Improved Encoding of Possibilistic Networks in CNF Using Quine-McCluskey Algorithm

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Abstract: Compiling Possibilistic Networks consists in evaluating the effect of evidence by encoding the possibilistic network in the form of a multivariable function. This function can be factored and represented by a graph which allows the calculation of new conditional possibilities. Encoding the possibilistic network in Conjunctive Normal Form (CNF) makes it possible to factorize the multivariable function and generate a deterministic graph in Decomposable Negative Normal Form (d-DNNF) whose computation time is polynomial. The challenge of compiling possibilistic networks is to minimize the number of clauses and the size of the d-DNNF graph to guarantee the lowest possible computation time. Several solutions exist to reduce the number of CNF clauses. We present in this paper several improvements for the encoding of possibilistic networks. We will then focus our interest on the use of Quine-McCluskey's algorithm (QMC) to simplify and reduce the number of clauses.

1 INTRODUCTION

Compiling possibilistic networks is inspired by the approaches used to compile Bayesian networks. Indeed, a large amount of research has already been done since the end of the 90s (Darwiche, 2003; Darwiche, 2004; Darwiche and Marquis, 2002). The goal of these works generally consists in the encoding of a Bayesian network into a multilinear function before the generation of a circuit with a minimal size. The evidence is then inserted in the circuit and the new conditional probabilities are computed. We can use the same reasoning with possibilistic networks although the theory of possibility is different from the theory of probability. These differences require adaptations of the existing solutions.

There are few papers that deal with the compiling of possibilistic networks. N. B. Amor *et al.* (Raouia et al., 2010) were the first to experiment with the compilation of qualitative possibilistic networks. The possibilistic network was encoded by using a conjunctive normal form, then a d-DNNF graph was generated. Evidence was introduced in the graph then an algorithm was applied to compute new conditional possibilities. This approach was compared to a new approach based on possibilistic logic. The results highlighted were better for the second approach. We proposed our solution to compile a possibilistic network by using the compiling of the junction tree. To do this we computed the junction tree and we generated a circuit which alternates the clusters and separators. Evidence is introduced in the circuit before applying an algorithm with two passes. An upward pass from the leaves to the root and a downward pass from the root to the leaves. This solution allows us to compute all conditional possibilities.

We propose in this paper several improvements of the encoding of the possibilistic network into CNF. Indeed, these improvements are not included in the first paper (Raouia et al., 2010), they allow us to strongly reduce the number of clauses during the encoding of the possibilistic network into CNF. The improvements of Bayesian networks have inspired our research such as determinism, Context-Specific Independence (CSI), ... For CSI we propose to use Quine-McCluskey (QMC) (McCluskey, 1956; Chavira and Darwiche, 2006) algorithm to reduce the number of clauses. This algorithm is used to find prime implicants and to minimize a Boolean function such as Karnaugh maps. Karnaugh maps are easier to develop and faster. Nevertheless, they are less efficient when the number of variables of the function grows. We have preferred QMC for this reason.

Afterwards, we will present possibilistic networks and the compiling of possibilistic networks by using CNF. We will present the improvements to reduce the

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number of clauses. Then we will study the algorithm of QMC and the use of this algorithm to reduce the number of clauses. In the last section, we will analyse the evaluation of the improvements.

2 POSSIBILISTIC NETWORKS

The theory of possibility proposed by L. A. Zadeh in 1978 (Zadeh, 1978) is an extension of the fuzzy sets theory that allows us to represent imprecise and uncertain knowledge. Indeed, the fuzzy sets theory only deals with imprecise knowledge. If we consider a universe Ω, we can define a possibility distribution π of the set of parts of Ω noted $P(\Omega)$ in $[0,1]$. If $\pi(x)=0$ for an event x, then the event is impossible. On the other side if $\pi(x)=1$ then the event is possible. The possibility of the disjunction of several events is the maximum of the possibilities of the events. Two measures are fundamental in this theory: the measure of possibility and the measure of necessity (certainty).

