Distance Transform in Parallel Logarithmic Complexity

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Abstract: Nowadays a huge amount of digital data are generated every moment in a broad spectrum of application domains such as biomedical imaging, document processing, geosciences, remote sensing, video surveillance, etc. Processing such big data requires an efficient data structure, encouraging the algorithms with lower complexity and parallel operations. In this paper, first, a new method for computing the distance transform (DT) as the fundamental operation in binary images is presented. The method computes the DT with the parallel logarithmic complexity $O(log(n))$ where n is the maximum diameter of the largest foreground region in the 2D binary image. Second, we define the DT in the combinatorial map (CM) structure. In the CM, by replacing each edge with two darts a smoother DT with the double resolution is derived. Moreover, we compute n different distances for the nD-map. Both methods use the hierarchical irregular pyramid structure and have the advantage of preserving topological information between regions. The operations of the proposed algorithms are totally local and lead to parallel implementations. The GPU implementation of the algorithm has high performance while the bottleneck is the bandwidth of the memory or equivalently the number of available independent processing elements. Finally, the logarithmic complexity of the algorithm speeds up the execution and suits it, particularly for large images.

1 INTRODUCTION AND MOTIVATION

The distance transform (DT) (Rosenfeld and Pfaltz, 1966) is a fundamental operation of many methods in pattern recognition and geometry. It is used in a wide range of applications such as skeletonization (Niblack et al., 1992), map matching robot self-Localization (Sobreira et al., 2019), image registration (Hill and Baldock, 2015), template matching (Prakash et al., 2008; Lindblad and Sladoje, 2014), Line Detection in Manuscripts (Kassis and El-Sana, 2019), Weather Analysis and Forecasting (Brunet and Sills, 2017), etc. The DT is applied to a binary image containing background and foreground regions. The result of the transform is a new gray-scale image whose foreground pixels have intensities representing the minimum distance from the background.

To compute the DT, the common algorithms (Rosenfeld and Pfaltz, 1966; Nilsson and Söderström, 2007; Fabbri et al., 2008) propagate the distances in linear-time $O(N)$, where N is the number of grid cells or pixels in a 2D binary image. In contrast, in this paper we propose a novel method that propagates the distances in an exponential way and computes the DT with $O(log(n))$ where n is the diameter of the largest connected component in the binary image. To this aim, we employ the hierarchical structure of the irregular graph pyramid (Kropatsch, 1995; Brun and Kropatsch, 2012; Banaeyan and Kropatsch, 2021). In addition, we define the DT over the combinatorial map (CM) that not only results in a finer resolution of the distance map but also provides different distances for different dimensions employing map-edit-distance (Combier et al., 2013) in analogy to the graph edit distance (Gao et al., 2010).

Currently we are working on the *Water's gateway to heaven* project[1] dealing with high-resolution X-ray micro-tomography (μCT) and fluorescence microscopy. The size of the images is more than 2000 in each dimension where we need the DT to separate cells, which are visually difficult to be separated. Therefore, fast computation of the DT with low complexity is required.

In this study, the proposed algorithm has logarithmic complexity and efficiently computes the *city block* (L_1 norm) distance metric. The next Section recalls the irregular graph pyramid and the combinatorial map. In Section 2, the new algorithm to com-

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pute the DT is presented. Section 3 introduces the DT over the combinatorial map and proposes a novel algorithm to compute the DT. Finally, Section 4 compares the execution of the proposed algorithm with the state-of-the-art.

1.1 Definitions

A digital image can be represented using a 4-adjacent neighborhood graph. Let $G = (V,E)$ be the neighborhood graph of image P where V corresponds to P and E relates neighboring pixels. Let the gray-value $g(v)$ of a vertex v be $g(p)$. The $contrast(e)$ is an attribute of an edge $e(u,v)$ where $u,v \in V$ and $contrast(e) = |g(u) - g(v)|$. In the binary images, the pixels (and corresponding vertices) have either of the two values 0 and 1. Similarly, the edge contrast has only two possible values 0 and 1.

