Abstraction of Prevention Conceived in Distributed Knowledge Base

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Abstract: This paper is concerned with prevented information common in distributed knowledge base. The distributed knowledge is a framework to reflect knowledge acquisition and retrieval in a seminar class with participants. The presented accounts are listened to, by participants such that the participants may acquire knowledge. To make the acquired knowledge formal, it is restricted to rule-type and assumed as logically causal or algebraically structural. For such formalized knowledge, the meaning of knowledge may be considered by a consistent fixed point of the mapping associated with knowledge rules. The fixed point approach has complexity if the mapping is nonmonotonic (not monotonic), because the fixed point is not always available for the mapping. In this paper, we face such complexity caused by the nonmonotonic mapping which is reasonable from interactions between positive and negative informations in acquiring knowledge. With the assumption of some consistent fixed point existence, retrieval derivation to the ruled knowledge is inductively designed such that it may be sound with respect to the fixed point meaning of knowledge. The derivation adopts negation by failure, making adjustments between succeeding and failing cases, inherently involved in acquired knowledge. As regards the whole system in accordance with the seminar class, distributed knowledge base is formalized by taking a direct product of knowledge regarded as acquired by each participant. To make complexity of mutually interactive communications simpler and to possibly formulate common prevention conceived in the distributed knowledge base, we present a mapping associated with distributed knowledge base containing common negatives. Then the meaning of the whole system with common prevention information may be theoretically defined by a consistent fixed point of a mapping. A consistent fixed point of the mapping does not always exist, however, if it is definable, retrieval derivation in distributed knowledge base may be designed with an oracle of assumed prevention information in such a distributed knowledge base.

1 INTRODUCTION

A community may be regarded as containing distributed knowledge base which is formed with knowledge of community members. Then some common knowledge can be abstracted, although formulation of knowledge arrangements is complex as a process. This paper aims at the formation of common knowledge, especially, preventive negative. This is motivated as a problem in a seminar circle which is empirical in the virtual reality by M. Sasakura, et al. (2021). The treatment of common negative may involve applicative and theoretical interests, while positive information can be enjoyed individually.

As regards knowledge with logic, there have been advanced methodologies.

(a) Nonmonotonic logic on reasoning and inference mechanism is now rather classical but should be still worthwhile examining (Hanks and McDermott, 1987; Reiter, 2001).
(b) With respect to modal logic which may be tools for epistemic aspects, first-order modal logic is formally dealt with (Fitting, 2002), and second-order abstraction is considered for modality such that ambiguity of knowledge may be discussed by B. Kooi (2016). Quantifiers over epistemic agents are introduced (Naumov and Tao, 2019), as well.
(c) With reference to computability, quantified modal logic is discussed (Rin and S.Walsh, 2016). Topological space may be taken for modality (Goldblatt and Hodkinson, 2020).
(d) For stepwise removal of objects, dynamic modality is presented (Bentham et al., 2022). Dynamic logic for action was presented by L. Spalazzi et al. (2000). Justified belief truth is discussed (Egre et al., 2021).

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(e) Reduction of decidability is compiled (Rasga et al., 2021) and inference theory is formulated (Tennant, 2021).

Concerning knowledge representation, this paper considers a rule set as knowledge, where each rule can be given by some algebraic expression reflecting logical implication or causal relation, based on algebra. The rule set causes nonmonotonic acquiring of knowledge. In this sense, the classical nonmonotonic logic is most relevant to the present paper, based on algebraic expressions. Some common prevention in distributed knowledge base (each of which is formed by algebraic expressions) causes complexity, to be detected.

For the behaviours of distributed systems, communication theories are embedded with the ideas of rewriting process and process algebra, as well as of sequence generations:
(a) Rewriting process with reductions for calculus is formulated by functionalism as in the work (C. Bertolissi et al., 2006).
(b) Mobile ambients are dealt with in process of behaviours (Cardelli and Gordon, 2000; Merro and Nardelli, 2005).
(c) Sequences traversing states in a distributed system may be covered by the method of automata (M. Droste et al., 2009).

Among the above processes, the presentation of the seminar (referred to, in this paper) is regarded as the sequence generated in the above third method. The seminar presentation of this paper contains a stream of photo images, which is related to the sequence generated by traversing states. On the other hand, the distributed system of knowledge is captured in a static sense. Just common, negative (as a meet of reports from the participants) can be examined at each site of participants.

