Application of Particle Detection Methods to Solve Particle Overlapping Problems

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Keywords: Particle Detection, Particle Overlap, non-Local Means, Laplacian of Gaussian.

Abstract: The study of fluid flows concerns many fields (e.g., biology, aeronautics, chemistry). To overcome the problems of flow disturbances caused by intrusive physical sensors, different methods of flow quantification, based on optical visualization, are particularly interesting. Among them, PTV (Particle Tracking Velocimetry) which allows the individualized tracking of tracers/particles, is of growing interest. Different numerical treatments will enable us to identify and track the particles. However, detection algorithms (e.g., Sobel, Canny, Robert, Gaussian, morphology) can be sensitive to noise and the phenomenon of overlapping particles in flow. In this work, we have focused on the detection part with the objective of improving it as much as possible. To quantify the performance of the different methods tested, synthetic images, with well-defined parameters have been generated. We compared the performances of the Laplacian of Gaussian (LoG) and the Difference of Gaussian (DoG) methods, with the traditional method of threshold binarization. In addition, we tested other techniques based on non-local means (NLM) and overlapping detector to improve the detection of particles in case of noisy images or overlapping particles. The results show that the LoG gives very good results in most cases, with additional improvement when using the NLM and the overlap detector.

1 INTRODUCTION

The study of fluid flows concerns different fields, such as aeronautics, automotive, biology, chemistry, food processing, geology, etc. The flows coupled, for example to chemical reactions, lead to various biological and mechanical phenomena, such as the circulation of blood in the human body, the distribution of oxygen, heat, and pressure of the breathable air in the lungs, the production of energy in various automotive engines, and so on. It is therefore very important to be able to know the velocity and/or the trajectory of the fluid in order to study the above-mentioned phenomena. Among the different methods of flow quantification, optical visualization methods are particularly effective and accurate, especially since they are low or non-intrusive and therefore have low, to no, impact on the flow being studied. The optical methods generally require the flow (the fluid) to be seeded with tracers, i.e. particles / "solid" objects, that can be either artificial (e.g., hollow glass beads), or natural (e.g. platelets, globules, and cells in biological fluids) (Scharnowski and Kähler, 2020). Through image processing, information such as position, size, concentration, displacement, velocity vector, etc. can be obtained. Over the last few decades, various techniques were developed for extracting, for instance, velocity vector field from the image frames. Among those methods, one can cite Particle Streak Velocimetry (PSV), Particle Image Velocimetry (PIV), and Particle Tracking Velocimetry (PTV). In the last few years, there has been quite a lot of improvements in these above-mentioned techniques to analyze the properties of objects at micro- and nano-scale (Baek and Lee, 1996; Lima et al., 2012). To be able to individualize each particle/fluid element and to follow

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AlKendi, W., Mahapatra, P., Alkindy, B., Guyeux, C. and Barthès, M.

Application of Particle Detection Methods to Solve Particle Overlapping Problems. DOI: 10.5220/0011852500003497

In Proceedings of the 3rd International Conference on Image Processing and Vision Engineering (IMPROVE 2023), pages 84-91 ISBN: 978-989-758-642-2: ISSN: 2795-4943

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its trajectory over time, PTV (Particle Tracking Velocimetry) is used (Scharnowski and Kähler, 2020; Ohmi and Li, 2000). Usually, a classical PTV algorithm applied on pre-processed images consists of, first, detecting the positions of each individual particle and then, matching and tracking these particles across the image frames (Baek and Lee, 1996; Kim and Lee, 2002). Over the years, different edge detectors have been used to identify particles. Initially, the particles were identified using single and dynamic threshold binarization methods. Going forward, the two-dimensional Gaussian regression technique was applied to the particle intensity values to estimate the sub-pixel particle centroid positions (Ohmi and Li, 2000; Heyman, 2019). However, with the increase in the complexity in the images, when the intensity and size of the particles varied, researchers went ahead to apply different detectors based on noise, gradient, template, and morphology. These edge detectors like Sobel, Canny, Robert, Prewitt, Gaussian, Overlap, and so on, are generally sensitive to the change in pixel gray levels (Katiyar and Arun, 2014). Some of these detectors output different results in terms of their sensitivity towards noise and in detecting false edges. Depending on the type of data input, different applications of these detectors might work better. Some will be good for larger and intense particles, whereas others will have a better chance of catching smaller and lighter ones (Janke et al., 2020). In order to track particles across image frames acquired over a time period, most of the algorithms are influenced by the probability relaxation algorithm taking into account the similar displacements exhibited by the nearest neighboring particles (Baek and Lee, 1996; Ohmi and Li, 2000). The particle matching and tracking were improved by using iterative matching schemes and Deep Learning networks (Janke et al., 2020; Heyman, 2019; Lee et al., 2019). However, with these probabilistic methods, it is very difficult to track multiple particles due to the occlusion/overlap of two or more particles (Qian et al., 2021). In this paper, we seek to improve the detection of particles for PTV applications. For this purpose, different detection methods are implemented, using Laplacian of Gaussian (LOG) and Difference of Gaussian (DOG), and compared to the traditional threshold binarization method (Lefta et al., 2022). Noise minimization has also been implemented, and finally, we also seek to solve as much as possible the problem of overlapping particles visualized during the detection and tracking of the latter.

