Mammography Unit Location: Reconciling Maximum Coverage and Budgetary Constraints

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Abstract: This work addresses the Bi-objective Mammography Unit Location-Allocation Problem. This problem consists in allocating mammography units satisfying two objectives and respecting the constraints of device capacity for screenings and the maximum travel distance for the service. The first objective function maximizes the coverage of exams performed by the allocated mammography devices, while the second function minimizes the total amount of equipment used. We introduce a mixed-integer linear programming bi-objective model to represent the problem and apply the Weighted Sum and Epsilon-constraint methods to solve it. The Epsilon-constraint method was able to generate better Pareto fronts. The instances used for testing come from real data from two Brazilian states obtained from the Brazilian Health Ministry.

1 BREAST CANCER IN AMERICAS: HEALTH CARE BUDGETS vs. HUMAN LIVES AT STAKE

There were 4 million cancer cases in the Americas in 2020, and this disease was responsible for 1.4 million deaths in these continents (PAHO, 2020). Also, according to this report, breast cancer is the second leading cause of death among women. Almost 500,000 new breast cancer cases and more than 100,000 deaths from breast cancer were registered in the Americas.

In Brazil, the situation is no different. In 2020, there were an estimated 66,280 breast cancer cases, representing an adjusted incidence rate of 43.73 cases per 100,000 women (INCA, 2019), with 11.84 deaths per 100,000 women (INCA, 2022).

When breast cancer is diagnosed in the early stages, 95% of women affected survive (Witten and Parker, 2018). On the other hand, mammography screening is the primary way to detect early-stage breast cancer (Xavier et al., 2016). Given this, the Brazilian Ministry of Health recommends that women aged 50-69 should have the screening biannually (Brasil, 2017). This institution also recommends that 20% of the female population aged 40-49 undergo yearly screening.

Although Brazilian public and private health services have a sufficient number of mammography units, screening is not accessible to all women (Miranda and Patrocinio, 2018; Amaral et al., 2017). A limiting factor to accessing mammography screening is the determination of the Brazilian Ministry of Health, which defines 60 km as the maximum distance a woman should travel (Brasil, 2017). In (Amaral et al., 2017), the authors showed that some cities have an oversupply of screenings, while others are not served by any equipment within a 60 km radius. A similar result is corroborated in (Rodrigues et al., 2019), whose authors identified an unequal distribution of mammography devices in Brazil, with a surplus of equipment in 17 states and a deficit in the others (9 states and the Federal District). Although other factors contribute to discouraging or even making screening infeasible, these studies show that the distance women should travel to undergo mammography screenings plays a key role in access to it.

Many studies have proposed mathematical programming formulations and heuristics to propose the best location and allocation for mammography units (Corrêa et al., 2018; de Campos et al., 2020; Souza

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et al., 2019; Souza et al., 2020; de Assis et al., 2022). These studies aim to maximize the coverage for screenings and present a tool to health managers to support them in deciding the best location for mammography units and their allocations (assignments), i.e., the set of locations that each mammography unit must serve. Although other studies address generic facility location-allocation problems for health care, no study was found in the literature to maximize coverage and minimize the number of mammography units needed for screenings. Reducing the number of mammography units needed is essential, given the high cost of these devices and the need for better management of public health costs. In order to fill this gap, we introduce the Bi-objective Mammography Unit Location-Allocation Problem (BOMULAP) in this article.

The main contributions of this work are the following:

i) Introducing of the BOMULAP aiming at maximizing the coverage for screenings and minimizing the infrastructure costs of health care;
ii) The implementation and comparison of two well-known solution methods of multi-objective optimization, the Weighted Sum Method and the Epsilon-green Constraint Method, to solve the BOMULAP;
iii) Application of the methodology to realistic cases extracted from Brazilian public health service databases.

The remainder of this article is organized as follows. Section 2 presents a literature review of the main articles about the Mammography Unit Location and Mobile Mammography Unit Routing problems considering the Brazilian reality. In Section 3.1, we introduce the problem under investigation. In Section 3.2, the bi-objective mathematical programming model is presented. Section 3.3 presents brief descriptions of the proposed solution methods for solving BOMULAP. In Section 4, the computational results of the proposed methods are presented and discussed. Finally, Section 5 presents the conclusions and future work.

2 SURVEYING THE HISTORY THUS FAR

In (Shariff et al., 2012), the authors tackle a health care location-allocation problem from Malaysia. They introduce a mathematical programming formulation for the problem and then develop a Genetic Algorithm (GA) to obtain good upper bounds in a reasonable time. The analysis of the obtained results establishes the superior computational performance of the Genetic Algorithm, which is able to find high-quality solutions for all the instances on the test-bed. On the other hand, the CPLEX commercial solver was unable to reach good quality solutions for networks displaying more than 809 nodes.