This paper then makes theoretical analyses and designs on:
(a) knowledge representation abstracted from the seminar participants,
(b) meaning of acquired knowledge and retrieval, and
(c) applications of meaning (with retrieval) to distributed knowledge system containing common negative.

The paper is organized as follows. Section 2 abstracts a theoretical method of sharing prevention knowledge, from the seminar in a virtual reality space. In Section 3, a rule set is formally given as algebraic expressions whose meaning may be defined by fixed point of a mapping associated with the rule set. A retrieval derivation for knowledge is detection and examination of the meaning which knowledge denotes. In Section 4, a distributed knowledge with common negative is dealt with, by expanding individual knowledge base and derivation. Concluding remarks are given in Section 5.

2 KNOWLEDGE ACQUISITION AND PROCESSING

The seminar (which we have implemented in virtual reality) conceives some knowledge processing problem. As in lecture and seminar, participants (who listen to the presenter's verbal accounts) may be regarded as acquiring knowledge. The class consists of images through video camera, to be organized as the streams to the seminar participants. After reviewing the presentation, the participants deliver reports to be gathered into a common virtual space.

Common Prevention in Distributed Knowledge Base

There is a motivated problem of this paper: After reporting, can the seminar participants hold a common knowledge? As common knowledge, preventive notices (which the seminar presents and participants would keep) are the subject of this paper. The distributed system is in general complex, owing to negatives: The more positive and negative knowledge are acquired, the more complicated the system is. We here assume that:
(a) Each participant organizes his/her understanding as knowledge base, with respect to the presentation by verbal accounts in seminar.
(b) Each participant can manage knowledge consisting of rules, where each rule is constructed in terms of positive and negative informations deriving conclusion.
(c) Negatives inferred by each knowledge are examined, to be common in the distributed system, where each knowledge may derive inferences, as the participant thinks of, after the seminar presentation.
(d) The knowledge acquisition (to be virtually implementable in the seminar) is schematized with human interaction.

That is, the verbal accounts or explanations are to be presented such that the participants can acquire knowledge of rule sets, where the participants supposedly acknowledge algebraic structure of knowledge. Among acquired knowledge, common prevention may be inferred. This scheme is arranged as in Table 1. On the common prevention, algebraic analysis is made for retrieval derivation to be designed.

Rule Set as Knowledge

The rule formation comes from logical or algebraic

Table 1: The acquired knowledge is formalized.

Presenter Verbal accounts	$\Longrightarrow$ Acquired	Participants Rules
Examined Alert	$\Longleftarrow$ Communicated	Negatives Prevention

property to understand knowledge. As regards positive and negative informations, theories on state spaces are given (Hornicher, 2021). Merge of distributed knowledge is formulated (Christoff et al., 2022). In this paper, common negative in a distributed system is paid attention to, conceived from the seminar problem so that the negative of logical or algebraic basis may be shared.

A rule is of the form r, in a formal way (which would be clearer in Section 3), represented by positive and negative information to derive a conclusion, which can be taken as knowledge acquisition:

$r = (Pos, Neg, con)$ where

$Pos, Neg \subseteq A$ (a domain)
$Pos ::= \{a\} \cup Pos$
$Neg ::= \{a^*\} \cup Neg$
(a^* stands for negative operation on A)
$con ::= a$ or a^* (for $a \in A$)

The form contains recursion of representation in Backus-Naur Form. If we see the structure as logical, we may interpret the knowledge as follows:

(a) If $Pos = \{a_1, \ldots, a_n\}$, Pos denotes $a_1 \wedge \ldots \wedge a_n$ with conjunction $\wedge$.

(b) If $Neg = \{b_1^*, \ldots, b_m^*\}$, Neg denotes $b_1^* \wedge \ldots \wedge b_m^*$.

(c) (Pos, Neg, con) denotes $Pos \wedge Neg \Rightarrow con$ with "$\Rightarrow$" (implication).

Then a set R of rules (where the rules are mutually conjunctive) is assumed as knowledge (which each participant acquires after the seminar). As auxiliary means, a subset of the set $\{a^* \mid a \in A\}$ may be reported to the presenter. It is considerably common negative information.

We analyze the meaning of knowledge and a distributed knowledge system with common negative. Then we show some way to see the meaning and to perform retrieval of the knowledge.

Although the following treatment on knowledge as rule set is algebraic, the rule set is interpreted logically as a conjunction of rules which are in accordance to logical implications with premises and conclusion in 3-valued intuitionistic logic.