2 METHODOLOGY

In this paper, we have proposed an algorithm to detect featureless micro or nanoparticles in a liquid flow, where the main objectives are to maximize the number of detected particles and minimize the problems related to overlapping. In order to evaluate and determine the accuracy of the algorithms, synthetic images, of known content, have been created. They allow us to vary only one parameter at a time and compare the results obtained at the end of the processing with the known and imposed parameters used for the generation of images.

2.1 Synthetic Images: Dataset

Three groups of synthetic two-dimensional $(1024 \times 768 \text{ px})$ images have been created, where each group consists of 40 images including 200 randomly distributed particles with different properties such as particle size, particle's light intensity, and Gaussian noise rate. Figure 1 shows samples of the synthetic dataset images for which we varied one or more parameters (e.g., size dispersion, background noise, illumination dispersion).

2.1.1 Dataset Creation

In order to test the implementation and robustness of our algorithm, we need to have images, especially synthetic images that are modelled on the basis of real-world experimental ones. These particle image recordings are based on different characteristics like, for instance, particle position, diameter, shape, dynamic intensity range, spatial density, image depth, flow patterns, noise in the image, etc. These synthetic images with well-defined particle locations also help us in quantifying the quality of our algorithms by providing measurement error estimation (Ohmi and Li, 2000; Raffel et al., 2018; Mohr et al., 2019).

The synthetic images required to test our detection algorithm are generated based on the following steps (Raffel et al., 2018; Thielicke, 2021):

- Size (height \times width) and background (black/white) of the images to be generated.
- Parameters like number N, diameter d_τ, and flow pattern of particles along with the type of noises (e.g., Gaussian, Salt and Pepper, Poisson) to be added in the images.
- Creation of particles based on their intensity, size and centroid positions (X_0, Y_0, Z_0) in the first image.



(a) Noise rate = 0%, $Z_0 = 0\%$, $d_{\tau} = 10$ px.



(d) Noise rate = 2%, $Z_0 = 0\%$, $d_{\tau} = 10$ px.



(b) Noise rate = 0%, $Z_0 = 0 - 20\%$, $d_{\tau} = 10 - 20$ px.



(e) Noise rate = 2%, $Z_0 = 0 - 20\%$, $d_{\tau} = 10 - 20$ px. Figure 1: Samples of the synthetic dataset.



(c) Noise rate = 0%, $Z_0 = 10 - 40\%$, $d_{\tau} = 10 - 20$ px.



(f) Noise rate = 2%, $Z_0 = 10 - 40\%$, $d_{\tau} = 10 - 20$ px.

- Size and centroid positions of each particle on the first image are generated using a random number generator.
- The peak intensity of each particle is based on the Gaussian intensity profile as a function of particle depth position (Z_0), efficiency with which the particle scatters the incident light (q) and the depth of view (ΔZ). Thus, the peak intensity of each particle $I_0(Z_0)$ is (Raffel et al., 2018):

$$I_0(Z_0) = q \exp\left(-\frac{Z_0^2}{(1/8)\Delta Z^2}\right)$$
(1)

• Next, the intensity profile across an individual particle's size boundary (diffusion effect) is calculated by a Gaussian intensity profile, which is a function of particle peak intensity $I_0(Z_0)$, particle image diameter d_{τ} (e⁻² intensity value of the Gaussian bell containing 95% of the scattered light), and particle position X, Y (in pixels), given by (Raffel et al., 2018):

$$I(X,Y) = I_0(Z_0) e^{\left(-\frac{(X-X_0)^2 - (Y-Y_0)^2}{(1/8)d_{\tau}^2}\right)}$$
(2)

• Finally, as per the flow pattern, a trajectory is generated for each particle in order to have their corresponding positions and intensities in subsequent image frames across a time period.

2.1.2 Dataset Properties

The following properties have been taken into consideration to validate the accuracy of the system in different scenarios which may occur in the future real dataset.

