An Application of Priority-Based Lightweight Ontology Merging

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Abstract: Merging multiple and frequently contradictory sources of information has been identified as a significant issue in the semantic web community. In addition, pieces of information to be combined are provided with uncertainty due, for instance, to the reliability of sources. To solve this, possibility theory offers a useful tool for representing and reasoning with uncertain, partial, and inconsistent information. In this paper, we concentrate on dance video processing, in which many inconsistent information sources exist. Therefore, we propose possibilistic merging operators for the dance *OWL2-EL* ontologies to deal with the conflicting dance sources. We represent an extension of $\mathcal{EL}$ within a possibility theory setting. It leverages a min-based operator to merge the ontologies based on possible distributions. Furthermore, the semantic fusion of these distributions has a natural syntactic counterpart when dealing with $\mathcal{EL}$ ontologies. The min-based fusion operator is recommended when distinct dance sources that provide information are dependent.

1 INTRODUCTION

Various sorts of knowledge originating from conflicting (or inconsistent) sources are affected by uncertainty. The problem of merging this knowledge is a key challenge in several applications, including distributed databases, multi-agent systems, and distributed information systems, i.e., (Konieczny and Pérez, 2002; Everaere et al., 2010; Hue et al., 2007; Konieczny and Pérez, 2011; Patricia et al., 2008; Haret and Woltran, 2019). Additionally, a knowledge base is composed of a set of pieces of information provided by sources. A set of formulas syntactically formulates the pieces of information. Normally, they are semantically represented by a set of interpretations. Therein, the syntactical approaches merge all the formulas to obtain one knowledge base that represents various different sources. Otherwise, the views of semantics consist of ranking all the interpretations and merging them using merging operators in order to obtain a unique order for all sources of information (Konieczny and Pérez, 2002).

Indeed, in terms of the problem of culture and its hesitant preservation, traditional dance management has attracted scientists' attention in recent years. Namely, we concentrate on Vietnamese traditional dances (for short, VTDs). Most VTDs are crucially stored in numerous distinct video sources that are in conflict. (Lam, 1994). Indeed, in the same dance, each different video has distinct movements. Furthermore, they manage the resulting large amount of heterogeneous digital content. Therefore, preserving VTDs is a remarkable problem that needs to be handled. This paper concentrates on merging the dance ontologies distributed from different sources. For this purpose, we realize that an application to manage those data sources is necessary and desirable. As a result, the approach of the OWL2-EL ontology is reasonably expected, as OWL2-EL is one of the essential tractable profiles of the W3C Ontology Web Language, which provides a powerful framework to compactly encode structured knowledge with low computational complexity. Notably, we are interested in $\mathcal{EL}$ Description Logics to represent VTD's ontology sources, perceiving further what is the motivation behind building an application *(semantic web using OWL2-EL)*.

On the other hand, possibility theory (Dubois and Prade, 2012), especially the qualitative possibility theory offers a natural framework to deal with ordinal uncertainty. It holds when there exists a preference ranking between pieces of information that reflects their reliability or compatibility with the available knowledge or when there exists a total preorder

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between them. Merging the pieces of information depends on the nature of the knowledge base such as the propositional knowledge bases (Konieczny and Pérez, 2002), prioritized knowledge bases (Delgrande et al., 2006), or weighted knowledge bases(Benferhat et al., 1993; Qi et al., 2006). Indeed, there exist several approaches to merge possibilistic logic bases (Benferhat et al., 1993; Benferhat et al., 1999; Benferhat et al., 2000). However, there exists a few works that merge possibilistic DLs, more precisely the lightweight DLs DL-Lite (Benferhat et al., 2013). Note that, there is to the best of our knowledge, no approaches for fusing possibilistic $\mathcal{EL}$ ontology. Therefore, merging $\mathcal{EL}$ ontology combine with possibility theory to handle inconsistent and uncertain problems is quite essential, especially, in the case of the dance video processing from several sources.

Moreover, one of the big problems regarding dance video processing is that the motions (postures and gestures) from performers/dancers are only partially specific and accurate. They have a range of approximations. For example, as a piece of evidence, *Posture:"the left hand is horizontal at shoulder level"*, meaning that the left hand of the dancer is horizontal with the shoulder (creating an angle of 90 degrees from the body), however, in fact, the dancer performed the left-hand posture with a different degree. Therefore, utilizing possibility theory in the dance domain is appropriate.

In this paper, we take advantage of the qualitative possibility theory in order to deal with qualitative and ordinal uncertainty. We concentrate on the lightweight description logics named $\mathcal{EL}$ (Baader et al., 2010), which offer a powerful expressiveness in expressing ontological knowledge and guarantee the tractability of reasoning processes, especially for instance and subsumption checking (Kazakov et al., 2012; Kazakov et al., 2003). This logic underlies the *OWL2 EL* profile, one of the three profiles proposed as sublanguages of the full *OWL2*. The $\mathcal{EL}$ description logic is suitable especially for medical applications such as based on using general core terminology of SNOMED CT[1], GALEN [2] and for biology applications such as Gene Ontology[3]. However, we concentrate primarily on the dance domain in this paper by representing a prominent example of VTD video processing. First, we will investigate the possibility theory with an extended fragment of $\mathcal{EL}$. Then, we will focus on the minimum operator for merging possibilistic $\mathcal{EL}$ ontologies.