3 KNOWLEDGE ORGANIZATION

In this section, a rule set is formally described. Each rule reflects algebraic, logical or causal relation such that a set of rules is regarded as knowledge.

3.1 A Rule Set

We firstly assume a domain A to construct a set of knowledgeable objects. We then make use of:

$$A^* = \{a^* \mid a \in A\}$$

where a^* is to be interpreted as negative of a.

A rule is a triplet of the form (Pos, Neg, con) where:

(a) $Pos \subseteq A$,

(b) $Neg \subseteq A^*$ and

(c) $con = a$ or a^* for $a \in A$.

The expression (Pos, Neg, a) or (Pos', Neg', a^*) may be adopted, if the form con (of the third item in the rule triplet) is $a \in A$ or $a^* \in A^*$, respectively. A name R is used to refer to a set of such triplets (a rule set).

Example 1. We illustrate a set R of rules:

(i) $(\{revised\}, \{fashioned^*\}, evolved)$

(ii) $(\{fashioned\}, \emptyset, ruled)$

(iii) $(\emptyset, \{ruled^*\}, revised)$

Many-valued logic provability is presented (Pawlowski and Urbaniak, 2018). From practical views, answer set programming and problem solving are compiled in 2-valued logic (Gebser and Schaub, 2016; Kaufmann et al., 2016).

Apart from answer set programming, 3-valued evaluation is studied with the unknown (the undefined), for preventive negative to be paid attention to. The 3-valued evaluation applicable to algebraic expressions contains originality with respect to complexity for both rule sets and retrieval. As a 3-valued evaluation, we adopt the following algebra:

Base Algebra of 3-Valued Domain

A bounded lattice $K = (\{f, unk, t\}, \bigvee, \bigwedge, \bot, \top)$ equipped with the partial order $\sqsubseteq$ and an implication $\Rightarrow$ may be taken:

(a) $\bot$ and $\top$ are the least and the greatest elements f and t, respectively, with respect to the partial order $\sqsubseteq$ such that $\bot = f \sqsubseteq unk \sqsubseteq t = \top$.

(b) The least upper bound (*join*) $\bigvee$ and the greatest lower bound (*meet*) $\bigwedge$ exist for any two elements of the set $\{f, unk, t\}$.

(c) The implication $\Rightarrow$ (a relation on the set) is defined in a way that $z \sqsubseteq (x \Rightarrow y)$ iff $x \bigwedge z \sqsubseteq y$.

t is the truth, f the falsehood and unk (the unknown as t or f) is the undefined for the truth value, where the partial order is defined, regarding the truth value.

Evaluation of Rule Set and Model

A valuation $V : A \rightarrow \{f, unk, t\}$ is assumed with the bounded lattice K. With respect to V, the value $eval_V(E)$ of an expression E is given.

$eval_V(a) = V(a)$ $(a \in A)$
$eval_V(a^*)$
$=$ if $eval_V(a) = f$ then t else f $(a^* \in A^*)$
$eval_V(con)$
$=$ if $con = a$ then $eval_V(a)$
else if $con = a^*$ then $eval_V(a^*)$

$eval_V(Pos) = \bigwedge\{V(a) \mid a \in Pos\}$
$eval_V(Neg) = \bigwedge\{V(a^*) \mid a^* \in Neg\}$

$eval_V((Pos, Neg, con))$
$=$ if $eval_V(Pos) \sqsubseteq eval_V(con)$
or $eval_V(Neg) \sqsubseteq eval_V(con)$ then t
else if $eval_V(con) = f$ then f else unk

$eval_V(R) = \bigwedge_{(Pos,Neg,con)\in R} eval_V((Pos, Neg, con))$

For a pair $(I, J) \in 2^A \times 2^A$ such that $I \cap J = \emptyset$, i.e., I and J are disjoint, the valuation

$$V(I, J) : A \rightarrow \{f, unk, t\}$$

is defined to be:

$V(I, J)(a)$
$=$ if $a \in I$ then t else if $a \in J$ then f
else unk

If the disjoint pair (I, J) $(I \cap J = \emptyset)$ causes $eval_{V(I,J)}(R) = t$, the pair is called a model of the rule set R.

3.2 Fixed Point Model

For a method to obtain a model of the given rule set, a fixed point of the mapping associated with the rule set may be taken, although we cannot always have such a fixed point, because of nonmonotonic functionality of the mapping.

