communication*, 71.

Yuen, P., Mokhtarian, F., Khalili, N., and Illingworth, J. (2000). Curvature and torsion feature extraction from free-form 3-d meshes at multiple scales. *IEE Proceedings-Vision, Image and Signal Processing*, 147(5):454–462.