A Robust Optimization for a Single Operating Room Scheduling Problem with Uncertain Durations

Yoshito Namba[1], Mari Ito[2] and Ryuta Takashima[1][a]

[1]Department of Industrial Administration, Tokyo University of Science, Chiba, Japan

[2]Center for Mathematical and Data Sciences, Kobe University, Hyogo, Japan

Keywords: Operating Room, Robust Optimization, Scheduling, Surgical Management.

Abstract: In order to improve the quality of patient care, efficient surgical management is significant for overall hospital management. This study proposes a robust optimization model that minimizes delay in surgery by taking the surgical sequence into account. We verified an influence of the risk-averse tendency on the schedule. In the numerical analysis, the schedule created by the robust optimization model was compared with that of the stochastic programming model. The results suggest that robust optimization models tend to avoid long delays.

1 INTRODUCTION

Efficient surgical management is important for the quality of patient care and hospital management. The quality of patient care is affected because of the long waiting time of patients owing to the delay from the scheduled end time of surgery. In terms of hospital management, surgeries account for most of the hospital revenue and expenditure (Jackson, 2002; Macario et al.; 1995). Therefore, an operating room schedule is created to improve its operating rate and reduce the cost of surgery.

In the scheduling flow of the operating room, the surgeon and patient decide the surgery date through mutual agreement. The surgeon then reports the estimated duration of surgery to the operating room manager. The manager decides when and in which operating room to perform the surgery, based on information such as the estimated duration of surgery. However, there is uncertainty regarding the duration of the surgery. Factors of uncertainty include the patient's condition, lack of information on the preoperative diagnosis, and the surgeon's skill. Surgery is often not performed according to the scheduled end time based on the reported duration. In addition, there may be a risk of delay, with surgery being delayed significantly from the scheduled end time. Long delays lead to increased overtime for surgical staff, not only increasing costs, but also reducing staff satisfaction. Therefore, to manage the operating room efficiently, robust scheduling that considers the uncertainty of the surgical duration is required. In the operating room scheduling, it is necessary to consider decision-making to avoid the risk of delay.

Operating room scheduling has been studied extensively (Cardoen et al., 2010; Gerchak et al., 1996; Lamiri et al., 2008). For example, Addis et al. (2016) proposed the operating room rescheduling by considering the uncertainty of patient arrival and the duration of surgery. Ito et al. (2019) formulated a single operating room scheduling problem that considers the uncertainty of the surgical duration. A risk measure called conditional value-at-risk (CVaR) was used to reflect the tendency toward delayed risk aversion. Another technique that reflects this scheduling trend is robust optimization. Aslani et al. (2021) proposed a robust optimization model with a radix constraint for the first-time and repeat patients in urology, considering the risk of a significant increase in the arrival of a number of first-time patients. Shi et al. (2019) formulated a robust optimization model for a home health care routing and scheduling problem with considering uncertain travel and service times. The authors compared the solutions obtained by the stochastic programming model and the robust optimization model. Denton et al. (2010) proposed an operating room scheduling model with robust optimization to address the

[a] https://orcid.org/0000-0001-5590-5008

Namba, Y., Ito, M. and Takashima, R.
A Robust Optimization for a Single Operating Room Scheduling Problem with Uncertain Durations.
DOI: 10.5220/0011733600003396
In *Proceedings of the 12th International Conference on Operations Research and Enterprise Systems (ICORES 2023)*, pages 180-184
ISBN: 978-989-758-627-9; ISSN: 2184-4372

uncertainty of the surgical duration. However, the previous study did not consider the sequence of surgeries in the operating room. When scheduled, in practice, it is necessary to consider the sequence of surgeries within the operating room, because it is more convenient to perform surgeries belonging to the same department consecutively when arranging surgical equipment and adjusting schedules.

In this study, we propose a robust optimization model that considers the sequence of surgeries and minimizes the delay. In the numerical analysis, the delay was calculated for uncertain surgical duration parameter sets. We compared it with a stochastic programming model to verify whether the risk-averse tendency is reflected in the schedule.

2 MATHMATICAL MODEL

2.1 Single Operating Room Scheduling

We propose a robust optimization model for the single-operating-room scheduling problem under uncertain parameter sets, the worst-case that results in maximum total surgical duration. Single operating room scheduling determines the procedures for surgeries in an operating room. The operating room scheduling model and the formulation of the maximum surgical duration problem, which is considered the main problem, is presented below. Note that the stochastic programming model is the model from Ito et al. (2022).

Notation

Index Sets

J: Set of surgeries.

D: Set of departments.

E_d: Set of surgeries belonging to the same department d, $d \in D$.

Parameters

w_j: Weight of surgery j, $j \in J$.

d_j: The time from the operating room opening to the time when surgery j should be completed, $j \in J$.