Mapping for Modeling of Rule Set

Given a rule R, a mapping $T_R : 2^A \times 2^A \rightarrow 2^A \times 2^A$ is defined to be

$$T_R(I, J) = (I', J')$$

such that the pair (I', J') is given for the pair (I, J) to T_R:

- if $(\forall(Pos, Neg, a) \in R.\ ((\exists b \in Pos.\ b \in J)$ or $(\exists c^* \in Neg.\ c \in I)))$ then $a \in J'$
- else if $(\exists(Pos, Neg, a) \in R.\ ((\forall b \in Pos.\ b \in I)$ and $(\forall c^* \in Neg.\ c \in J)))$ and $(\forall(Pos', Neg', a^*) \in R.\ ((\exists b \in Pos'.\ b \in J)$ or $(\exists c^* \in Neg'.\ c \in I)))$ then $a \in I'$
- else $(\exists(Pos, Neg, a) \in R.\ ((\forall b \in Pos.\ b \notin J)$ and $(\exists b' \in Pos.\ b' \notin I))$ and $(\forall c^* \in Neg.\ c \in J)))$ and $(\forall(Pos', Neg', a^*) \in R.\ ((\exists b \in Pos'.\ b \in J)$ or $(\exists c \in Neg^*.\ c \in I)))$ entails $a \notin I' \cup J'$

If $T_R(I, J) = (I, J)$ such that $I \cap J = \emptyset$, the pair is called a consistent fixed point of T_R.

Proposition 1. *If (I, J) is a consistent fixed point of T_R then $eval_{V(I,J)}(R) = t$.*

Proof. All the rules of R are examined. For each $a \in A$, we check the evaluation of R with respect to the valuation $V(I, J)$.

(i) When $a \in I$,

$$(\forall(Pos, Neg, a) \in R.\ eval_{V(I,J)}((Pos, Neg, a)) = t).$$

Because $a \in I$,

$$(\forall(Pos', Neg', a^*) \in R.\ ((\exists b \in Pos'.\ b \in J) \text{ or } (\exists c^* \in Neg'.\ c \in I))).$$

It follows that

$$eval_{V(I,J)}(Pos') = f \text{ or } eval_{V(I,J)}(Neg') = f.$$

Thus $eval_{V(I,J)}((Pos', Neg', a^*)) = t$.

(ii) Assume that $a \in J$. Then

$$(\forall(Pos, Neg, a) \in R.\ ((\exists b \in Pos.\ b \in J) \text{ or } (\exists c^* \in Neg.\ c \in I))).$$

It follows that

$$eval_{V(I,J)}(Pos) = f \text{ or } eval_{V(I,J)}(Neg) = f.$$

Thus $eval_{V(I,J)}((Pos, Neg, a)) = t$. On the other hand,

$$eval_{V(I,J)}(a^*) = t$$

such that $eval_{V(I,J)}((Pos', Neg', a^*)) = t$.

(iii) In case that $a \notin I \cup J$:

$$(\exists(Pos, Neg, a) \in R.\ ((\forall b \in Pos.\ b \notin J) \text{ and } (\exists b' \in Pos.\ b' \notin I)) \text{ and } (\forall c^* \in Neg.\ c \in J))) \text{ and } (\forall(Pos', Neg', a^*) \in R.\ ((\exists b \in Pos'.\ b \in J) \text{ or } (\exists c^* \in Neg'.\ c \in I))).$$

It follows that

$$eval_{V(I,J)}(Pos) = unk \text{ and } eval_{V(I,J)}(Neg) = t.$$

On the assumption of $a \notin I \cup J$, $eval_{V(I,J)}(a) = unk$. Thus $eval_{V(I,J)}((Pos, Neg, a) = t$.

For any rule of the assumed form

$$(Pos', Neg', a^*) \in R,$$

$eval_{V(I,J)}(Pos' \cup Neg') = f$, while $eval_{V(I,J)}(a^*) = f$. Thus $eval_{V(I,J)}(Pos', Neg', a^*) = t$. □

Example 2. Assume the rule set R as in Example 1. Then, we have a model as fixed points of T_R:

$$(\{revised, evolved\}, \{fashioned, ruled\})$$

Thus, the consistent fixed point (I, J) of T_R is a model of R.

Given a set R of rules, the following derivation is an amended version of negation by failure.

Retrieval Derivation

suc and $fail$ are succeeding and failing derivations, inductively defined as follows.