(Beheshtifar and Alimoahmmadi, 2015) handle a multi-objective location-allocation problem for clinic facilities. The authors propose four objective functions for their mathematical program, based on the well-known *p*-median problem, bringing the underlying assumption of full demand coverage. The first objective function minimizes the sum of demand-weighted distance costs. The second objective function minimizes the standard deviation of the distance between the patients and the health care clinics. The third objective function maximizes the suitability of the selected locations for the new facilities. Finally, the fourth objective function minimizes the costs of acquiring land space and installing new health care clinic facilities. The Non-dominated Sorting Genetic Algorithm II (NGSA II) is used to yield the Pareto-efficient front. The analysis of the obtained results shows that it is possible to reduce the average distance to provide access to health care services at the expense of larger land acquisition costs, generally speaking.

In (Corrêa et al., 2018), the authors present four formulations for the MULAP. The models are based on the classical *p*-median problem. The second differs from the first in allowing violation of the maximum distance that a mammography device can be assigned to serve a locality. In the last two models, the number of women traveling to be served is considered, and the maximum distance constraint in the last one is relaxed. The authors tested the formulations using the cities of 12 health regions that are up to 100 km away from the city of Ouro Preto. No feasible solutions were found in the established processing time in the formulations that impose maximum distance. One of the reasons for this is that several cities in the state of Minas Gerais are more than 60 km away from those that host mammography devices.

In (Souza et al., 2019), the authors did a case study of the MULAP considering the State of Rondônia, Brazil. They present two mathematical programming models based on the maximum coverage. The first model considers that a mammography device can only serve a city if it fully meets its demand. The second model relaxes the constraint of fully meeting the demand. They showed that the second formulation better uses the devices' screening capacity. The authors suggest using mobile mammography devices in cities not served by fixed devices.

In (Sá et al., 2019), the authors did a case study of the MULAP to the State of Espírito Santo, Brazil, using the binary mathematical programming model of Souza et al. (2019). They treated two scenarios. In the first, the location of mammography devices already installed is not changed, while in the second scenario, there is permission for free relocation of these mammography devices. They analyzed the acquisition of new equipment in these two scenarios and showed how many new mammography devices would be needed to cover all the demands for mammography screenings in the state.

In (Souza et al., 2020), the authors did a case study of MULAP in the state of Minas Gerais, Brazil, which has the largest number of cities in Brazil. They used the binary mathematical programming formulation of (Souza et al., 2019). In addition, as MULAP is NP-hard (Church and ReVelle, 1974), they developed a Variable Neighborhood Search-based algorithm (Hansen et al., 2017) to handle large instances of MULAP, such as the one in this state of the Brazilian federation. They showed that the proposed VNS algorithm finds good-quality solutions in reduced computational time.

A case study of MULAP in Minas Gerais and Rondônia states is also done in (de Campos et al., 2020). They used the relaxed formulation of (Souza et al., 2019), which allows the demand of a city to be partially covered by a host city, and also developed a Simulated Annealing-based algorithm (Kirkpatrick et al., 1983). The authors analyzed two scenarios to determine the minimum amount of devices needed to maximize the coverage for mammography screenings in the Minas Gerais state, considering 6758 screenings per year as the productivity of mammography equipment. In the first scenario, the relocation of existing equipment is allowed, while in the second, this relocation is not allowed. They showed that in the first scenario, it is possible to cover 99.97% of the demand in Minas Gerais without purchasing any new equipment, while in the second scenario, it would be necessary to purchase 77 devices to achieve this coverage rate.

(Rosa et al., 2020a) included a new constraint in the mathematical programming model of (Souza et al., 2019) to treat MULAP. In this model, women can only be served by cities within the same health micro-region in which they reside. The authors also analyzed the gradual acquisition of new devices to maximize coverage by screenings. They showed that with cities clustered into health micro-regions, the number of equipment needed to maximize coverage is higher than without this requirement. This result shows that the existing clustering of cities may need to be revised.

(Rosa et al., 2020b) introduced the Mammography Mobile Unit Routing Problem (MMURP). The problem consists of maximizing the demand met and reducing the distance traveled by mobile mammography units in Minas Gerais state. They proposed a hierarchical constructive heuristic algorithm, wherein a solution is considered better in the first level when it meets a greater number of screenings. If two solutions have the same number of screenings, the chosen solution is the one with the shorter distance traveled. The authors showed that in the region studied, which involves 444 cities, it would be possible to provide almost 360 thousand more mammography screenings than is currently provided in the state.