Possibilistic networks (Benferhat et al., 1999; Borgelt et al., 2000) are adaptations of Bayesian networks to the theory of possibility. There are two families of possibilistic networks: qualitative possibilistic networks based on the minimum and quantitative possibilistic networks based on the product. We use qualitative possibilistic networks if we are interested in the order of the state of the variables and we use quantitative possibilistic networks if we want to compare quantitative values such as Bayesian networks. Several conditioning operators exist in possibility theory. In this research, we will use the operator of conditioning proposed by D. Dubois and H. Prade (Dubois and Prade, 1988):

$$\Pi(A|B)=\begin{cases}\Pi(A,B) & \text{if } \Pi(A,B)<\Pi(B),\\ 1 & \text{if } \Pi(A,B)=\Pi(B).\end{cases} \tag{1}$$

We can define a possibilistic network as follows:

Definition 2.1. If $G=(V,E,\Pi)$ is a directional acyclic graph where V is the set of variables of the graph, E the set of edges and Π the set of possibility distributions of the variables, then we have the factoring property:

$$\Pi(V)=\bigotimes_{X\in V}\Pi(X/U). \tag{2}$$

U represents the parents of the variable X. The operator $\bigotimes$ is the minimum or the product. The Conditional Possibility Table (CPT) of the variable X given the parent variables U is a tabular which represents all the configurations of the variables and the conditional possibilities $\Pi(X/U)$.

3 ENCODING THE POSSIBILISTIC NETWORK INTO CNF

A possibilistic network can be represented by a multivariable function noted f whose variables are of two categories: the indicator variables and parameter variables. For all states of a variable $X=x$ we have an indicator variable λ_x. We have for each parameter of a CPT $\pi(X|U)$ a parameter variable $\theta_{x|u}$ where u is an instantiation of U that represents the parents of the variable X and x is an instantiation of X. The multivariable function f gathers in one expression all the instantiations of the variables. It makes easier the evaluation of evidence and the computation of queries to retrieve the new values of the conditional possibilities.

Definition 3.1. If P is possibilistic network, $V=v$ an instantiation of the variables, $U=u$ an instantiation of the parents of the variable X and $X=x$ an instantiation of X, then the multivariable function f of P is:

$$f=\bigoplus_{v}\bigotimes_{xu\sim v}\lambda_x\otimes\theta_{x|u} \tag{3}$$

In the above formula, $xu\sim v$ represents an instantiation of the variable X and its parents U compatible with the instantiation v. The operator $\bigoplus$ is the function maximum and $\bigotimes$ is the function minimum or the product. In this research, we will use for $\bigotimes$ the function minimum because we are interested only in qualitative possibilistic networks.

We present in the following table 1 an example of a possibilistic network:

Table 1: Example: $A\rightarrow B$.

A	B	
true	true	1 ($\theta_{b\|a}$)
true	false	0.2 ($\theta_{\bar{b}\|a}$)
false	true	0.1 ($\theta_{b\|\bar{a}}$)
false	false	1 ($\theta_{\bar{b}\|\bar{a}}$)

A	
true	1 (θ_a)
false	0.1 ($\theta_{\bar{a}}$)

The multivariable function f of the possibilistic network is as follows:

$$f=\lambda_a\otimes\lambda_b\otimes\theta_a\otimes\theta_{b|a}\oplus\lambda_a\otimes\lambda_{\bar{b}}\otimes\theta_a\otimes\theta_{\bar{b}|a}\\ \oplus\lambda_{\bar{a}}\otimes\lambda_{\bar{b}}\otimes\theta_{\bar{a}}\otimes\theta_{\bar{b}|\bar{a}}\oplus\lambda_{\bar{a}}\otimes\lambda_b\otimes\theta_{\bar{a}}\otimes\theta_{b|\bar{a}} \tag{4}$$

With $\oplus$ the maximum and $\otimes$ the minimum. We can generate the circuit associated with the multivariable function f called MIN-MAX circuit because of the choice we made for the operators:

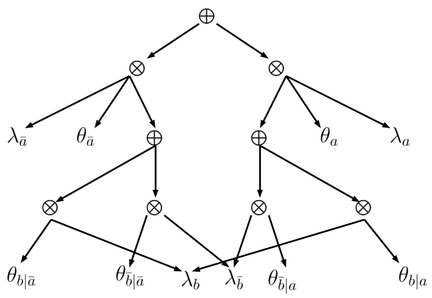


Figure 1: MIN-MAX circuit of the example.