(1) $\emptyset : suc$.

(2)

if $\exists(Pos, Neg, a) \in R$.
$((\forall(Pos', Neg', a^*) \in R.\ Pos' \cup Neg' : fail)$ and $Pos \cup Neg \cup G : suc)$ then $\{a\} \cup G : suc$.

(3) If $\{a\} : fail$ and $G : suc$ then $\{a^*\} \cup G : suc$.

(4) If $\forall(Pos, Neg, a) \in R$. $(Pos \cup Neg : fail)$ then $\{a\} : fail$.

(5) If $\{a\} : fail$ then $\{a\} \cup G : fail$.

(6) If $\{a\} : suc$ then $\{a^*\} \cup G : fail$.

Example 3. Assume the rule set R as in Example 1. Then we have a succeeding derivation.

1. For "$\{evolved\} : suc$" to hold,

   "$\{revised\} \cup \{fashioned^*\} : suc$"

   must hold, since there is $(\{revised\}, \{fashioned^*\}, evolved) \in R$.
2. For "$\{revised\} \cup \{fashioned^*\} : suc$" to hold, "$\{revised\} : suc$" must hold, since we can see "$\{fashioned\} : fail$".
3. For "$\{revised\} : suc$" to hold, "$\{ruled^*\} : suc$" must hold, since there is $(\emptyset, \{ruled^*\}, revised) \in R$.
4. We can see "$\{ruled\} : fail$". The check of whether "$\{ruled^*\} : suc$" or not may be reduced to the case for "$\emptyset : suc$" (initially assumed).

Thus we can display the following scheme:

$\{evolved\}$
|
with $(\{revised\}, \{fashioned^*\}, evolved) \in R$
|
$\{revised\} \cup \{fashioned^*\}$
|
with $\{fashioned\} : fail$
|
$\{revised\}$
|
with $(\emptyset, \{rules^*\}, revised)$
|
$\{ruled^*\}$
|
with $\{ruled\} : fail$
|
$\emptyset :\ suc$

The derivation is sound with respect to possibly existing, consistent fixed point model in the sense:

Proposition 2. *Assume that (I, J) is a consistent fixed point of T_R.*

(1) *If $\{a\} : suc$ then $a \in I$.*

(2) *If $\{a\} : fail$ then $a \in J$.*

Proof. (1) If $\{a\} : suc$ then

$$(\forall(Pos', Neg', a^*) \in R.\ Pos' \cup Neg' : fail).$$

It follows that

$(\exists b \in Pos'.\{b\} : fail)$, or
$(\exists c^* \in Neg' : \{c^*\} : fail)$(i.e.,$\exists c^* \in Neg'.\{c\} : suc)$.

By induction hypothesis, there is $b \in J$ or $c \in I$, for the assumed derivation $\{a\} : suc$. With respect to $\{a\} : suc$, two cases are to be examined.
(a) If $(\emptyset, \emptyset, a) \in R$, as a basis, it is evident that $a \in I$.
(b) If $(Pos, Neg, a) \in R$ such that $Pos \cup Neg : suc$, as induction step, we can reason that

$$(\forall b \in Pos.\ b \in I) \text{ and } (\forall c^* \in Neg.\ c \in J),$$

from the condition that

$$(\forall b \in Pos.\{b\} : suc) \text{ and } (\forall c^* \in Neg.\{c\} : fail).$$

This completes the induction step, such that $a \in I$.
(2) If $\{a\} : fail$ then

$$(\forall(Pos, Neg, a) \in R.\ Pos \cup Neg : fail).$$

When there is no rule of the form (Pos, Neg, a), then $a \in J$. Otherwise, it follows that

$$(\exists b \in Pos.\{b\} : fail) \text{ or } (\exists c^* \in Neg.\{c^*\} : fail)$$

(the latter of which is caused by $\exists c^* \in Neg.\{c\} : suc$). By induction, $b \in J$ or $c \in I$. This concludes that $a \in J$. □

Negation by Failure
In the Retrieval Derivation, the item (2) contains the point: For "$\{a\} : suc$" to hold, the following condition is required:

$$(\forall (Pos', Neg', a^*) \in R.\ Pos' \cup Neg' : fail)$$

This condition contains complexity, which the original negation by failure rule (K. Clark, 1978) does not need. The condition comes from the evaluation for the triplet with the implication in:

$$Pos \cup Neg \text{ implies } con.$$

The meaning of the implication is concerned with 3-valued domain, as above mentioned for the definition of $eval_V(\bullet)$. Then:

(i) If $\{a\} : fail$ then $\{a^*\} : suc$.