The remainder of this paper is structured as follows: we describe a prominent example in Section 2 briefly. In the next section, *(Section 3*, we give a refresher on the $\mathcal{EL}$ logic. After that, we condense the possibility theory over $\mathcal{EL}$ interpretations in Section 4. Section V discusses the fragment of $\mathcal{EL}$ combined with possibility distribution. A presentation of the min-based merging of π-$\mathcal{EL}_{\perp}^{+}$ possibility distributions and syntactical merging of π-$\mathcal{EL}_{\perp}^{+}$ are in section 6 and section 7, respectively. To illustrate our proposal, a noticeable example related to merging $\mathcal{EL}$ ontologies for dance video processing is presented in Section 8. Finally, Section 9 concludes the paper.

[1]http://www.snomed.org/

[2]http://www.opengalen.org/index.html

[3]http://geneontology.org/

2 DESCRIPTION OF A PROMINENT DANCE EXAMPLE

This section contains a brief description of a potential (prominent) example of our approach to classifying and merging a collection of dance videos issued from different sources. More preciously, we concentrated on Vietnamese traditional dances (VTDs), in which existing a large number of fundamental motions in dance videos. Motions carry important information which is of a multi-fold nature. At present, there are many different dataset sources to represent a Vietnamese dance, hence, merging the dance video remains to be an active research area. A question arises as to how to merge dance knowledge (motions, stories in dances, others) from several different video sources.

As known, a video is a sequence of frames (images). Moreover, a set of frames can be referred to as a dance segment (or a dance step) in a composing dance. Each motion corresponds to an image extracted from a video. In this section, we present the fundamental features of VTDs to represent merging dance $\mathcal{EL}$ ontologies. In this example, we focused on representing VTDs motions through dance orientations

Regarding orientation features (Lam, 1994)(Tran et al., 2003), it is one of the most significant characteristics in VTDs because the motions, postures, and gestures are always described explicitly through the orientations with body parts written in almost of all documents. For instance, the left hand has a direction in orientation 7, the right hand is in orientation 1. In (Tran et al., 2003), the experts of the dance domain in Vietnam are split fundamental orientations into eight directions as Figure 1, denoted by $VNOri\{i\}$ with $i = [1,8]$. Namely, $VNOri1$ is the direction of the dancer opposite the spectator (in front of the audience), it is also used for the first preparation step of performing.

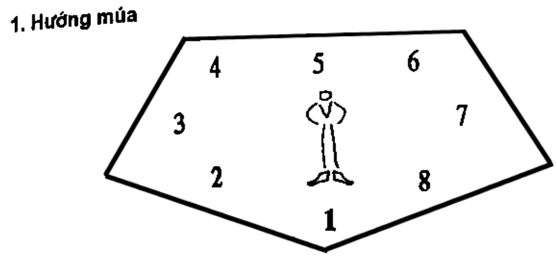

Figure 1: Orientation (direction) features of VTDs.

In this paper, we concentrate primarily on representing orientation characteristics in VTDs into the merging operators proposed. Therefore, we would use these features to model and illustrate the merging operators throughout the examples of this paper. Our main purpose is to represent how to merge possibilistic ontologies with dance video processing, in this case Vietnamese traditional dances with orientation features.

3 A REFRESHER ON $\mathcal{EL}$ LOGICS

Our approach is built on a foundation of a lightweight Description Logic (DL) framework to encode the ontology. We now provide a brief description of the DL $\mathcal{EL}$ (Baader et al., 2005).

Let N_C, N_R, N_I be three pairwise disjoint sets where N_C denotes a set of atomic concepts, N_R denotes a set of atomic roles and N_I denotes a set of individuals. The $\mathcal{EL}$ concept expressions are built according to the following syntax:

$$C ::= \ \top \mid N_C \mid C \sqcap C \mid \exists r.C.$$

where $C, D \in N_C$, $a, b \in N_I$, and $r \in N_R$.

An $\mathcal{EL}$ ontology $O = \langle \mathcal{T}, \mathcal{A} \rangle$ (a.k.a. knowledge base) comprises two components, the TBox (Terminological Box denoted by $\mathcal{T}$) and ABox (denoted by $\mathcal{A}$). The TBox consists of a set of General Concept Inclusion (GCI) axioms of the form $C \sqsubseteq D$, meaning that C is more specific than D or simply C is subsumed by D, $C \equiv D$ which is a shortcut for $C \sqsubseteq D$ and $D \sqsubseteq C$. The ABox is a finite set of assertions on individual objects of the form $C(a)$ or $r(a,b)$.

The semantics is given in terms of interpretations $I = (\Delta^I, \cdot^I)$, which consist of a non-empty interpretation domain Δ^I and an interpretation function $\cdot^I$ that maps each individual $a \in N_I$ into an element $a^I \in \Delta^I$, each concept $A \in N_C$ into a subset $A^I \subseteq \Delta^I$, and each role $r \in N_R$ into a subset $r^I \subseteq \Delta^I \times \Delta^I$.

Table 1: Syntax and semantics of $\mathcal{EL}$.