This first approach is not sufficient to simplify the multivariable function. However, the factoring and the simplification of the function can reduce the size of the circuit and therefore the computation time for inference. The solution proposed by A. Darwiche in (Darwiche, 2002) consists in encoding the Bayesian network by using propositional logic into CNF. Firstly we associate an indicator variable with each state of the variables, then we create parameter variables for all CPTs. The encoding of the possibilistic network is performed by creating the clauses of CNF.

There are two main steps in the process of encoding:

1. The first one consists in encoding the state of the variables as follows:
   (a) For all variables X of the possibilistic network with states $x_1,...,x_n$ we create the following clauses:
   $$\lambda_{x_1} \vee ... \vee \lambda_{x_n} \tag{5}$$
   (b) Then we generate the clauses:
   $$\begin{array}{c} \lambda_{x_1} \vee \lambda_{x_2} \\ \lambda_{x_1} \vee \lambda_{x_3} \\ \vdots \\ \lambda_{x_{n-1}} \vee \lambda_{x_n} \end{array} \tag{6}$$
2. The second step consists in encoding the parameters of the CPT.
   (a) For all parameters of a CPT we generate the clauses that encompass the indicators and the parameter as follows:
   $$\lambda_{x_i} \wedge \lambda_{y_j} \wedge ... \wedge \lambda_{z_l} \rightarrow \theta_{x_i|y_j,...,z_l} \tag{7}$$
   (b) Then we create for each indicator the clauses with the parameter and the indicator as follows:
   $$\begin{array}{c} \theta_{x_i|y_j,...,z_l} \rightarrow \lambda_{x_i} \\ \theta_{x_i|y_j,...,z_l} \rightarrow \lambda_{y_j} \\ \vdots \\ \theta_{x_i|y_j,...,z_l} \rightarrow \lambda_{z_l} \end{array} \tag{8}$$

Several improvements are possible, firstly if a parameter is null, it can be omitted. To do this, we replace step 2 with only one clause $\neg\lambda_{x_i} \vee \neg\lambda_{y_j} \vee ... \vee \neg\lambda_{z_l}$. Another improvement is to use only one variable in the encoding for the same value of parameters in a CPT. It was demonstrated by (Chavira and Darwiche, 2005) that it is possible in this case to avoid the generation of the clauses of step 2(b).

We can generate the Boolean circuit in d-DNNF before its transformation into a MIN-MAX circuit. A recursive algorithm presented in (Petiot, 2021) allows us to compute the new conditional possibilities for all the variables of the possibilistic network. As for the Bayesian network (Chavira and Darwiche, 2006), we were also interested in the simplification of the clauses for the same value of a parameter. The objective was to remove in the clauses the variables that are not useful, those for which a change doesn't affect the parameter. This context-specific independence represents independences that are true only for some contexts. We can use this property to improve the computation time and reduce the size of the d-DNNF graph. Indeed, the authors of (Chavira and Darwiche, 2006) demonstrate empirically and analytically that search algorithms used to generate the d-DNNF graph provide often better results after this improvement. We propose to use the QMC algorithm usually used to simplify Boolean functions.

4 IMPROVEMENT OF THE ENCODING IN CNF WITH THE QMC ALGORITHM

Quine Mc-Clukey algorithm was proposed in 1956 (McCluskey, 1956) to minimize a Boolean function represented by binary coding. This algorithm has two main steps. The goal of the first step is to find all implicants also called generators. We perform several groups of terms defined by the number bits equal to one in the terms, then we combine the terms of each adjacent group to find new implicants. For example, if the have the term 0000 in the first group and the term 0100 in the second group this generates the implicant 0X00. The X represents the changing bit. We continue the iteration with the groups until there is no change in the groups. Then we pass to the second step. In this step, we eliminate all implicants which are not prime implicants.

We have generalized the QMC algorithm to variables with more than two states. The states of a variable are encoded by using the number of bits defined by the formula $log(\frac{|A|}{log2})$ where $|A|$ is the number of states of the variable A. We perform a processing of all prime implicants for non-Boolean variables to merge generators. This algorithm allows us to simplify the clauses and also to reduce the number of clauses during the encoding of the possibilistic networks. We present the following example to illustrate this improvement of the encoding:

Table 2: CPT with 3 variables.