(ii) In case that $\{a^*\} : fail$, it may be possible that $\{a\} : suc$. (Therefore, if $\{a\} : suc$ then $\{a^*\} : fail$.)

This amended negation by failure is well enough to escape from infinite procedure for failure such that succeeding derivation contains only finite steps of executions.

On the other hand, the rule set R

$$R = \{(\emptyset, \{b^*\}, a), (\emptyset, \{a^*\}, b)\}$$

induces 2 consistent fixed points of T_R:

$$(\{a\}, \{b\}) \text{ and } (\{b\}, \{a\})$$

Because of no finite failure for $\{b\}$, we cannot have $\{a\} : suc$. At the same time, we cannot obtain $\{a\} : fail$ such that it is no case of $\{b\} : suc$.

4 DISTRIBUTED KNOWLEDGE BASE

In this section, distributed knowledge base is just a tuple of knowledges each of which is described in Section 3. But the negative from each knowledge must be common. How we can have the meaning of distributed knowledge base with common negatives is presented to apply to the theory for prevention alert in distributed systems. Then we can have retrieval derivation by means of oracle of assumed prevention, with respect to the meaning of distributed knowledge base.

4.1 Tuple of Rule Sets

As distributed knowledge base (which may be regarded as constructed by seminar participants), a tuple of rule sets

$$\tau = \langle R_1, \ldots, R_n \rangle \ (n \geq 1)$$

is used. To represent a distributed knowledge base, we extend the method of how each rule set denotes a knowledge over a set A.

A valuation $V_i : A \to \{f, unk, t\}$ $(1 \leq i \leq n)$ is assumed with the bounded lattice K, as in the case of Section 3.

The evaluations of R_k with the way as below are to be executed for $1 \leq k \leq n$.

Evaluation of Rule Sets Tuple and Model
With $(V_1, \ldots, V_n)$, the value $Eval_{(V_1,\ldots,V_n)}(\tau)$ is given by:

$$\begin{aligned} & Eval_{(V_1,\ldots,V_n)}(\tau) \\ & = Eval_{(V_1,\ldots,V_n)}(\langle R_1, \ldots, R_n \rangle) \\ & = (eval_{V_1}(R_1), \ldots, eval_{V_n}(R_n)) \end{aligned}$$

For a pair $(I, J) \in 2^A \times 2^A$ such that $I \cap J = \emptyset$, i.e., I and J are disjoint, the valuation

$$V_k(I, J) : A \to \{f, unk, t\} \ (1 \leq k \leq n)$$

is used as $V(I, J)$ in Section 3.

If with the disjoint pairs (I_k, J_k) $(1 \leq k \leq n)$ for a tuple τ of rule sets,

$$Eval_{(V_1(I_1,J_1),\ldots,V_n(I_n,J_n))}(\tau) = (t, \ldots, t)$$

then the pairs are called a model of τ.

Just taking a direct product of pairs each of which may be defined by the mapping of T_{R_k} $(1 \leq k \leq n)$, we have a mapping of Tr_τ.

Mapping for Modeling of Rule Sets Tuple
Given a tuple of rule sets $\tau = \langle R_1, \ldots, R_n \rangle$, a mapping

$$Tr_\tau : (2^A \times 2^A)^n \to (2^A \times 2^A)^n$$

is defined to be

$$\begin{aligned} & Tr_\tau((I_1, J_1), \ldots, (I_n, J_n)) \\ & = (T_{R_1}(I_1, J_1), \ldots, T_{R_n}(I_n, J_n)) \\ & = ((I'_1, J'_1), \ldots, (I'_n, J'_n)) \end{aligned}$$

such that each pair (I'_k, J'_k) is given for the pair (I_k, J_k) with T_{R_k} (as T_R in Section 3):