In the neighborhood graph of the binary image, the edges with zero and one contrast are defined as **zero-edge**, e_0, and **one-edge**, e_1, respectively. Therefore, the edges of the graph are partitioned into $E = E_0 \cup E_1$ where $e_0 \in E_0$ and $e_1 \in E_1$.

1.1.1 Irregular Pyramids

Irregular pyramids (Kropatsch, 1995) are a stack of successively reduced smaller graphs where each graph is built from the graph below by selecting a specific subset of vertices and edges. Two basic operations are used to construct the pyramid: edge contraction and edge removal. In the edge contraction, an edge $e = (v,w)$ is contracted while its two endpoints, v and w, are identified and the edge is removed. The edges that were incident to the joined vertices will be incident to the resulting vertex after the operation. In edge removal, an edge is removed without changing the number of vertices or affecting the incidence relationships of other edges. In each level of the pyramid, the vertices and edges disappearing in the level above are called *non-surviving* and those appearing in the upper-level *surviving* ones.

Definition 1 (Contraction Kernel (CK))**.** *A CK is a tree consisting of a surviving vertex as its root and some non-surviving neighbors with the constraint that every non-survivor can be part of only one CK (Banaeyan and Kropatsch, 2022a).*

An edge of a CK is denoted by the directed edge and points towards the survivor.

1.1.2 Combinatorial Pyramids

A combinatorial pyramid is a hierarchy of successively reduced combinatorial maps (Brun and Kropatsch, 2003; Brun and Kropatsch, 2012). In the CM each edge is encoded by two half-edges where each half-edge is called a *dart*, $d \in \mathcal{D}$ where $\mathcal{D}$ is a finite set of darts. The CM encodes the edges around each vertex by using the α and the σ as an *involution* and a *permutation* on the set of $\mathcal{D}$, respectively. The σ encodes consecutive edges around the same vertex while turning counterclockwise. The clockwise orientation is denoted by σ^{-1}. The α provides a one-to-one mapping between consecutive darts forming the same edge such that $\alpha(\alpha(d)) = d$.

2 LOGARITHMIC DT USING THE IRREGULAR PYRAMID

In the linear algorithms (Nilsson and Söderström, 2007) the DT is propagated between one vertex (pixel) and its adjacent vertex (pixel) in each step of the propagation. Consider a 1D grid of N pixels aligning in a horizontal line. In order to propagate the DT from the most-left pixel to the most-right pixel, $N-1$ steps are needed. However, thanks to the hierarchical structure of the pyramid with logarithmic height, such propagation can be performed only in $log(N)$ steps as we will see in Section 2.3. In the pyramid, two vertices of a connected component that are not adjacent (and may be far from each other) at the base level, may become adjacent at the upper levels of the pyramid.

2.1 Initialization

To compute the DT, the first step is an initialization procedure where the endpoints of the E_1 receive $DT = 1$ and the remaining vertices receive the $DT = \infty$. Note that, the proposed algorithm computes the DT for both background and foreground regions simultaneously. This is the reason why in the initialization step we assign the $DT = 1$. The common algorithms for computation of the DT consider the background as a region with $DT = 0$. However, to convert the DT of the proposed algorithm to the common algorithms, it needs only to substitute the $DT = 0$ of the background pixels.

2.2 Selecting the CKs

Selecting the CKs is the main procedure in constructing the pyramid. To this aim, we use the proposed method in (Banaeyan et al., 2022). First, an index is assigned to each vertex. Using the total order set defined over the indices, each vertex has a unique integer index, Idx(.). Each non-surviving vertex selects one surviving vertex with maximum Idx of its neighborhood (Banaeyan et al., 2022):

$$v_s = \text{argmax}\{Idx(v_s)|\ v_s \in \mathcal{N}_0(v), |\mathcal{N}_0(v)| > 1\} \quad (1)$$

where

$$\mathcal{N}_0(v) = \{v\} \cup \{w \in V | e_0 = (v,w) \in E_0\} \quad (2)$$

The E_1 are the edges between two different CCs and the E_0 are the edges inside a CC. Since the E_1 have their $DT = 1$, we do not contract the E_1, and therefore, the CKs are selected only from the E_0. In addition, each edge $e_0 = (v,w)$ of the CKs has an orientation from v to w where the w has the largest index among the neighbors, $Idx(w) = \max\{Idx(u)|u \in \mathcal{N}_0(v)\}$.