A	B	C	$\pi(A\|B,C)$
a_1	b_1	c_1	1 ($\boldsymbol{\theta_1}$)
a_1	b_1	c_2	1 ($\boldsymbol{\theta_1}$)
a_1	b_2	c_1	1 ($\boldsymbol{\theta_1}$)
a_1	b_2	c_2	0.2 (θ_2)
a_2	b_1	c_1	0 (nothing)
a_2	b_1	c_2	0.6 (θ_3)
a_2	b_2	c_1	1 ($\boldsymbol{\theta_1}$)
a_2	b_2	c_2	0.5 (θ_4)
a_3	b_1	c_1	0.3 (θ_5)
a_3	b_1	c_2	0.6 (θ_3)
a_3	b_2	c_1	1 ($\boldsymbol{\theta_1}$)
a_3	b_2	c_2	1 ($\boldsymbol{\theta_1}$)

The first encoding consists in using only one variable for all identical parameters in the CPT. We can also reduce the number of clauses for parameters equal to zero. We obtain the following result:

Table 3: Encoding in CNF of the CPT.

$$\lambda_{a_1} \wedge \lambda_{b_1} \wedge \lambda_{c_1} \rightarrow \theta_1$$
$$\lambda_{a_1} \wedge \lambda_{b_1} \wedge \lambda_{c_2} \rightarrow \theta_1$$
$$\lambda_{a_1} \wedge \lambda_{b_2} \wedge \lambda_{c_1} \rightarrow \theta_1$$
$$\lambda_{a_1} \wedge \lambda_{b_2} \wedge \lambda_{c_2} \rightarrow \theta_2$$
$$\neg\lambda_{a_2} \vee \neg\lambda_{b_1} \vee \neg\lambda_{c_1}$$
$$\lambda_{a_2} \wedge \lambda_{b_1} \wedge \lambda_{c_2} \rightarrow \theta_3$$
$$\lambda_{a_2} \wedge \lambda_{b_2} \wedge \lambda_{c_1} \rightarrow \theta_1$$
$$\lambda_{a_2} \wedge \lambda_{b_2} \wedge \lambda_{c_2} \rightarrow \theta_4$$
$$\lambda_{a_3} \wedge \lambda_{b_1} \wedge \lambda_{c_1} \rightarrow \theta_5$$
$$\lambda_{a_3} \wedge \lambda_{b_1} \wedge \lambda_{c_2} \rightarrow \theta_3$$
$$\lambda_{a_3} \wedge \lambda_{b_2} \wedge \lambda_{c_1} \rightarrow \theta_1$$
$$\lambda_{a_3} \wedge \lambda_{b_2} \wedge \lambda_{c_2} \rightarrow \theta_1$$

We can use the QMC algorithm to simplify the clauses generated for parameter θ_1 in bold in table 2. We can see in the above table that the variable A isn't Boolean. This variable has 3 states. We need two bits to encode the three states of the variable. We propose the coding 00 pour a_1, 01 for a_2, 10 for a_3. The variables B and C require only one bit as they are Boolean: 0 for b_1, 1 for b_2, 0 for c_1, 1 for c_2. We obtain the following result:

Table 4: Encoding of the variables for θ_1.

A	B	C	Binary coding
00	0	0	0000
00	0	1	0001
00	1	0	0010
01	1	0	0110
10	1	0	1010
10	1	1	1011

We can associate an identifier to each binary coding. If we apply the QMC algorithm, we obtain:

Table 5: Step n°1 - combinations of terms.

Identifiers	Column 1
0	0000
1	0001
2	0010
6	0110
10	1010
11	1011

Combinations	Column 2
0,1	000X
0,2	00X0
2,6	0X10
2,10	X010
10,11	101X

Then, we must extract the prime implicants:

Table 6: Step n°2 - find prime implicants.

Implicants \ Identifiers	0	1	2	6	10	11
000X	x	**X**	-	-	-	-
00X0	x	-	x	-	-	-
0X10	-	-	x	**X**	-	-
X010	-	-	x	-	x	-
101X	-	-	-	-	x	**X**

In the previous table, we must look for columns with only one x. The line associated with the x of each selected column gives us prime implicants. For example, columns 1, 6, and 11 have only one x, then we can deduce the prime implicants $000X$, $0X10$, $101X$. The other implicants are redundant, they don't provide more information. Therefore, we must perform a post-processing linked to the number of states of the variables. The variable A has three states and we can see in prime implicants $0X10$ and $101X$ that the variable A has the values $0X$, and 10. X replaces 0 and 1. We can deduce that we have all states of the variable A so we can change $0X10$ by $XX10$. The clauses generated are as follows:

Table 7: Encoding improvement by using prime implicants for parameter θ_1.