Syntax	Semantics
$C \sqsubseteq D$	$C^I \subseteq D^I$
r	$r^I \subseteq \Delta^I \times \Delta^I$
a	$a^I \in \Delta^I$
$C \sqcap D$	$C^I \cap D^I$
$\top$	Δ^I
$\exists r.C$	$\{x \in \Delta^I \mid \exists y \in \Delta^I s.t.(x,y) \in r^I, y \in C^I\}$

A summary of the syntax and semantics of $\mathcal{EL}$ is shown in Table 1. An interpretation I is said to be a model of (or satisfies) an axiom ϕ in the form of the left column in the table, denoted by $I \models \phi$, when the corresponding condition in the right column is satisfied. For instance, $I \models C \sqsubseteq D$ if and only if $C^I \subseteq D^I$. Similarly, I satisfies a concept (resp. role) assertion, denoted by $I \models C(a)$ (resp. $I \models r(a,b)$), if $a^I \in C^I$ (resp. $(a^I, b^I) \in r^I$). An interpretation I is a model of an ontology O if it satisfies all the axioms and assertions in O. An ontology is said to be consistent if it has a model. Otherwise, it is inconsistent. An axiom ϕ is entailed by an ontology, denoted by $O \models \phi$, if ϕ is satisfied by every model of O. We say that C is subsumed by D w.r.t. an ontology O iff $O \models C \sqsubseteq D$. Similarly, we say that a is an instance of C w.r.t. O iff $O \models C(a)$.

In this work, we assume that the input ontologies are provided in a specific normal form, to which we apply completion rules for classification. We assume that each source is in the *normal form* (Baader et al., 2005). We define it as follows:

Definition 1 (Normal form of $\mathcal{EL}$): *An $\mathcal{EL}$ TBox is in* normal form *if all concept inclusions have one of the following forms:*

$$A \sqsubseteq B, A \sqcap B \sqsubseteq C, A \sqsubseteq \exists r.B, \exists r.A \sqsubseteq B$$

where $A, B \in N_C$.

Note that, this assumption will transform the complex concept into the $\mathcal{EL}$ normal form (with the atomic concept) before the merging process is performed.

4 POSSIBILITY THEORY OVER $\mathcal{EL}$ INTERPRETATIONS

Possibility theory (Dubois and Prade, 2012) is a theory devoted to representing and reasoning with uncertain and inconsistent ontologies. In the following, we define the basic notions of this theory. Let Ω be a universe of discourse and $I = (\Delta^I, .^I) \in \Omega$ be the $\mathcal{EL}$ interpretations.

4.1 Possibility Distribution

A possibility distribution is the main block of the possibility theory, which is a function denoted by π. It is a mapping from Ω to the unit interval $[0,1]$. It assigns to each interpretation $I \in \Omega$ a possibility degree $\pi(I)$ ranged between 0 and 1, reflecting its compatibility or consistency w.r.t the available knowledge. We say that I is totally possible (i.e., fully consistent with available knowledge) when $\pi(I) = 1$ and is impossible (i.e., fully inconsistent) when $\pi(I) = 0$. Finally, given two interpretations I and I', we say that I is more consistent or more compatible than I' if $\pi(I) > \pi(I')$. A possibility distribution π is normalized if $\exists I \in \Omega$ s.t $\pi(I) = 1$, otherwise, π is subnormalized. The concept of normalization is important since it reflects the presence of conflicts in the set of available information.

4.2 Possibility and Necessity Measures

Given a possibility distribution π, standard possibility theory offers two measures from 2^{Ω} to the interval $(0,1]$ which discriminate between the plausibility and the certainty regarding an event $A \subseteq \Omega$.

A possibility measure $\Pi(A) = sup\{\pi(I) : I \in A\}$ evaluates to what extent A is compatible or plausible w.r.t available knowledge encoded by π. When $\Pi(A) = 1$ and $\Pi(\bar{A}) = 0$, we say that event A is certainly true. Furthermore, when $\Pi(\bar{A}) \in]0.1[$ we say that A is somewhat certain. Finally, we say that there exists a total ignorance about A if $\Pi(M) = 1$ and $\Pi(\bar{A}) = 1$. The possibility measure satisfies the following properties:

$$\forall A \in \Omega, \forall F \in \Omega, \Pi(A \cup F) = \max(\Pi(A), \Pi(F))$$

and

$$\forall A \in \Omega, \forall F \in \Omega, \Pi(A \cap F) \leq \min(\Pi(A), \Pi(F))$$

A necessity measure $N(A) = 1 - \Pi(\bar{A})$, which is the dual function of the possibility measure Π, evaluates to what extent A is certainty entailed from available knowledge encoded by π. When $N(A) = 1$, we say that A is certain. When $N(A) \in]0,1[$, we say that A is somewhat certain. When $N(A) = 0$ and $N(\bar{A}) = 0$, we say that there is a total ignorance about A.