- Size: particle size is designed as the diameter in pixels, where some images contain fixed particle sizes while others contain a range of sizes. In this work, we present results for a fixed size of 10px, or for a range of sizes between 10px and 20px within the same image. This choice of particle size variation range is larger, on purpose than the classical size variations of hollow glass beads commonly used as tracers.
- Intensity: it is the intensity of the light directed towards the particle which varies according to Equations 1 and 2. To change the intensities, we varied the value of the particle depth position Z_0 . In this work, the variation of Z_0 is in a range of 0 40, i.e. corresponding to an intensity variation on the shade of gray between 71 and 255.
- Noise: Gaussian noise has been applied to the images. Results presented here are either for no noise or for a variance of 2%.
- Shape: generally, all the particles are circular. However, some of them may have a polygon shape due to the high noise rate.

• Displacement: in this example, we have fixed a linear displacement of 2 pixels between 2 consecutive images. Other types of displacement (with a gradient, with a rotation etc.) as well as other displacement values have been tested, but are not presented here.

Table 1 gathers the different parameters tested and compared in this article, while Figure 1 shows examples of synthetic images obtained with parameters of this table.

Table 1: Dataset properties: images of size $1024 \times 768 \ px$ each having 200 particles and a variation in particle intensity, diameter, and noise. Particles are imposed with a linear displacement of 2 px across subsequent image frames.

Z ₀ variation (%)	d_{τ} (px)	Noise variation (%)	# Images
0 %	10	0	10
	10	2	10
	10 to 20	0	10
		2	10
	10	0	10
0 - 20 %		2	10
	10 to 20	0	10
		2	10
10 - 40 %	10	0	10
	10	2	10
	10 to 20	0	10
		2	10

2.2 Detection

The detection stage is one of the most important one since all the upcoming stages depend on the accuracy of detecting the particles. Figure 2 illustrates the general diagram of the proposed detection stage. Image binarization, filters, and non-local means denoising algorithm have been applied to the dataset with automatic configuration determination considering maximizing the number of detected particles while satisfying the proposed particle diameter sizes.

2.2.1 Simple Detection Methods

After loading the images as gray-scale ones, some well-known methodologies have been tested for detecting the particles according to the predetermined diameter sizes. For instance, image threshold binarization, Difference of Gaussian (DoG), and Laplacian of Gaussian (LoG).

Over the years, the basic and standard method for individual particle detection has been the Single Threshold Binary method. The binary method is in



Figure 2: General diagram of the detection stage.

general related to the visual properties of the particles (Ohmi and Li, 2000). Once the input image has been pre-processed and converted into a grayscale image (intensity value of each pixel is between 0 and 1), the user can set a threshold value of intensity which indicates to ignore all the particles having an intensity above the threshold value. A Gaussian sub-pixel location estimation method can be used to precisely estimate the location of the center of each particle by using a normalized auto-correlation followed by a Gaussian fitting in order to achieve sub-pixel accuracy. The integral coordinates of the centroid location (X_0, Y_0) can be achieved from the maximum of the auto-correlation peak (C). The exact coordinates of each particle (X,Y) with sub-pixel accuracy can be calculated (Raffel et al., 2018).

Difference of Gaussian (DoG) filter is a two-stage edge detection process. In the first step, the DoG performs edge detection by applying a Gaussian blur (eq. (3)) on the input image for a specified value of standard deviation (σ_1). This results in a blurred version of the input image. This image is subtracted from the less blurry input image resulting from the application of another Gaussian blur with a more sharper value of standard deviation (σ_2). This difference helps in detecting the pixel values when they cross zero (i.e. zero crossings) i.e. when negative becomes positive and vice versa, thus, helping in focusing on the edges or areas of pixels having some variation around their neighbors.

$$G(x,y;\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[-\frac{x^2 + y^2}{2\sigma^2}\right]$$
(3)

The LoG is an edge detection algorithm to locate boundaries and extract features by identifying pixel intensity variance within the image. Generally, it is derivative from the Gaussian filter for noise removal by smoothing the image and Laplacian operator. The Laplacian operator helps in highlighting the regions/ areas of rapid intensity change in the image, i.e. when the pixel values go from negative to positive or vice versa (i.e.zero-crossings), therefore, is a very common method used for edge detection. Equation 4, based on Marr and Hidreth work (Marr and Hildreth, 1980) shows the mathematical representation of the 2D LoG function, centered on zero:

$$LoG(x,y) = -\frac{1}{\pi\sigma^4} \left[1 - \frac{x^2 + y^2}{2\sigma^2} \right] e^{-\frac{x^2 + y^2}{2\sigma^2}}$$
(4)

It is worth mentioning that this method has been used to detect particles within clear images. In other words, images with no noise, and due to the presence of different noise ratios in the rest of the images, more advanced algorithms have been chosen to handle the noise elimination process as explained in the next section.