- if $(\forall (Pos, Neg, a) \in R_k.\ ((\exists b \in Pos.\ b \in J_k)$ or $(\exists c^* \in Neg.\ c \in I_k)))$ then $a \in J'_k$
- else if
$(\exists (Pos, Neg, a) \in R_k.\ ((\forall b \in Pos.\ b \in I_k)$ and $(\forall c^* \in Neg.\ c \in J_k)))$ and
$(\forall (Pos', Neg', a^*) \in R_k.\ ((\exists b \in Pos'.\ b \in J_k)$ or $(\exists c^* \in Neg'.\ c \in I_k)))$ then $a \in I'_k$
- else $(\exists (Pos, Neg, a) \in R_k.\ ((\forall b \in Pos.\ b \notin J_k)$ and $(\exists b' \in Pos.\ b' \notin I_k))$ and $(\forall c^* \in Neg.\ c \in J_k)))$ and $(\forall (Pos', Neg', a^*) \in R_k.\ ((\exists b \in Pos'.\ b \in J_k)$ or $(\exists c \in Neg^*.\ c \in I_k)))$ entails $a \notin I'_k \cup J'_k$

The fixed point of the mapping Tr_τ may be obtained such that each R_k $(1 \leq k \leq n)$ is modelled.

Proposition 3. *If*

$$Tr_\tau((I_1,J_1),\ldots,(I_n,J_n)) = ((I_1,J_1),\ldots,(I_n,J_n))$$

such that $I_k \cap J_k = \emptyset$ $(1 \leq k \leq n)$ *then*

$$Eval_{(V(I_1,J_1),\ldots,V(I_n,J_n))}(\tau) = (t,\ldots,t),$$
$$i.e., eval_{V(I_k,J_k)}(I_k,J_k) = t \ (1 \leq k \leq n).$$

Proof. For each R_k, the proof of Proposition 1 can be applied, where each T_{R_k} is independent of another T_{R_j} $(k \neq j)$. Tr_τ is defined in terms of such mutually independent T_{R_k}. Therefore the above fixed point of Tr_τ is a tuple of the consistent fixed points of T_{R_k} $(1 \leq k \leq n)$. Thus the above fixed point of Tr_τ is associated with

$$Eval_{(V(I_1,J_1),\ldots,V(I_n,J_n))}(\tau)$$

which is a tuple of n values of t. □

The fixed points (I_k,J_k) (for $1 \leq k \leq n$) are components of the (tuple) fixed point of τ.

4.2 Retrieval in Knowledge Base

Fixed point of

$$Tr_\tau : (2^A \times 2^A)^n \rightarrow (2^A \times 2^A)^n,$$

$((I_1,J),\ldots,(I_n,J))$ $(I_k \cap J = \emptyset, 1 \leq k \leq n)$ can be knowledge acquisitions of individuals, where prevention information is common.

Since each (I_k,J) is independent of another $(I_{k'},J)$, retrieval derivation to each R_k is available, which is defined with an oracle O supposed to be a common negative knowledge, following the derivation given in Section 3.

Retrieval Derivation to R_k with Oracle O

(1) $\emptyset : suc$.

(2) If $a \notin O$ where there is $(Pos,Neg,a) \in R_k$ such that $((\forall (Pos',Neg',a^*) \in R_k.\ Pos' \cup Neg' : fail)$ and $Pos \cup Neg \cup G : suc)$ then $\{a\} \cup G : suc$.

(3) If $\{a\} : fail$ and $G : suc$ then $\{a^*\} \cup G : suc$.

(4) If $a \in O$ such that $\forall (Pos,Neg,a) \in R_j.(Pos \cup Neg : fail)$ then $\{a\} : fail$.

(5) If $\{a\} : fail$ then $\{a\} \cup G : fail$.

(6) If $\{a\} : suc$ then $\{a^*\} \cup G : fail$.

The above retrieval derivation may contain soundness with respect to (I_k,J), in the sense:

Proposition 4. *Assume that for*

$$\tau = \langle R_1,\ldots,R_n \rangle$$

$((I_1,J),\ldots,(I_n,J))$ *is a fixed point of* Tr_τ, *where*

$$I_k \cap J = \emptyset \ (1 \leq k \leq n).$$

For the derivation to R_k *with the oracle* $Q \subseteq J$, *we have:*

(1) *If* $\{a\} : suc$ *then* $a \in I_k$.

(2) *If* $\{a\} : fail$ *then* $a \in J$.

Proof. Because O is supposedly a subset J, the reason why soundness is held is almost the same as the proof of Proposition 2 in Section 3. But we describe formally in the similar way.

(1) If $\{a\} : suc$ with $a \notin O$ then

$$\forall (Pos',Neg',a^*) \in R.\ Pos' \cup Neg' : fail.$$

Thus $(\exists b \in Pos'.\{b\} : fail)$ or $(\exists c^* \in Neg.\{c^*\} : fail)$ (i.e., $\exists c^* \in Neg'.\{c\} : suc$).