A necessity measure N satisfies the following properties:

$$\forall A \subseteq \Omega, \forall L \subseteq \Omega, N(A \cap L) = min(N(A), N(L))$$

and

$$\forall A \subseteq \Omega, \forall L \subseteq \Omega, N(A \cup L) \geq max(N(A), N(L))$$

It is faithful to note that not all the subsets of Ω represent axioms in $\mathcal{EL}$, indeed the disjunction is not allowed in $\mathcal{EL}$ language. Now, we are able to determine the possibility and the necessity measures associated with the axioms (ϕ) of $\mathcal{EL}$ language. Where the possibility measure of ϕ is defined as follow:

$$\forall I \in \Omega, \Pi(\phi) = \max\{\pi(I) : I \models \phi\}$$

and its associated necessity measure is defined as follows:

$$\forall I \in \Omega, \Pi(\phi) = \max\{\pi(I) : I \not\models \phi\}$$

with $I \not\models \phi$ means that I is not a model of the axioms ϕ.

5 POSSIBILISTIC $\mathcal{EL}$

Since the practical knowledge of the VTD has some opposing statements. i.e., *Hand* and *Leg* is separate, *LeftHand* is disjoint with *RightHand*. Hence, in this section, we take advantage of the $\mathcal{EL}_{\perp}^{+}$ (Giordano et al., 2009; Kazakov et al., 2014) as a fragment of $\mathcal{EL}$. The set of $\mathcal{EL}_{\perp}^{+}$ concepts can be extended as follows:

$$C ::= \ \top \mid \perp \mid N_C \mid C \sqcap C \mid \exists r.C$$

where $r \in N_R$. Namely, the $\mathcal{EL}_{\perp}^{+}$ is added by a bottom concept ($\perp$) that its semantics is an empty set ($\emptyset$). i.e., $LeftHand \sqcap RightHand \sqsubseteq \perp$. Moreover, it extends in role inclusions $r_1 \circ \ldots \circ r_n \sqsubseteq r$, where $r_1, \ldots, r_n$, and $r \in N_r$ (see (Giordano and Dupré, 2020) for more details). For this setting, we use the "strict" $\mathcal{EL}_{\perp}^{+}$ normal form that is extended with the axioms as follows: $A \sqsubseteq B, A \sqcap B \sqsubseteq C, A \sqsubseteq \exists r.B, \exists r.A \sqsubseteq B, A \sqcap B \sqsubseteq \perp$ where $A, B \in N_C$.

Now, we define the syntax and semantics of the possibilitic extension of $\mathcal{EL}_{\perp}^{+}$, denoted by π-$\mathcal{EL}_{\perp}^{+}$. Moreover, we also present an illustration of the posibilistic ontology based on Figure 1.

5.1 Possibilistic π-$\mathcal{EL}_{\perp}^{+}$ Ontology

A possibilistic $\mathcal{EL}_{\perp}^{+}$ ontology, denoted by O_{π}, is defined by $O_{\pi} = \{(\phi_i, \alpha_i), i = 1..n\}$ where ϕ is $\mathcal{EL}_{\perp}^{+}$ axioms and $\alpha \in (0,1]$ its certainty degree, meaning that $N(\phi_i) \geq \alpha_i$. Note that the higher is α the more the axiom is certain. Only the axioms having a degree strictly greater than 0 are explicitly represented in the ontology. However, when the axioms have a degree equal to 1 then O_{π} coincides with the standard $\mathcal{EL}_{\perp}^{+}$ ontology, which is denoted by O, with $O = \{\phi_i, i = 1..n\}$.

Example 1: *In order to illustrate explicitly ontology merging, we take advantage of existing operators of the $\mathcal{EL}_{\perp}^{+}$ setting to represent the orientation characteristics of VTDs and relevant primary concepts (body-parts concepts).*

Firstly, we take an account of TBox of VTDs Ontology 1 denoted by $\mathcal{TO}_{\pi_1}$, including:

$$\mathcal{TO}_{\pi_1} = \begin{cases} Orientations \sqsubseteq VTD-Movements, 0.8 \\ VNOri1 \sqsubseteq Orientations, 0.9 \\ VNOri2 \sqsubseteq Orientations, 0.95 \\ VNOri7 \sqsubseteq Orientations, 0.87 \\ VNOri8 \sqsubseteq Orientations, 0.93 \\ Hands \sqsubseteq BodyParts, 1.0 \\ LeftHand \sqsubseteq Hands, 0.78 \\ RightHand \sqsubseteq Hands, 0.89 \\ LeftHand \sqcap RightHands \sqsubseteq \perp, 0.85 \\ Shoulders \sqsubseteq BodyParts, 0.98 \\ RightShoulder \sqsubseteq Shoulders, 1.0 \\ LeftShoulder \sqsubseteq Shoulders, 0.89 \\ LeftHand \sqsubseteq \exists hasPosIn.VNOri8, 0.75 \\ RightHand \sqsubseteq \exists hasPosIn.VNOri2, 0.83 \\ LeftShoulder \sqsubseteq \exists hasPosIn.VNOri7, 0.8 \\ RightShoulder \sqsubseteq \exists hasPosIn.VNOri3, 0.9 \\ RightLeg \sqsubseteq \exists hasPosIn.VNOri1, 1.0 \end{cases}$$

Based on the description of section II regarding orientations features, we implement the concepts of VTD as in $\mathcal{TO}_{\pi_1}$ and also implement one relation (a role) between body parts concepts and dance orientations concepts such as:

- **hasPosIn:** gives for each body part corresponding to a specific orientation, for example, the left shoulder has a posture in orientation 3.