Conditions	Prime implicants	Clauses
$\lambda_{a_1} \wedge \lambda_{b_1} \wedge \lambda_{c_1}$	$\lambda_{a_1} \wedge \lambda_{b_1}$	$\lambda_{a_1} \wedge \lambda_{b_1} \rightarrow \theta_1$
$\lambda_{a_1} \wedge \lambda_{b_1} \wedge \lambda_{c_2}$	$\lambda_{b_2} \wedge \lambda_{c_1}$	$\lambda_{b_2} \wedge \lambda_{c_1} \rightarrow \theta_1$
$\lambda_{a_1} \wedge \lambda_{b_2} \wedge \lambda_{c_1}$	$\lambda_{a_3} \wedge \lambda_{b_2}$	$\lambda_{a_3} \wedge \lambda_{b_2} \rightarrow \theta_1$
$\lambda_{a_2} \wedge \lambda_{b_2} \wedge \lambda_{c_1}$		
$\lambda_{a_3} \wedge \lambda_{b_2} \wedge \lambda_{c_1}$		
$\lambda_{a_3} \wedge \lambda_{b_2} \wedge \lambda_{c_2}$		

Finally, we obtain the following encoding for the CPT:

Table 8: Final result of the encoding of the CPT.

$$\begin{gathered}
\lambda_{a_1} \wedge \lambda_{b_1} \rightarrow \theta_1 \\
\lambda_{b_2} \wedge \lambda_{c_1} \rightarrow \theta_1 \\
\lambda_{a_3} \wedge \lambda_{b_2} \rightarrow \theta_1 \\
\lambda_{a_1} \wedge \lambda_{b_2} \wedge \lambda_{c_2} \rightarrow \theta_2 \\
\neg\lambda_{a_2} \vee \neg\lambda_{b_1} \vee \neg\lambda_{c_1} \\
\lambda_{a_2} \wedge \lambda_{b_1} \wedge \lambda_{c_2} \rightarrow \theta_3 \\
\lambda_{a_2} \wedge \lambda_{b_2} \wedge \lambda_{c_2} \rightarrow \theta_4 \\
\lambda_{a_3} \wedge \lambda_{b_1} \wedge \lambda_{c_1} \rightarrow \theta_5 \\
\lambda_{a_3} \wedge \lambda_{b_1} \wedge \lambda_{c_2} \rightarrow \theta_3
\end{gathered}$$

5 EXPERIMENTATION

To evaluate the improvements proposed for the encoding of possibilistic networks into CNF we used several existing Bayesian networks. Indeed, there is unfortunately no testing set available for possibilistic networks, so we proposed to transform several Bayesian networks into possibilistic networks to perform the evaluation. Another solution would be to generate random possibilistic networks but we prefer to use existing datasets. For this research, we used the Bayesian networks: Asia (Lauritzen and Spiegelhalter, 1988), Earthquake and Cancer (Korb and Nicholson, 2010), Survey (Scutari and Denis, 2014), Alarm (Beinlich et al., 1989), HEPAR II (Onisko, 2003), Child (Spiegelhalter and Cowell, 1992) and WIN95PTS.

We present in the following table a description of the Bayesian networks:

Table 9: Bayesian networks description.

Networks	Nodes	Edges	Param.	Size	Type
Cancer	5	4	10	Small	SC
Earthquake	5	4	10	Small	SC
Asia	8	8	18	Small	MC
Survey	6	6	21	Small	MC
Child	20	25	230	Medium	MC
Alarm	37	46	509	Medium	MC
Win95PTS	76	112	574	Large	MC
Hepar II	70	123	1453	Large	MC

In the first column, we have the name of the Bayesian network, then the number of nodes, the number of edges, the number of parameters, the size and the type of a network. Indeed, the Bayesian networks are classified into 3 categories: small if they have fewer than 20 nodes, medium if they have between 20 and 50 nodes and large if they have more than 50 nodes. A possibilistic network singly connected (SC) or a polytree is a directed acyclic graph whose non-oriented graph is a tree. The non-oriented graph of a possibilistic network multiply connected (MC) has several cycles.