By induction hypothesis, there is $b \in J$ or $c \in I$, for the assumed derivation $\{a\} : suc$.

2 cases are also to be examined:

(a) If $(\emptyset,\emptyset,a) \in R$, as a basis, it is evident that $a \in I$.

(b) If $(Pos,Neg,a) \in R$ such that $Pos \cup Neg : suc$, as induction step, we can reason that

$(\forall b \in Pos.\ b \in I)$ and $(\forall c^* \in Neg.\ c \in J)$,

from the condition that

$(\forall b \in Pos.\ \{b\} : suc)$ and $(\forall c^* \in Neg.\ \{c\} : fail)$.

This completes the induction step, such that $a \in I$.

(2) If $\{a\} : fail$ with $a \in O$ then

$$\forall (Pos,Neg,a) \in R.\ Pos \cup Neg : fail.$$

When there is no rule of the form (Pos,Neg,a), then $a \in J$. Otherwise, it follows that

$$\exists b \in Pos.\{b\} : fail \text{ or } \exists c^* \in Neg.\ \{c^*\} : fail$$

(the latter of which is caused by $\exists c^* \in Neg.\{c\} : suc$). By induction, $b \in J$ or $c \in I$. This concludes that $a \in J$. □

Proposition 4 shows that the derivations to R_k with oracle O is sound with respect to the consistent fixed point (I_k,J) of T_{R_k}, where the oracle O, an assumed subset of the set J can be regarded as common prevention in the distributed knowledge base

$$\tau = \langle R_1,\ldots,R_n \rangle.$$

Example 4. Assume the rule set

$$R_1 = \{\ (\emptyset,\emptyset,c),\ (\{c\},\{d^*\},a),\ (\{b\},\emptyset,\{a^*\})\ \}$$

The set is modeled by $(\{a,c\},\{b,d\})$.

Assume the rule set

$$R_2 = \{\ (\emptyset,\emptyset,c),\ (\{b,c\},\emptyset,a),\ (\emptyset,\emptyset,a^*)\ \}.$$

The set R_2 is modeled by $(\{c\},\{a,b\})$. For the tuple $\langle R_1,R_2 \rangle$, $\{b\}$ is a common prevention.

5 CONCLUSION

Motivated by a seminar relation between the presenter and participants, we settle knowledge acquisitions and distributed knowledge base.

We established theoretical results:

(a) A triplet of a positive set, a negative set and conclusion as a rule, is adopted, such that a rule set may be regarded as acquired knowledge whose evaluation is made in 3-valued domain: The rule-set evaluation is based on 3-valued bounded lattice structure.

(b) This evaluation involves complex difficulty caused by strong negatives. We conclude that a fixed point of a mapping associated with a rule set may be a model (which evaluates the rule set as true in the 3-valued domain). A fixed point does not always exist, however, it may be a model if it must be consistent (that is, no case is evaluated as both positive and negative), leaving the unknown (undefined) as it is.

(c) A retrieval derivation is made by amending negation by failure, to be sound with respect to the model which is a consistent fixed point.

This is another theoretical dealing with the algebraic/logical complex, compared with the papers (Yamasaki and Sasakura, 2021a; Yamasaki and Sasakura, 2022). Then we deal with distributed knowledge base containing common prevention, abstractly reflecting the seminar system with the reports on negatives from participants.

(d) We think of a tuple of triplets (for rule sets) as distributed knowledge base, which is regarded as a theoretical representation of a seminar system.

(e) The evaluation of distributed knowledge base is made such that we may have some model by a consistent fixed point of a mapping associated with distributed knowledge base, although such a fixed point does not always exist. This mapping contains the mappings associated with acquired knowledges (virtually of participants).

(f) With some assumed prevention, distributed retrieval derivations are available by means of strategies containing individual derivation for knowledge as briefly pointed out in (c).

As in the treatment of communication (Yamasaki and Sasakura, 2021b), if both positive and negative are common, then the whole distributed knowledge base should be treated as single knowledge base. In this case, Section 3 gives a theoretical base. Owing to implementation of communication among R_k $(1 \leq k \leq n)$, the retrieval derivation must be essentially complex. If we could have access to another R_j from R_k only by reference to, a possible fixed point and the given procedural aspects of retrieval derivation may be abstractly simpler.

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