(Rosa et al., 2021) approached the MMURP through the Iterated Greedy algorithm (Stützle and Ruiz, 2018) and named it Smart IG. The Randomized Variable Neighborhood Descent (RVND) procedure is executed to refine the solution. Of the 853 cities in Minas Gerais state, 579 were analyzed. The results showed that the developed algorithm found solutions that fully meet the demand of the studied region, and these results are superior to those obtained through the constructive algorithm by its previous work (Rosa et al., 2020b).

In (de Assis et al., 2022), the authors treated the MULAP allowing the partial fulfillment of the demands for screenings of the cities. They proposed an algorithm based on the General Variable Neighborhood Search (GVNS) (Hansen et al., 2017). The initial solution is built by a procedure based on the construction phase of the GRASP metaheuristic (Festa and Resende, 2018). They also introduced a new representation for the problem solution, in which it is possible to individualize the screenings performed by each mammography device. They compared the results of the proposed algorithm with those of the Simulated Annealing by (de Campos et al., 2020) and showed that GVNS obtained better solutions in some instances.

In (de Freitas Almeida et al., 2022), the authors deal with the location-allocation of Magnetic Resonance Imaging (MRI) machines. The main objective is to maximize the coverage for MRI exams considering the new equipment acquisitions and equity in regional service supply in Brazil. They propose three mathematical programming formulations for the problem. The first one aims to maximize the demand covering; the second minimizes the cost of acquiring new MRI machines. The third and final one aims to minimize the traveled distance by the patients. Several distinct scenarios are investigated for different numbers of new MRI units acquired. The analysis of the obtained results shows that for full demand coverage, a total of 812 new MRI machines would be required, where 753 of those new MRI units would be allocated to cities with no MRI machines available. The authors also consider addressing stochastic demand components and decomposition methods as solution strategies for future work.

3 MATERIALS AND METHODS

3.1 Problem Statement

The Bi-objective Mammography Unit Location-Allocation Problem discussed here, denoted by BOMULAP, has the following characteristics:

- There is a set N of cities to be covered by mammography screenings;
- There is a set of p mammography units to be allocated to the set N;
- Each mammography unit has a capacity to perform Γ screenings annually;
- Each city j has a demand δ for mammography screenings;
- Only cities with hospital infrastructure are candidates to host mammography units;
- Each city that hosts a mammography machine can only meet the demand of cities that are no more than R km away from it;
- A city cannot be served by more than one host city;
- A host city can only serve another city if it is able to meet all of its demand.

The objectives are to maximize the coverage of mammography screenings and minimize the number of mammography units needed.

3.2 BOMULAP ILP Fomulation

Through equations (1) to (8), we introduce the bi-objective integer linear programming formulation that defines the BOMULAP. This formulation transforms into an objective function, the constraint used in (Souza et al., 2020) that limits the number of mammography units.

Initially, we introduce the parameters and decision variables of the formulation.

Model Parameters:

d_{ij} : Distance from city i to city j;
δ_j : Annual demand for mammography screenings in city j;
Γ : Annual mammography screening capacity of a mammography unit;
p : Maximum number of mammography units allowed;

R : Maximum distance for service;
ξ_i : Parameter that assumes the value of 1 if the city i has hospital infrastructure to host a mammography device and 0 otherwise;
S_{ij} : $\{(i,j) \in N \times N \mid [(d_{ij} \wedge d_{ji}) \leq R] \wedge \xi_i = 1\}$ is the adjacency set, filtered by the maximal service distance R and the infrastructure availability ξ_i.

Decision Variables:

x_{ij} : 1 if women of city j are served by an equipment located in city i and 0 otherwise, $\forall (i,j) \in S_{ij}$;
y_i : number of mammography units located in city i, $\forall i \in N \mid \xi_i = 1$.

Problem Formulation:

$$\max f_1(x) = \sum_{(i,j)\in S_{ij}} \delta_j \cdot x_{ij} \quad (1)$$

$$\min f_2(y) = \sum_{i \in N \mid \xi_i = 1} y_i \quad (2)$$

$$\text{s.t.:} \sum_{(i,j)\in S_{ij}} \delta_j \cdot x_{ij} \leq \Gamma \cdot y_i, \quad \forall i \in N \mid \xi_i = 1 \quad (3)$$