2.3 Contracting the Selected CKs

In a CK, the adjacent edges are *dependent* and cannot be contracted at the same time. Two dependent edges by definition are adjacent edges sharing one endpoint. Those edges not sharing an endpoint are defined as *independent* edges.

Proposition 1. *A path of length N, can be contracted at maximum in* $[log_2(N)] + 1$ *steps.*

Proof. In the path of length N, every other edges have no endpoints in common and hence they are independent. As a result, such independent edges are contracted in one step. In the resulting induced graph, again, every other edges are independent and they can be contracted in one step. By iterating such procedure, the path of length N is contracted at maximum in $[log_2(N)] + 1$ steps. The number of required steps is equal to $log_2(N)$ when $N = 2^n$. □

Fig. 1 shows a 1D grid containing 16 vertices. The 1D grid is considered as the path with a length of 15 and it can be contracted in 4 steps. In each step the oriented edges are independent and they are contracted simultaneously. The priorities of the contractions are encoded by numbers 1 to 4 and the oriented edges are independent.

2.4 Logarithmic DT in 1D Grid

To compute the DT in 1D, the irregular pyramid is constructed in the bottom-up fashion. To this aim, the independent edges are identified based on the priorities of contractions. During the construction procedure, only the independent edges having two unknown DT at their endpoints are contracted. After the contractions, the vertices with known DT propagate their DT to their adjacent vertices at each level of the pyramid. Such propagation iterates until we reach to the top of the pyramid where there is no edge remaining for the contraction and all the vertices at this level have their own DT.

The next step is to traverse the irregular pyramid in the top-down procedure. In the top-down processing, each vertex inherits its DT to the same vertex at a level below. Afterward, the distances are propagated into their adjacent vertices. Such procedures iterate in each level until we reach to the base where all the vertices receive their own DT.
Note that the DT in each level of the irregular pyramid is propagated as follows:

$$D(v_i) = \min\{D(v_i), D(v_j) + |Idx(v_i) - Idx(v_j)| \\ |\ v_j \in \mathcal{N}(v_i)\} \quad (3)$$

Algorithm 1: Computing DT in a grid structure.

```
    Input: Neighborhood Graph: $G = (V,E)$
 2: Initialization: $DT = \infty$, $\forall v \in V$
    $DT = 1$, $\forall v \in E_1$
 4: Propagating the distances to adjacent neighbors
    Selecting the CKs (Bottom-up traversing)
 6: While ($DT = \infty$ in the current level)
    Contracting the edges
 8: Propagating the distances to adjacent neighbors
    end (Top of the Pyramid)
10: For ($j = L$ downto $1$) (Top-down traversing)
    Imitate the DT from $L \rightarrow L-1$
12: Propagating the distances to adjacent neighbors
    end
```

Fig. 2 shows the computing of the DT in 1D grid by using the irregular pyramid. The Alg. 1 summarizes the steps of computing the DT in the grid structure by using the irregular pyramid.

2.5 Logarithmic DT in 2D Grid

Consider the binary image has M rows and N columns such that $(1,1)$ is the coordinate of the pixel ($p \in P$) at the upper-left corner and (M,N) at the lower-right corner. The corresponding 4-adjacent neighborhood graph of the binary image has MN vertices. An index $Idx(.,.)$ of each vertex is defined (Banaeyan and Kropatsch, 2022b):

$$Idx : [1,M] \times [1,N] \mapsto [1, M \cdot N] \subset \mathbb{N} \quad (4)$$

$$Idx(r,c) = (c-1) \cdot M + r \quad (5)$$

where r and c are the row and column of the pixel, respectively. The Alg. 1 is used for computing the DT in the 2D grid as well. Here, The DT is propagated as