2.2.2 Non-Local Means Algorithm

With the different noise rates within the dataset, the demand for a more efficient algorithm is required to increase image quality without affecting the edges of the particles, hence improving the accuracy of the particle detection process.

Non-local means (NLM) is an image processing algorithm for image denoising. it is a more advanced technique for removing Gaussian noise from scientific images that arise from electronic components (e.g., microscope and MRI) which affects the process of extracting information from these images. The NLM algorithm searches the image space to calculate the mean within non-local regions. In other words, it is not calculating the mean based on a local group of similar pixels (e.g., 4x4 or 9x9) as proposed by other researchers within the same field (Buades et al., 2005). Instead, it is assigning the center weight CW (the weight of the pixel to be denoised) according to region similarity from all over the image, regardless of their locations.

Finally, contour detection has been used to detect the boundary of the particles and extract information about their shape, which aims to identify the particle center and diameter to be registered within the system.

2.2.3 Overlapped Particles Detection

Since the particles are distributed randomly on images, this may occurring overlapping particles in some areas. Figure 3 shows overlapped particles example. To detect these overlapped particles, we used template matching which is a high-level machine vision technique for determining the similarities of the template image matrix within the source image matrix (Brunelli, 2009). Prior to the use of the Template Matching technique, we carried out two main preprocessing steps. Firstly, automatically extract a good shape template from the same source image depending on a predefined particle diameter size, or by calculating all particle diameter sizes within the image and selecting the diameter size that occurs mostly. Secondly, extracting the particles' overlapped segments and estimating the number of the particles depending on the detected segment diameter and the extracted template diameter size. The following formula shows the process of estimating the number of overlapped particles within the segment.

$$z = x \div y \tag{5}$$

No. of particles =
$$\begin{cases} 2 \text{ if } z \leq 2 \\ [z] \text{ otherwise.} \end{cases}$$
(6)

where x is the diameter size of the extracted segment and y is the diameter size of the template.



Figure 3: Overlapped particles example.

3 DETECTION RESULTS

The algorithm has been programmed in Python, a general-purpose programming language, which has several powerful modules in this research field, which will aid in the enhancement of the simplicity and scalability in future research.

3.1 Detection

Several experiments and comparisons have been carried out to determine the most beneficial methods for handling the process of detecting the maximum number of particles in respect of practicability, efficiency, and execution time. In the first part, we compare three detection methods. In the second part, we improve the results by treating the noise in the images. Finally, in the third part, we refine the detection thanks to an algorithm allowing to minimize the problems related to the overlapping of the particles on the same image.

3.1.1 DoG, LoG, and Binarization

As illustrated in Section 2, three techniques: DoG, LoG, and image binarization have been taken into consideration in the process of experimenting with the dataset from table 1.

In addition, automatic configurations determination has been developed to find the optimal configuration in the range of 0-255 configurations that aim in reaching the objectives. Table 2 gathers the results obtained by processing the synthetic images with the 3 detection methods. We can see that the LOG method gives the best results in the vast majority of cases. However, the quality of the results decreases when images are noisy. In order to minimize the noise and to improve the detection, we used an algorithm, presented in the next section.

In addition, Figure 4 shows an example of the performance comparison sample of the mentioned methods configurations on one of the test case images (Figure 4(a)) from table 2 ($Z_0 = 10 - 40$, D = 10 - 20 px, Noise = 2%). As can be seen from Figure 4(e), Laplacian of Gaussian (LoG) method was able to detect more number of particles (N_p : 176/200) as compared to DoG method (Figure 4(b),(c)). Although the classical Binary threshold method detects a huge number of particles, it is clear from Figure 4(d), that in case of a noisy image, the Binary method also detects noise as particles. In this case, we note that another concern of detection is related to the phenomenon of overlapping of particles, particularly visible here when the value of diameter detected is higher than the maximum diameter generated on our synthetic images. This can also be seen for example on Figure 5.

3.1.2 Non-Local Means Algorithm

To minimize the effect of noise on the particle detection part, non-Local Means Algorithm for gray images has been used in the proposed method with some aiding approaches, for instance, Gaussian blur and adaptive Gaussian thresholding, with variance algorithm settings that fit dataset variations in terms of the following algorithm configurations: h-parameter deciding filter strength. A higher h value leads to better noise removal but also removes particle edges. Template Window size and Search Window size: should be an odd value.