$$\sum_{(i,j)\in S_{ij}} x_{ij} \leq 1, \quad \forall j \in N \quad (4)$$

$$x_{ij} \leq x_{ii}, \quad \forall (i,j) \in S_{ij} \quad (5)$$

$$y_i \leq p \cdot x_{ii}, \quad \forall i \in N \mid \xi_i = 1 \quad (6)$$

$$x_{ij} \in \{0,1\}, \quad \forall (i,j) \in S_{ij} \quad (7)$$

$$y_i \in \mathbf{Z}^+. \quad \forall i \in N \mid \xi_i = 1 \quad (8)$$

The objective functions (1) and (2) aim at maximizing the total demand for mammography screenings and minimizing the total number of mammography units, respectively. Inequalities (3) are standard bin-packing constraints ensuring that the capacity of each mammography unit for annual screenings must be uphold. Constraints (4) indicate that each city j needs to be served by some mammography machine installed in city i if the pair (i, j) is adjacent (service-able) or not to be served at all. Constraints (5) force a mammography unit installed in city i to handle the local demand at least, also working as a strong version of the well-known fixed-charge constraints, strengthening the formulation. Inequalities (6) tie together the mammography service availability to the allocation of mammography units in a given city i. Finally, constraints (7) and (8) specify the decision variables' feasible domains. Please, recall that despite the distance matrix d_{ij} is not directly used in the formulation, this parameter is required to implement the adjacency set S_{ij}, and therefore its definition is needed for completeness.

3.3 Two Multi-Objective Competing Philosophies

In this Section, the exact methods of multi-objective optimization proposed for the solution of the BOMULAP are presented.

3.3.1 The Epsilon-Constraint Method

The Epsilon-constraint method (Ritzel et al., 1994) transforms a multi-objective problem into a mono-objective problem. One objective is chosen to be optimized, and the others are transformed into additional inequality constraints in the model.

In the model discussed in this article, the objective function $f_1(x)$ described by Eq. (1) was chosen to be maximized while the $f_2(y)$, described by Eq. (2), was added as a constraint in the model, $\sum_{i \in N} y_i \leq p$. Therefore, the formulation to be solved for every selected value of p becomes:

$$\max \quad f_1(x) = \sum_{(i,j) \in S_{ij}} \delta_j \cdot x_{ij} \tag{9}$$

Subject to (3)-(8) and:

$$\sum_{i \in N \mid \xi_i = 1} y_i \leq p \tag{10}$$

where p is then prescribed from 1 to 100 in order to cover the full demand for mammography screenings and build the desired Pareto front.

3.3.2 The Weighted Sum Method

The Weighted Sum Method (WSM) (Zadeh, 1963) consists of assigning weights to the objective functions of a multi-objective problem, thus transforming it into a single-objective problem. The sum of all weights must be equal to 1.

Hence, for this approach, the objective functions (1) and (2) are linearly combined with the aid of a weight $\lambda \in [0,1]$, resulting:

$$\max \quad \lambda \left(\frac{1}{\sum_{j \in N} \delta_j}\right) f_1(x) - (1 - \lambda) \left(\frac{1}{p}\right) f_2(y) \tag{11}$$

Function (11) is then maximized after the proper prescribing of λ, in order to yield the Pareto efficient front.

4 COMPUTATIONAL EXPERIENCE: WHY ALL THIS EXTRA BURDEN PAYS OFF

4.1 A Realistic Test-Bed Based on Real-World Problems

In order to test the two solution methods, we used the instances related to the states of Espírito Santo (ES) and Rondônia (RO) available in (Sá et al., 2019) and (Souza et al., 2019), respectively. Table 1 presents the main characteristics of these instances. In this table, the columns *State*, *nC*, *p*, δ, *R*, and Γ represent, respectively, the State of the Brazilian federation, its number of cities, the number of mammography units existing in this State, the existing demand for mammography screenings, the maximum distance allowed between an equipment host city and the cities it serves, and the annual capacity of mammography screenings for each device.

Table 1: Instance Characteristics.

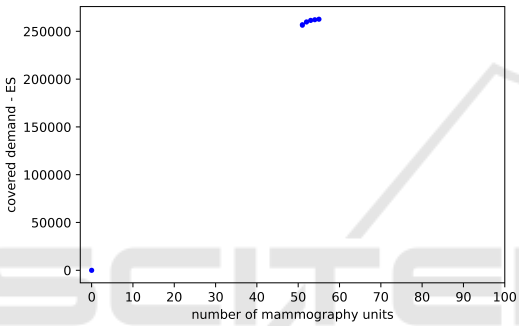
State	*nC*	*p*	δ	*R*	Γ
ES	78	30	262732	60	5069
RO	52	8	120636	60	5069

For the test environment, AMPL software with Cplex 20.1.0.0 was used to run the two solution methods proposed in the previous section. These methods were tested on a computer equipped with 1 Intel(R) Core(TM) i7-10750H CPU @ 2.60GHz (12 threads, fully utilized), 16 GB RAM and Windows 11 Home system.