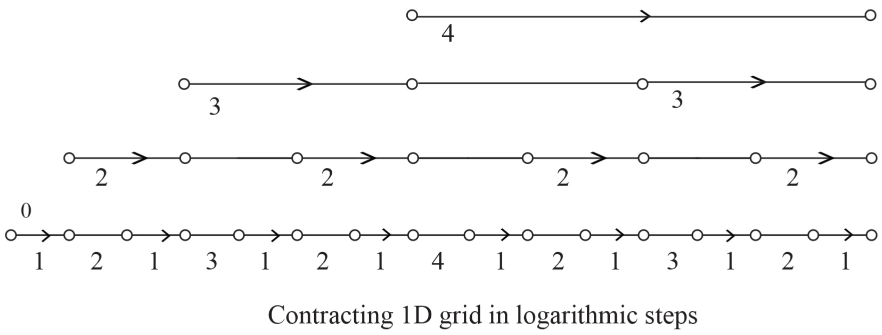


Figure 1: Edge contractions in the logarithmic way.

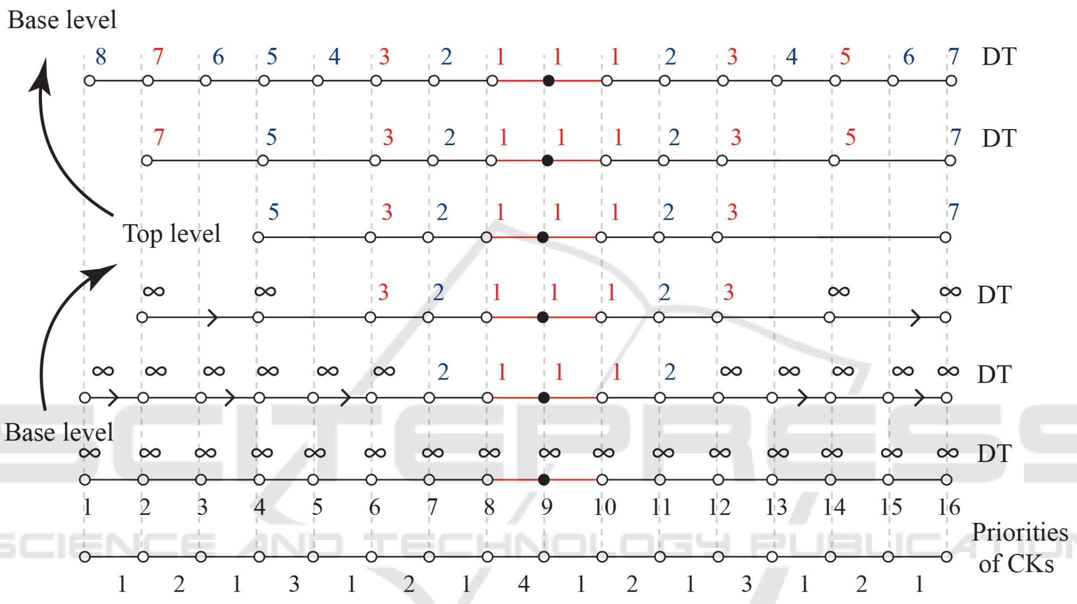


Figure 2: Computing of the DT in 1D grid.

follows:

$$D(v_i) = \min\{D(v_i), D(v_j) + \begin{cases} 1 & if\ T = 1 \\ \frac{T}{M} & if\ T \neq 1 \end{cases} \} \tag{6}$$

where

$$v_j \in \mathcal{N}(v_i), \quad T = |Idx(v_i) - Idx(v_j)| \tag{7}$$

An example of computing the DT in a 2D binary image is shown in Fig. 3.