We have converted the conditional probability tables of these Bayesian networks into conditional possibility tables. The structures of the graphs are the same.

The conversion of conditional probability tables into conditional possibility tables is a well-known problem and solutions have been proposed by researchers. One of the existing solution was proposed by D. Dubois *et al.* in (Dubois et al., 1993; Dubois et al., 2004). This solution guarantees the respect of the fundamental property $\Pi \geq P$. Indeed, the solution that consists in performing a normalisation of the probabilities to define the possibility values is not compatible with the previous property. The solution proposed by D. Dubois *et al.* to compute possibilities by using probability is as follows:

$$\pi_i = \begin{cases} 1 & \text{if } i = 1, \\ \sum_{j=i}^{n} p_j & \text{if } \pi_{i-1} > \pi_i, \\ \pi_{i-1} & \text{else.} \end{cases} \tag{9}$$

We can now convert the Bayesian networks into possibilistic networks to evaluate all improvements of the encoding in CNF. The computer used for the experimentation has a I5-8250U processor with 8 Go of RAM and an OS Windows 11. We used the tool c2d of professor A. Darwiche (Darwiche, 2001; Darwiche, 2004; Darwiche and Marquis, 2002) with arguments "-dt_method 0 -dt_count 25". This tool processes a set of clauses and generates a minimized d-DNNF graph. We used the generated graph to perform the inference after the injection of evidence in the d-DNNF graph. To do this, we apply a recursive algorithm proposed in (Petiot, 2021) to compute all conditional possibilities of the variables. This algorithm has two steps and requires two registers per node of the graph noted u and d. The first step consists in evaluating each node from the leaves to the root to fill the registers u. The second step is from the root to the leaves and allows us to compute the registers d. The algorithm is as follows:

1. Upward-pass
   (a) Compute the value of the node v from the leaves to the root and store it in register $u(v)$;
2. Downward-pass
   (a) Initialization: if v is the root node then initialize the register $d(v) = 1$ else set $d(v) = 0$;
   (b) Compute register d: for each parent p of the node v compute the register $d(v)$ as follows:
      i. if p is a node $\oplus$:

$$d(v) = d(v) \bigoplus d(p) \tag{10}$$

      ii. if p is a node $\otimes$:

$$d(v) = d(v) \bigoplus \left[d(p) \otimes \left[\bigotimes_{i=1}^{n} u(v_i^p) \right] \right] \tag{11}$$

The nodes v_i^p are the other children of p.

We have performed several improvements to avoid computing all operations. For example, during the upward pass, if we have a parent node of type $\otimes$ with a register $u(v) = 0$, then we avoid the computing of the other children because the value of the register u will not change. We can do the same improvement for the operator $\oplus$.

Another improvement can be performed if there are for the same parent two children with a register $u(v) = 0$ we define a flag true for the parent node. During the downward pass we take into account this new flag and if the node is a product node, we avoid the step ii.

The efficiency of this approach resides in the generation of a d-DNNF graph only once during the offline phase. The d-DNNF graph is not computed again during the online phase.

We present in table 10 for each possibilistic network and each improvement: the number of clauses, the number of nodes of the generated d-DNNF graph, the number of edges of the generated d-DNNF graph, the offline execution time and the online execution time. The solution $S0$ is the initial solution without any improvement, the solution $S1$ represents the solution that takes into account the parameters equal to zero ($\theta = 0$). The solution $S2$ uses only one variable for parameters with the same value. Finally, the solution $S3$ is the solution that exploits the context-specific independence by using Quine-McCluskey algorithm. The computation times are in seconds (s).

Table 10: Results of the encoding in CNF.