Tables 3 show different settings used in the noise removal process when the noise variance is 2%.

It is worth mentioning that the detected number

Table 2: Comparison of the detection results obtained from the synthetic images of table 1. Results in the last three columns Bin. DoG and LoG are given as percentages of detected particles using respectively classical binarization, DoG and LoG method.

7. (0%)	d (py)	Noise	Bin.	DoG	LoG
Z ₀ (%)	<i>u</i> _τ (px)	(%)	(%)	(%)	(%)
	10	0	67.5	95	98
0		2	125	77.5	96
0	10 to 20	0	71	91	99.5
		2	97.5	90	94
0 to 20	10	0	70	96.5	98
		2	60	96.5	93
	10 to 20	0	24	88	97.5
	10 to 20	2	60	85	96.5
10 to 40	10	0	64	70.5	78.5
		2	87.5	77	80.5
	10 to 20	0	40.5	72.5	82
	10 10 20	2	97.5	80	88

Table 3: Parameters values and results of using NLM on 2% Gaussian noise image.

Parameters	Setting 1	Setting 2	Setting 3
h	15	35	45
Template Window size	7	7	7
Search Window size	21	51	71
Result (%)	243.5	94.5	93.5

of particles represents the total number when using the system without the overlap detection tool. Hence, some overlapped particles are detected and counted as a single particle. Moreover, the satisfaction of the predetermined particle diameter sizes have been taken into consideration as a constraint when conducting the experiments.

Experimental results show the efficiency of the algorithm which improves the denoising performance and enhances image quality without affecting particle edges. Hence, increasing the detection accuracy within the noisy dataset. Figure 6 shows the performance of the NLM algorithm.

3.1.3 Overlap Detector Tool

Experiments have been carried out with the usage of the overlap detector tool to maximize the detected number of particles and enhance the overall performance of the system. Open-Source Computer Vision Library (OpenCV) provides different Single template matching comparison methods, for instance, TM_SQDIFF, TM_SQDIFF_NORMED, TM_CCORR, TM_CCORR_NORMED,



Figure 4: Performance comparison sample of DoG, LoG, and image binarization methods applied on a synthetic image with 200 particles (*N*), with a Diameter (d_{τ}) and variation of particle depth position ΔZ_0 . intensity variation of $10px \pm 100\%$ and 10 - 40%, respectively.



Figure 5: Occurrence of particle overlapping leading to detection of particle with diameter higher than the maximum diameter. Smaller images on the right highlights detachment circles that includes one or more particles overlapped with each other detected as one single particle.

TM_CCOEFF, and TM_CCOEFF_NORMED. These methods have been experimented with to decide the most applicable method in our situation. Additionally, the function minMaxLoc has been used to find the minimum and maximum result values with their locations to extract the particles that fit the template from the overlapped region and achieve the objectives.

Table 4 shows the number of particles detected when using the overlap detector tool. Final experiments outcomes showed highly beneficial detection rates achieved 99% within most of the datasets that has no variation in the particle diameter size. However, some of the overlapped particles could not be detected because of their coordinates lying on the borders within the images. Figure 7 shows a result example of using the overlap detector tool and statistics.

Table 4: The result of using overlap detector tool.

	ΔZ ₀ (%)	d _t (px)	Noise (%)	Without Overlap tool (%)	With Overlap tool (%)
	0	10	0	89 85	99 95
	0 to 20	10	0	93	99
		10	2	92.5	98.5
	10 to 40 10	10	0	87	99.5
		10	2	76	91

4 CONCLUSIONS

There can be no doubt that detecting the microscopic featureless particles in liquids with various properties is challenging. Although different research contexts in this field, it was found that using Laplacian of Gaussian and non-local means algorithms with automatic configuration determination is highly beneficial in the detection stage and aims the precise determination of the particle center and diameter. Furthermore, the overlap detector tool enhanced the outcomes and showed a highly beneficial estimation rate yields maximizing the detected number of particles for the current research synthetic dataset. However, further research must be conducted to enhance the detection rates of different noise ratio datasets, including the usage of a multi-template matching technique to handle the analysis process of different particle diameter sizes within the same image.



(a) Noise rate= 0%.





(c) NLM algorithm performance.

Figure 6: The performance of the NLM algorithm.



(a) When using overlap detector tool: detection accuracy of 99%.



(b) Without using overlap detector tool : detection accuracy of 89%.

Figure 7: The performance of the overlap detector tool.



(c) The statistics of using the overlap detector tool.

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