In this ontology, each axiom is attached with a necessity degree reflecting its certainty with the available knowledge. The axiom ($Orientations \sqsubseteq VTD-Movements, 0.8$), means that dance orientations may be considered as one of the fundamental motions (movements) of VTD with a possibilistic degree greater or equal to 0.7. Furthermore, the axiom ($RightLeg \sqsubseteq \exists hasPosIn.VNOri1, 1.0$) is a fully certain axiom because the possibility of this motion is attached to a degree equal to 1.0, meaning that the right leg of the performer has a posture in orientation 1 that is undoubting.

5.2 From π-$\mathcal{EL}_{\perp}^{+}$ Ontology to π-$\mathcal{EL}_{\perp}^{+}$ Possibility Distribution

As in standard $\mathcal{EL}$, the semantics of the possibility $\mathcal{EL}_{\perp}^{+}$ontology is defined by the possibility distribution, denoted by π_{O_π}, defined over the set of all interpretations of $\mathcal{EL}$ language. The possibility distribution can be interpreted in two ways: i) a numerical interpretation when there is a real sense, ii) an ordinal interpretation when there exists a preference ranking between pieces of information. In this paper, we will focus on the latter interpretation which is appropriate when there exists a total pre-order or preference ranking between pieces of information reflecting their reliability or consistency with the available knowledge. The possibility distribution π_{O_π} assigns to each interpretation $I \in \Omega$ a possibility degree $\pi(I) \in (0,1]$ reflecting what extent this latter satisfies the axioms of the ontology. In the following, we will define the possibility distribution associated with $\mathcal{EL}_{\perp}^{+}$ontology.

Definition 2: $\forall I \in \Omega$

$$\pi_{O_\pi}(I) = \begin{cases} 1 \ \ if \forall(\phi_i, \alpha_i) \in O_\pi, I \models \phi_i \\ 1 - \max\{\alpha_i : (\phi_i, \alpha_i) \in O_\pi, I \not\models \phi\} \ ow. \end{cases}$$

where ow is otherwise.

Example 2: *We will continue with the Example 1 to illustrate explicitly for Definition 2. Let π_1 be the possibility distribution associated with $\mathcal{TO}_{\pi_1}$ and let $\{I_1, I_2, I_3\}$ be three interpretations in $\mathcal{TO}_{\pi_1}$. Assuming that I_1, I_2, I_3 are satisfied most of the axioms except some axioms as follows:*

- *For I_1 **is NOT satisfied (unsatisfied)** following axioms:* $I_1 \not\models (LeftHand \sqsubseteq \exists hasPosIn.VNOri8, 0.75)$ $I_1 \not\models (LeftShoulder \sqsubseteq \exists hasPosIn.VNOri7, 0.8)$
- *For I_2 **is NOT satisfied (unsatisfied)** following axioms:* $I_2 \not\models (RightHand \sqsubseteq \exists hasPosIn.VNOri2, 0.83)$ $I_2 \not\models (RightLeg \sqsubseteq \exists hasPosIn.VNOri1, 1.0)$
- *For I_3 **is SATISFIED ALL** of the axioms*

From Definition 2, possibility distributions of I_1, I_2, I_3 as follows:

$$\Rightarrow \begin{cases} \pi_{\mathcal{TO}_{\pi_1}}(I_1) = (1 - \max(0.8, 0.75) = 0.2) \\ \pi_{\mathcal{TO}_{\pi_1}}(I_2) = (1 - \max(0.83, 1.0) = 0) \\ \pi_{\mathcal{TO}_{\pi_1}}(I_3) = 1.0 \ , for \ I_3 \models \phi_i \end{cases} \tag{1}$$

An ontology is said to be consistent if there exists at least one interpretation that satisfies all the axioms of the ontology, i.e., $\pi_{O_\pi}(I) = 1$. Otherwise, the ontology is inconsistent and their inconsistency is defined by the following expression:

$$\forall I \in \Omega, Inc(O_\pi) = 1 - \max\{\pi(I)\}$$

Therefore, from Example 2 and (1), we also said that the VTD ontology $\mathcal{TO}_{\pi_1}$ is consistent because as the above expression is $\pi_{\mathcal{TO}_{\pi_1}}(I_3) = 1.0$ as well as

$$Inc(\mathcal{TO}_{\pi_1}) = 1 - \max(0.2, 0, 1) = 0$$

$\Rightarrow \mathcal{TO}_{\pi_1}$ is consistent

6 MIN-BASED MERGING OF π-$\mathcal{EL}_\perp^+$ POSSIBILITY DISTRIBUTIONS

Several fusion operators have been proposed in order to manage the problem of merging n uncertain pieces of information represented by n possibility distributions (Benferhat et al., 1997; Benferhat and Kaci, 2003; Benferhat et al., 2013), these fusion operators aim to obtain a unique possibility distribution from the set of possibility distributions. As we cited above, the pieces of information can be represented syntactically by a set of weighted formulas and semantically by a set of interpretations.