$\overline{p}_j, \underline{p}_j$: Upper and lower bounds on the duration of surgery j, $j \in J$.

τ : Constant control conservative. Set how conservatively you want to control the worst-case scenario from the decision -maker's perspective. This represents the number of surgeries for which the upper bound of the surgical duration is reached.

Variables

p_j: Duration of surgery j, $j \in J$.

c_j: Finishing time of surgery j, $j \in J$.

t_j: Delay in surgery j from the expected end time, $j \in J$.

z_{ij}: Surgery precedence binary variable, where $z_{ij} = 1$ if surgery i is processed before surgery j, $z_{ij} = 0$ otherwise, $i, j \in J, i \neq j$.

α, β_j: dual variables, $j \in J$.

Formulation

$$\text{Minimize} \sum_{j \in J} w_j t_j \tag{1}$$

Subject to

$$\sum_{i \in J \setminus \{j\}} p_i z_{ij} + p_j \leq c_j, \forall j \in J, \tag{2}$$

$$t_j + d_j \geq c_j, \forall j \in J, \tag{3}$$

$$z_{ij} + z_{ji} = 1, \forall i \neq j \in J, \tag{4}$$

$$z_{ij} + z_{jk} + z_{ki} \leq 2, \forall i \neq k \neq j \in J, \tag{5}$$

$$\left| \sum_{j \in J} z_{ij} - \sum_{j \in J} z_{i'j} \right| = 1, \quad \forall i \neq i' \in E_d, \forall d \in D, \tag{6}$$

$$\sum_{j \in J} \left(p_j - \underline{p_j} \right) \geq \alpha \tau + \sum_{j \in J} \left(\overline{p_j} - \underline{p_j} \right) \beta_j, \tag{7}$$

$$\frac{1}{\overline{p_j} - \underline{p_j}} \alpha + \beta_j \geq 1, \forall j \in J, \tag{8}$$

$$\overline{p}_j \leq p_j \leq \underline{p_j}, \forall j \in J, \tag{9}$$

$$\alpha, \beta_j, c_j, t_j \geq 0, \ \forall j \in J, \tag{10}$$

$$z_{ij} \in \{0, 1\}, i \neq j \in J. \tag{11}$$

In the formulation above, the objective function (1) minimizes the delay in surgery j from the expected end time. Constraint (2) defines the surgery completion time according to the surgery sequencing relationships. Constraint (3) determines the delay in surgery. Constraints (4) and (5) ensure the feasibility of the surgery sequence by eliminating cyclic sequences. Constraint (6) sequentially allocates surgeries i and i' because surgeries i and i' are in the same department, and hence, it is more convenient to perform surgeries in the same department consecutively when arranging surgical equipment and adjusting schedules. Constraints (7) and (8) are the objective function values for the dual problem. Constraint (9) bounds the surgical duration using upper and lower bounds on the duration of surgery j. Constraint (10) is a non-negative constraint. Constraint (11) is a binary constraint:

2.2 Surgical Duration Uncertainty

As discussed in the Introduction, real-world surgical durations are often subject to uncertainties. A robust optimization model that considers uncertainty may be more suitable and reasonable for decision making. Our study involved uncertainty regarding surgical duration. We assumed that the uncertain surgical duration $\tilde{q}_j$ for each surgery j is with respect to the uncertainty set, without assumptions on distribution. The formulations are as follows:

Formulation

$$\text{Maximaize} \sum_{j \in J} \tilde{q}_j \tag{12}$$

Subject to

$$\tilde{q}_j = p_j - \underline{p_j},\ \forall j \in J, \tag{13}$$

$$\sum_{j \in J} \left(\frac{\tilde{q}_j}{\overline{p_j} - \underline{p_j}} \right) \leq \tau, \tag{14}$$

$$0 \leq \tilde{q}_j \leq \overline{p_j} - \underline{p_j},\ \ \forall j \in J. \tag{15}$$

In the above formulation, the objective function (12) defines the maximum surgical duration. Constraint (13) sets the left side of constraint (9) to zero and makes constraint (15) a nonnegative constraint to create a dual problem. The left side expresses an upper bound on the number of surgeries that will achieve their worst-case upper bound on surgical duration. Constraint (14) controls excessively conservatively, which is a weakness of robust optimization.

3 NUMERICAL ANALYSES

3.1 Data and Analysis Procedures

We solve the single operating room scheduling problem using Gurobi 9.5.1. The computational equipment is an Intel(R) Core (TM) i7-7500U CPU @ 2.90 GHz 8.00 GB. Specifically, there is one operating room, five surgeries, and the lower bound $\underline{p_j}$ of the surgical duration is defined as $\mathbb{E}[p_j] - \sigma_j$, and the upper bound $\overline{p_j}$ is defined as $\mathbb{E}[p_j] + \sigma_j$.