	Networks	Clauses	Nodes	Edges	Comp. (s)	Offline (s)	Online (s)
S0	Earthquake	74	101	164	0.150	0.178047	0.000108
	Cancer	74	103	168	0,159	0,190165	0,000119
	Asia	136	197	368	0,305	0,338114	0,000216
	Survey	146	205	416	0,302	0,335912	0,000277
	Child	1294	3846	16628	1.703	1.979594	0.036785
	Alarm	3443	16852	64561	3.622	6.260184	0.657594
	WIN95PTS	7848	18821	55859	11.668	24.418201	4.667845
	Hepar II	13511	48649	162392	10.988	31.373391	1.894660
S1	Earthquake	74	101	164	0.164	0.196493	0.000099
	Cancer	74	105	170	0,167	0,200351	0,000109
	Asia	124	176	326	0,293	0,327516	0,000192
	Survey	146	205	414	0,319	0,350377	0,000248
	Child	1288	3513	15936	1.910	2.102726	0.036518
	Alarm	3428	11899	40877	4.470	5.957962	0.463513
	WIN95PTS	6368	14652	49933	8.821	12.522713	1.986387
	Hepar II	13511	19892	82789	7.154	11.085583	**0.327567**
S2	Earthquake	30	75	108	0.067	0.097955	0.000084
	Cancer	30	110	159	0,066	0,099515	0,000093
	Asia	48	174	287	0,142	0,182561	0,000152
	Survey	53	216	382	0,115	0,147993	0,000193
	Child	437	1604	3536	0.670	0.738978	0.008184
	Alarm	895	4788	10823	1.536	2.163117	0.432953
	WIN95PTS	1300	14338	32590	1.128	5.918466	3.779214
	Hepar II	2329	43363	99974	3.491	18.122137	1.834362
S3	Earthquake	28	73	102	0.055	0.091184	**0.000078**
	Cancer	27	108	152	0,064	0,093174	**0,000087**
	Asia	48	171	279	0,174	0,208398	**0,000124**
	Survey	43	208	355	0,086	0,118943	**0,000192**
	Child	404	1553	3350	0.797	0.885231	**0.007140**
	Alarm	712	4600	10689	1,562	2.290760	**0.116903**
	WIN95PTS	763	6074	14805	1.152	32.588744	**0.473409**
	Hepar II	1836	33534	74279	2.229	16.288390	1.102634

We propose the following graph to compare the number of clauses for all improvements:

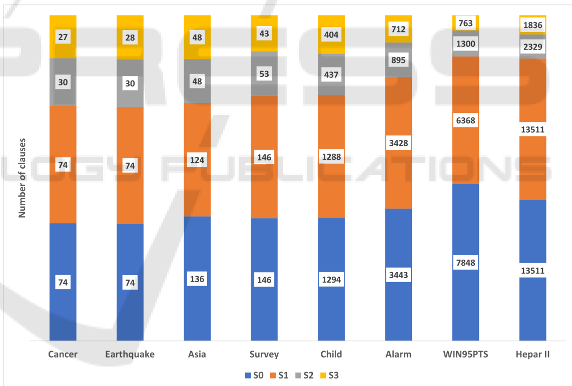


Figure 2: Comparison of the number of clauses.

In this graph, we can see the difference of the number of clauses between solution $S0$ and solution $S3$. The clauses are drastically reduced. We can see the improvement of each network in the following graph:

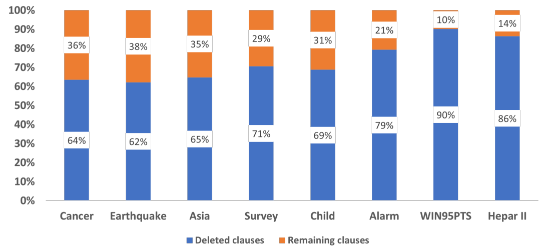
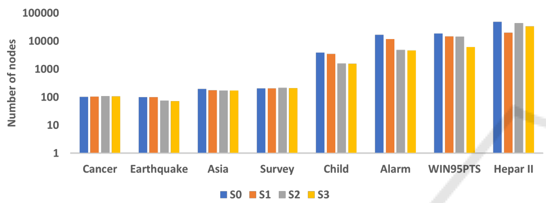
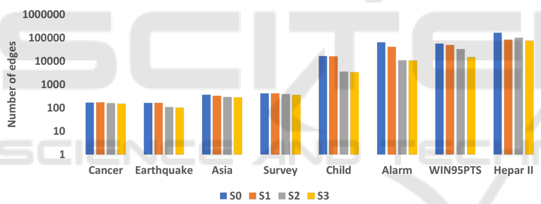


Figure 3: Percentage of improvement for the encoding between solutions $S0$ and $S3$.