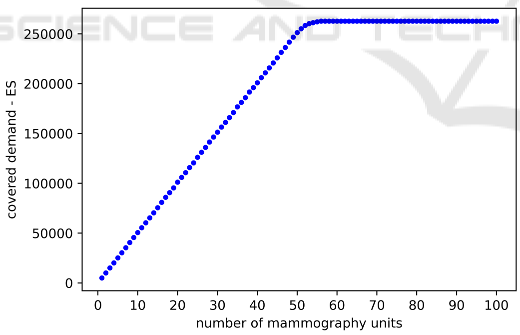
4.2 Analyzing the Obtained Results

For the generation of the Pareto front by the Weighted Sum Method, we ran the model 100 times, adding $\Delta_\lambda = 0.01$ to the value of λ at each run, starting it with the value $\lambda = 0$. In turn, to generate the Pareto front by the Epsilon-constraint method, we also ran the model 100 times for the ES and RO instances, adding in one unit the value of p at each execution and starting p with the value 1. We performed the executions until the gap of 1%. The data of the instances of the ES and RO states were obtained through the Brazilian government's website (DATASUS, 2021) and the Google Maps API, considering travel by car.

Figure 1(a) and Figure 2(a) show the Pareto fronts of BOMULAP generated by the Weighted Sum Method in the ES and RO instances, respectively. In these figures, the horizontal axes represent the number of allocated mammography units, and the vertical axes represent the total demands met. In turn, Figure 1(b) and Figure 2(b) illustrate the Pareto fronts of BOMULAP by the Epsilon-constraint method.



(a) Weighted Sum Method.

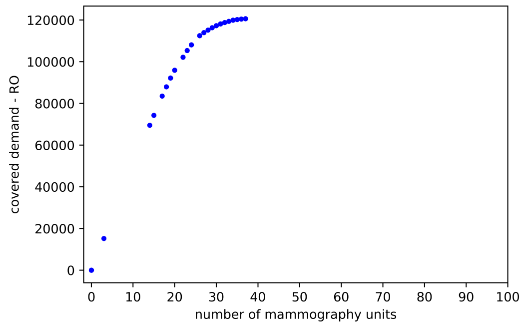


(b) Epsilon-constraint method.

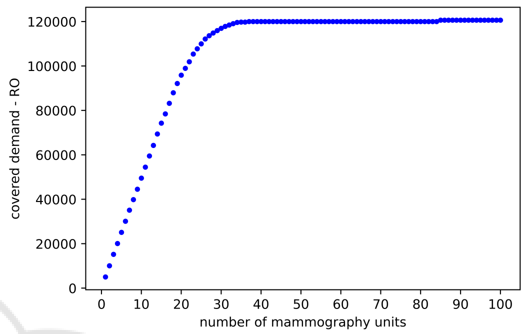
Figure 1: Pareto fronts for the ES instance.

It is observed that after a given number of installed mammography devices, increasing them is no longer worthwhile. In fact, even though there is a demand to be met, the infrastructure and maximum distance constraints prevent further improving the maximal coverage, despite the eventual availability of mammography units. An alternative to overcome this situation is integrating the MULAP with the MMURP.

Regarding the two competing multi-objective philosophies, the Weighted Sum scheme is particularly vulnerable to our way of handling the trade-offs



(a) Weighted Sum Method.



(b) Epsilon-constraint Method.

Figure 2: Pareto fronts for the RO instance.

between maximum coverage and infrastructure deployment since this scheme prefers not to spend any budget unless the concerns of unattended demands are relatively higher. On the other hand, the Epsilon-Constraint scheme can always yield some coverage, even though a small number of mammography units is allowed. Therefore, for the features of our specific application, the Epsilon-Constraint technique is certainly preferable when compared to the devised alternative since it is capable of defining the efficient front with superior resolution, favoring an enhanced decision-making.

5 INSIGHTS AND UPCOMING WORK

This paper presents the Weighted Sum and Epsilon-constraint methods for solving BOMULAP. In all instances, the Epsilon-constraint method provides better Pareto fronts than the Weighted Sum method.

The results presented in this paper can assist health managers in their decision-making, such as deciding where to relocate existing mammography devices and/or purchase new ones.

Future work intends to apply other exact methods, like the Parallel Partitioning Method (Lemesre et al., 2007), and solve instances from other Brazilian states.

Since BOMULAP is NP-hard, developing heuristic methods, such as Non-dominated Sorting Genetic Algorithm II (NSGA-II) (Deb et al., 2002), to deal with large instances of the problem is another suggestion.

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