Table 1: Two types of instances

Instance 1					
Surgery j	1	2	3	4	5
$\mathbb{E}[p_j]$(min)	120	120	120	120	120
σ_j(min)	20	40	60	80	100
Instance 2					
Surgery j	1	2	3	4	5
$\mathbb{E}[p_j]$(min)	160	140	120	100	80
σ_j(min)	20	40	60	80	100

Then, $\mathbb{E}[p_j]$ and σ_j represent the expected value and standard deviation of the duration of surgery j, respectively. The conservative τ is varies from 1 to 0—5. These two types of instances are listed in Table 1. As shown in Table 1, instance 1 has the same expected surgical duration for all surgeries. In contrast, the standard deviations were different. In instance 2, the standard deviation is the same as that in instance 1, but the expected value is different. All weights w_j are 1. The time from the operating room opening to the time when surgery j should be completed, d_j is 8 h or 480 min. Here, d_j means the regular opening time of the operating room; it is desirable that all surgeries be completed within the closing time.

We compared the schedule created using the robust optimization model with that derived using the stochastic programming model. The occurrence probability of the 1000 scenarios used in the stochastic programming model was assumed to follow a uniform distribution. The surgical duration in each scenario followed a log-normal distribution.

3.2 Results

The results of comparing the two models for each instance are shown in Tables 2 and 3. All instances are solved within 10 seconds of CPU time. Tables 2 and 3 show the sequence of surgeries, expected delay, and number of scenarios in which the delay is greater than or equal to 1000 min for schedules created using

Table: 2 Results of instance 1.

Model	Constant controlling conservative, τ	Surgical sequence	Expected delay (min)	Number of parameters with significant delays
Stochastic programming	-	2, 1, 3, 4, 5	170.07	6
Robust optimization	0	5, 4, 3, 2, 1	206.17	23
	1	5, 1, 2, 3, 4	191.26	15
	2	4, 1, 2, 3, 5	172.46	6
	3	4, 1, 2, 3, 5	172.46	6
	4	2, 1, 3, 4, 5	170.07	6
	5	2, 1, 3, 4, 5	170.07	6

Table 3: Results of instance 2.

Model	Constant controlling conservative, τ	Surgical sequence	Expected delay time(min)	Number of parameters with significant delays
Stochastic programming	-	3, 4, 5, 2, 1	181.17	25
Robust optimization	0	5, 4, 3, 1, 2	190.69	25
	1	5, 4, 3, 2, 1	185.08	24
	2	5, 4, 3, 2, 1	185.08	24
	3	5, 4, 3, 2, 1	185.08	24
	4	5, 4, 3, 2, 1	185.08	24
	5	2, 1, 3, 4, 5	205.78	8

the stochastic programming model and robust optimization model at each control conservative τ. From Table 2, the expected delay of the schedule created using the stochastic programming model and robust optimization model when τ = 5 is the lowest.

The scenario in which the delay was more than 1000 minutes was also the lowest. The number of parameters with a significant delay of more than 1000 min reached a maximum at τ = 0. In summary, as τ increases, the results of the robust optimization model approach those of the stochastic programming model. This indicates that the robust optimization model without distribution assumptions performs as well as the stochastic programming model, depending on the setting of the control conservative τ.

According to Table 3, the schedule created using the stochastic programming model exhibits the lowest expected delay. The number of parameter sets with a delay of more than 1000 min were the minimum in the schedule created by the robust optimization model when τ = 5. In addition, while the expected delay increases as τ increases, the number of parameter sets in which a delay of more than 1000 minutes occurs decreases.

These results suggest that the robust optimization model performs as well as the stochastic programming model, and tends to avoid significant delays under certain conditions. Thus, it is suggested that robust optimization models may be able to reflect the risk-averse tendencies of operating room managers in their schedules.

4 CONCLUDING REMARKS

In this study, we proposed a robust optimization model that minimizes the delay in surgery by considering the sequence of surgery. We also verified whether the risk-averse tendency is reflected in the schedule. The numerical analysis suggests that robust optimization models tend to avoid long delays. From the numerical analysis, compared to stochastic programming models, the robust optimization model is more effective for operating room managers who desire to avoid long delays.

In future work, we will consider the relationship between conservatism, delay and duration of surgery set in a robust optimization model. We will clarify this relationship by performing a numerical analysis by increasing the set of surgical durations, which is the input. We will expand the settings from a single operating room to multiple operating rooms and use real data to refine the schedules.

ACKNOWLEDGEMENTS

This work was supported by the Japan Society for the Promotion of Science KAKENHI Grant [number JP 21K14371]. The authors would like to thank Manabu Hashimoto of National Cancer Center Hospital East and Hirofumi Fujii of National Cancer Center for their valuable comments.

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