In this paper, we focus on the fusion operators, i.e., we will study the syntactic approach which consists of merging possibilistic $\mathcal{EL}_\perp^+$ ontology. However, in this section, we will focus on the semantic approach which consists of combining the possibility distributions associated with each possibilistic $\mathcal{EL}_\perp^+$ ontology using the min-based operator. Note that the min-based operator holds when the sources of information are dependent. Let $\{\pi_1, ..., \pi_n\}$ be a set of possibility distributions provided by n sources of information. All the sources use the same scale to represent uncertainty and share the same domain of interpretations $\{\Delta_1 = \ldots = \Delta_n\}$.

Definition 3: *Let $list(I) = \{\pi_1(I), \ldots, \pi_n(I)\}$ be a list that contains n possibility values. The min-based operator is a mapping from $list(I)$ to the unit interval $[0.1]$ and it is defined as follow:*

$$\forall I \in \Omega, \pi_\oplus(I) = \min(list(I))$$

Example 3: *To demonstrate a case of inconsistency, we represent the TBox of VTDs Ontology 2 identified by $\mathcal{TO}_{\pi_2}$ with specific probabilities as follows:*

$$\mathcal{TO}_{\pi_2} = \begin{cases} Orientations \sqsubseteq VTD-Movements, 0.8 \\ VNOri2 \sqsubseteq Orientationsm, 0.95 \\ VNOri3 \sqsubseteq Orientations, 0.80 \\ VNOri8 \sqsubseteq Orientations, 0.93 \\ Hands \sqsubseteq BodyParts, 1.0 \\ LeftHand \sqsubseteq Hands, 0.78 \\ RightHand \sqsubseteq Hands, 0.89 \end{cases}$$

$$\mathcal{TO}_{\pi_2} = \begin{cases} LeftHand \cap RightHands \sqsubseteq \perp, 0.85 \\ Shoulders \sqsubseteq BodyParts, 0.98 \\ RightShoulder \sqsubseteq Shoulders, 1.0 \\ LeftShoulder \sqsubseteq Shoulders, 0.89 \\ LeftHand \sqsubseteq \exists hasPosIn.VNOri8, 0.75 \\ RightHand \sqsubseteq \exists hasPosIn.VNOri2, 0.85 \\ LeftShoulder \sqsubseteq \exists hasPosIn.VNOri7, 0.8 \\ RightShoulder \sqsubseteq \exists hasPosIn.VNOri3, 0.3 \\ LeftLeg \sqsubseteq \exists hasPosIn.VNOri1, 0.78 \end{cases}$$

In the same way, regarding the possibilistic $\mathcal{EL}_\perp^+$ontology $\mathcal{TO}_{\pi_2}$, let consider the three interpretations, namely I_1, I_2, and I_3 being the axioms in $\mathcal{TO}_{\pi_2}$. Assuming that I_1, I_2, I_3 are satisfied most of the axioms except some axioms as follows:

- *For I_1 **is NOT satisfied (unsatisfied)** a following axiom: $I_2 \not\models LeftHand \sqsubseteq \exists hasPosIn.VNOri8, 0.75$*
- *For I_2 **is NOT satisfied (unsatisfied)** following axioms:*
  $I_2 \not\models LeftShoulder \sqsubseteq Shoulders, 0.89$
  $I_2 \not\models LeftShoulder \sqsubseteq \exists hasPosIn.VNOri7, 0.8$
- *For I_3 **is NOT satisfied (unsatisfied)** a following axioms: $I_2 \not\models LeftLeg \sqsubseteq \exists hasPosIn.VNOri1, 0.78$*

From Definition 2, possibility distributions of I_1, I_2, I_3 as follows:

$$\Rightarrow \begin{cases} \pi_{\mathcal{TO}_{\pi_2}}(I_1) = (1 - max(0.75) = 0.25) \\ \pi_{\mathcal{TO}_{\pi_2}}(I_2) = (1 - max(0.89, 0.8) = 0.11) \\ \pi_{\mathcal{TO}_{\pi_2}}(I_3) = (1 - max(0.78) = 0.22) \end{cases} \quad (2)$$

From the expression as Example 2, we also said that the VTD ontology $\mathcal{TO}_{\pi_2}$ is inconsistent because as the above expression is

$Inc(\mathcal{TO}_{\pi_2}) = 1 - max(0.25, 0.11, 0.22) = 0.75$

$\Rightarrow \mathcal{TO}_{\pi_2}$ *is inconsistent*

According to Definition 3 with min-based operator, we can **merge the two possibility distributions** represented respectively in Example 2 and Example 3.

Example 4: *We continue with Example 2 and Example 3. The merged possibility distribution, denoted by ($\pi_{\oplus VTD}$), is the act of merging two possibility distributions ($\pi_{\mathcal{TO}_{\pi_1}}$) and ($\pi_{\mathcal{TO}_{\pi_2}}$) represented respectively in Example2 and Example3. Let $\{I_1, I_2, I_3\}$ be three interpretations. The result of the min-base operator is*
$\Rightarrow$ $\pi_{\oplus}(I_1) = \min(list(I_1)) = \min(0.2, 0.25) = 0.2$, *and* $\pi_{\oplus}(I_2) = \min(list(I_2)) = \min(0, 0.11) = 0$, *and* $\pi_{\oplus}(I_3) = \min(list(I_3)) = \min(1.0, 0.22) = 0.22$.