We can see that the improvement reaches sometimes 90% of deleted clauses with only 10% of remaining clauses. We also present the number of nodes and edges of the d-DNNF graph generated by using the c2d tool:



(a) Comparison of the number of nodes.



(b) Comparison of the number of edges.

Figure 4: Comparison of the d-DNNF graph.

The results are better with solution S3 (QMC). The number of clauses is significantly reduced. As a result the graph generated has often fewer nodes and fewer edges. We compared the c2d computation time in the global offline computation time. We obtain the following graph:

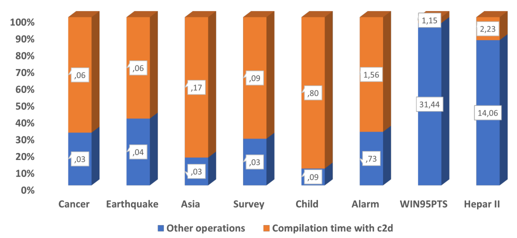


Figure 5: Comparison of c2d computation time and the offline computation time in seconds.

The above figure shows that the computation time of c2d is very important compared to the offline computation time for small and medium networks. That means that the choice of the compiling tool is very important for offline computation.

Then we have compared the computation time for online inference with two other approaches: the compiling of the junction tree of a possibilistic network (CJT) (Petiot, 2021) and the message-passing (MP) algorithm adapted to possibilistic networks (Petiot, 2018).

Table 11: Comparison of computation time in seconds.

Networks	S3 (s)	Cluster	Size	CJT (s)	MP (s)
Asia	**0,000124**	5	3	0,000186	0,005246
Cancer	**0,000087**	3	3	0,000092	0,001322
Earthquake	**0,000078**	3	3	0.000083	0,001463
Survey	**0,000192**	3	3	0,000212	0,002166
Child	**0.007140**	16	4	0,014618	0,011534
Alarm	0.116903	26	5	3,293882	**0,024094**
Win95PTS	0.473409	50	9	4,072174	**0,120225**
Hepar II	1.102634	57	7	0,293119	**0,084800**

We present a more synthetic view of the above results in the following graph. We show for all possibilistic networks the computation time and the number of parameters.

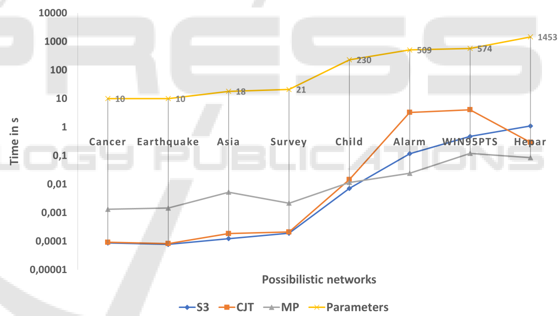


Figure 6: Comparison of the computation time and network size.

Our evaluation shows that the solution S3 provides the best results for small and sometimes medium possibilistic networks. The best results for large possibilistic networks are obtained by using message-passing algorithm. The solution S3 provides globally better results than the compiling of a junction tree (CJT). Other improvements are possible. For example, A. Darwiche proposes the reduction of the number of clauses by using *eclauses* (Chavira and Darwiche, 2005) or the deletion during the encoding of clauses 2(a) and 2(b) if the variable has no parents.

6 CONCLUSION

In this research, we have experimented several solutions to reduce the number of clauses during the compiling of a possibilistic network. We used the QMC algorithm to take into account the context-specific independence. As a result, we have simplified the clauses, even reduced the number of clauses. The use of c2d tool for the first allows us to generate minimized d-DNNF graph. The goal was to obtain the smallest possible graph to ensure an optimal computation time.

The proposed approach significantly reduces the number of clauses as well as the computation time during the encoding of the possibilistic networks. The online computation time depends on the quality of the compilation tool used to generate the d-DNNF graph. Our assessment of inference is satisfactory for small possibilistic networks. In our future works, we would like to improve our approach for large networks. We would like to compare new compiling tools such as ACE, DSHARP, and D4. We also wish to evaluate at least two other approaches for compiling possibilistic networks, the first being the logarithmic encoding of variables and the second the method of factors. Then we will evaluate quantitative possibilistic networks.

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