The merged possibility distribution should satisfy the following properties: Let consider the two interpretations $I \in \Omega$ and $I' \in \Omega$,

$$\forall I, I' \in \Omega, \begin{cases} \text{When } \pi(I) \leq \pi(I'), \text{ then } \pi_{\oplus}(I) \leq \pi_{\oplus}(I') \\ \text{When } \pi(I) = 1, \text{ then } \pi_{\oplus}(I) = 1. \end{cases}$$

Based on the Example 4, note that the use of a minimum operator for merging two normalized possibility distributions leads to sub-normalized possibility distribution. Indeed, to solve this problem, let's consider the following expression: $\forall I \in \Omega, L(\pi_{\oplus}) = \max\{\pi_{\oplus}(I)\}$. This function states that there exists at least one interpretation that is satisfied by all sources. Let (π_{O_π}) be a possibility distribution and ($\pi_{N_\oplus}$) be their normalized possibility distribution.

Definition 4: *The normalized possibility distribution is defined by the following expression:*

$$\forall I \in \Omega \;\; and \;\; L(\pi_{\oplus}) > 0, \pi_{N_{\oplus}}(I) = \begin{cases} 1 \;\; if \;\; \pi_{\oplus}(I) = L(\pi_{\oplus}) \\ \pi_{\oplus}(I) \;\; otherwise. \end{cases}$$

In the following, we will use the Definition 4 to represent the normalized possibility distributions.

Example 5: *Let us continue with Example 4. Let I_1, I_2, I_3 be three interpretations. Let $\pi_{N_\oplus}$ be the normalized possibility distribution of $\pi_{\oplus}$. Then*
$\Rightarrow \pi_{N_\oplus}(I_1) = 0.2$, $\pi_{N_\oplus}(I_2) = 0$ *and* $\pi_{N_\oplus}(I_3) = 1.0$.
Therein, $\pi_{N_\oplus}(I_3) = 1.0$ *since* $L(\pi_{\oplus}) = \max\{0.2, 0, 0.22\} = 0.22$ *and* $\pi_{\oplus}(I_3) = L(\pi_{\oplus}) = 0.22$.

Intuitively, the normalized possibility distribution consists of comparing the merged possibility distributions with the function $L(\pi_{\oplus})$. When $\pi_{\oplus}(I)$ is equal to $L(\pi_{\oplus})$, then the normalized possibility distribution is equal to 1.0. However, when $\pi_{\oplus}(I)$ is different to $L(\pi_{\oplus})$, then the normalized possibility distribution is equal to the merged possibility distribution.

7 SYNTACTICAL MERGING OF π-$\mathcal{EL}_{\perp}^{+}$ ONTOLOGY

In this section, we will define the syntactical counterpart of merging π-$\mathcal{EL}_{\perp}^{+}$ ontology. Let $\{O_{\pi_1}, ..., O_{\pi_n}\}$ be a set of π-$\mathcal{EL}_{\perp}^{+}$ ontology provided by n different sources. Each O_{π_i} is associated with a possibility distribution π_i.

Definition 5: *Let* $(\phi_i, \alpha_i) \in O_{\pi_1}$ *and* $(\varphi_i, \beta_i) \in O_{\pi_2}$. *The syntax of the min-based operator $\oplus$ is defined by the following expression:*

$$\begin{aligned} O_{\oplus} &= O_{\pi_1} \cup O_{\pi_2} \cup \{\phi_i \wedge \varphi_i, \max(\alpha_i, \beta_i)\} \\ &= O_{\pi_1} \cup O_{\pi_2} \end{aligned}$$

Now, based in Definition 5, we are able to merge $\mathcal{TO}_{\pi_1}$ and $\mathcal{TO}_{\pi_2}$ represented respectively in Example 2 and Example 3 in order to obtain $O_{\oplus VTD}$. Furthermore, their semantics is defined in Example 4. The result of merging possibilistic $\mathcal{EL}_{\perp}^{+}$ ontology will be presented in the next section.

Note that the merged possibilistic $\mathcal{EL}_{\perp}^{+}$ ontology can be **an inconsistent ontology**, since merging two consistent possibilistic $\mathcal{EL}_{\perp}^{+}$ ontologies does not always lead to a consistent merged ontology. In the following, we will define how to determine the consistency degree of the merged possibilistic $\mathcal{EL}_{\perp}^{+}$ ontology:

Definition 6: *Let $O_{\pi > \alpha}$ be a sub-ontology that contains axioms having a degree greater than alpha. The consistency degree of $\mathcal{EL}_{\perp}^{+}$ ontology, denoted by Inc(O_π), is syntactically defined by the following expression:*

$$Inc(O_\pi) = \max\{\alpha : O_{\pi_\alpha} \;\; is \; the \; inconsistency\}$$

8 THE RESULT OF MERGING DANCE $\mathcal{EL}$ ONTOLOGIES

In this section, we present the result of merging $\mathcal{EL}_{\perp}^{+}$ ontologies based on a potential example introduced in section 2 and represented their possibility distributions in Example 2 and Example 3. We would also present a condensed summary of the result of min-based merging through Table 2.

To sum up, from Definition 3 and 4, we have a merged possibilistic distribution table (semantics) of the result min-based merging of π-$\mathcal{EL}_{\perp}^{+}$ possibility distributions as follows:

Table 2: The result of min-based merging.

Min-based merging of π-$\mathcal{EL}$ possibility distributions				
I_i	$\pi_{\mathcal{T}O_{\pi_1}}$	$\pi_{\mathcal{T}O_{\pi_2}}$	$\pi_{\oplus VTD}$	$\pi_{N_{\oplus VTD}}$
	(1)	(2)	$min(list(I))$	
I_1	0.2	0.25	0.2	0.2
I_2	0	0.11	0	0
I_3	1.0	0.22	0.22	1.0

The following is an unambiguous explanation regarding Table 2 of processing min-based merging $\mathcal{EL}$ ontologies including: first of all, possibilistic distributions ($\pi_{\mathcal{T}O_{\pi}}$) of $\mathcal{T}O_{\pi_1}$ and $\mathcal{T}O_{\pi_2}$ presented in Column 1 and Column 2. Secondly, Column 3 is the merged possibilistic distribution based on the min-based merging operator ($\pi_{\oplus VTD}$) in Definition 3. Finally, (3) Column 4 is the normalized possibility distribution with

$$L(\pi_{\oplus VTD}) = max(\pi_{\oplus VTD}(I))$$
$$= max(0.2, 0, 0.22) = 0.22$$

introduced in Definition 4, therefore, axiom I_3, in this case, is equal to 1.0 (normalization) because

$$\pi_{\oplus VTD}(I) = L(\pi_{\oplus VTD}) = 0.22$$

In terms of the syntactical merging of $\mathcal{EL}_{\perp}^{+}$ontologies, we have the following merged results:

Let $O_{\oplus VTD}$ be a merged possibilitic $\mathcal{EL}_{\perp}^{+}$ ontology in which based on merging VTD probabilistic Ontology 1 ($\mathcal{T}O_{\pi_1}$) and VTD probabilistic Ontology 2 ($\mathcal{T}O_{\pi_2}$) represented in above parts of this paper. We here apply Definition 5 to consider syntactical merging of π-$\mathcal{EL}_{\perp}^{+}$ Ontology, we then obtain the results defined $O_{\oplus VTD}$ as follows:

$$\mathcal{T}O_{\oplus VTD} = \begin{cases} Orientations \sqsubseteq VTD - Movements, 0.8 \\ VNOri1 \sqsubseteq Orientations, 0.9 \\ VNOri2 \sqsubseteq Orientationsm, 0.95 \\ VNOri3 \sqsubseteq Orientations, 0.80 \\ VNOri7 \sqsubseteq Orientations, 0.87 \\ VNOri8 \sqsubseteq Orientations, 0.93 \\ Hands \sqsubseteq BodyParts, 1.0 \\ LeftHand \sqsubseteq Hands, 0.78 \\ RightHand \sqsubseteq Hands, 0.89 \\ LeftHand \cap RightHands \sqsubseteq \perp, 0.85 \\ Shoulders \sqsubseteq BodyParts, 0.98 \\ RightShoulder \sqsubseteq Shoulders, 1.0 \\ LeftShoulder \sqsubseteq Shoulders, 0.89 \\ LeftHand \sqsubseteq \exists hasPosIn.VNOri8, 0.75 \\ RightHand \sqsubseteq \exists hasPosIn.VNOri2, 0.85 \\ LeftShoulder \sqsubseteq \exists hasPosIn.VNOri7, 0.8 \end{cases}$$

$$\mathcal{T}O_{\oplus VTD} = \begin{cases} RightShoulder \sqsubseteq \exists hasPosIn.VNOri3, 0.9 \\ RightLeg \sqsubseteq \exists hasPosIn.VNOri1, 1.0 \\ LeftLeg \sqsubseteq \exists hasPosIn.VNOri1, 0.78 \end{cases}$$

Following the Definition 5, a merged possibilistic $\mathcal{EL}_{\perp}^{+}$ ontology is TBox union of the other possibilistic ontologies and the maximum selection of the possibilities between those axioms for the same concepts. For an explicit explanation of the maximum alternative, considering the same concept

$$RightShoulder \sqsubseteq \exists hasPosIn.VNOri3$$

has a possibility set to 0.9 in $\mathcal{T}O_{\pi_1}$ being greater than a possibility 0.3 of $\mathcal{T}O_{\pi_2}$, therefore, we in this case select the concept of $\mathcal{T}O_{\pi_1}$ with ($max(\alpha_i, \beta_i) = max(0.9, 0.3) = 0.9$) satisfied by Definition 5. Other axioms are considered similar.

9 CONCLUSION

In this paper, we investigated a fragment of lightweight description logic $\mathcal{EL}$, named $\mathcal{EL}_{\perp}^{+}$, with the possibility theory. This logic is suitable for applications such as biology and medicine. First, we provided their syntax and semantics. Then, we studied the merging $\mathcal{EL}_{\perp}^{+}$ possibility distributions using the minimum operator. Furthermore, we defined the merging possibilistic $\mathcal{EL}_{\perp}^{+}$ ontology using the same operator.

Finally, we also represented a potential example of merging the prioritized $\mathcal{EL}$ ontologies regarding the dance domain. It would be a significant foundation to build a universal application for Vietnamese traditional dance management from many different sources.

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