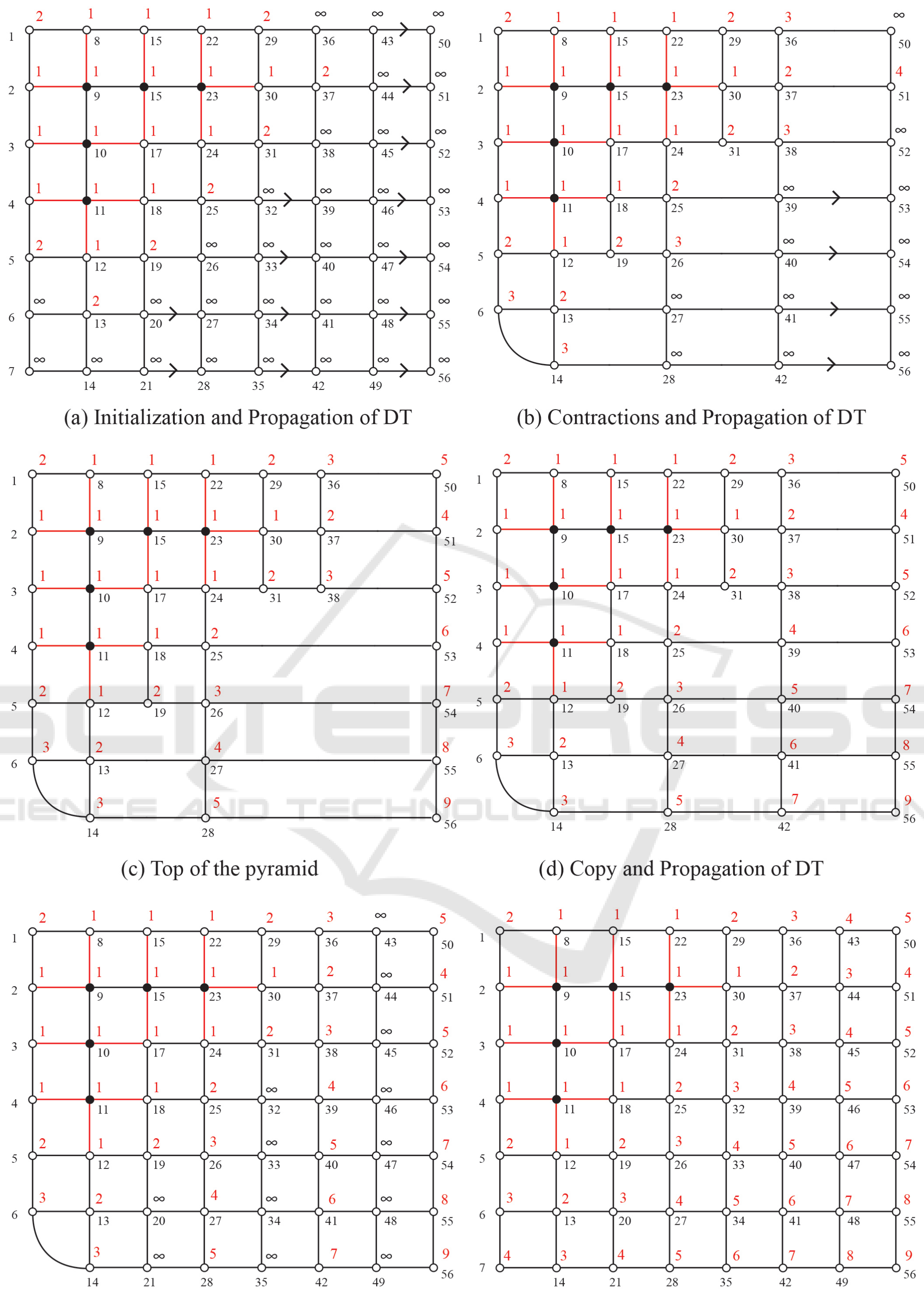
3 DEFINING THE DT IN A COMBINATORIAL MAP

The distance transform [1] computes for every pixel/voxel of an image/object how far it is from the closest obstacle, or boundary, or background. Different metrics can be used. In a topological data structure like a graph, a combinatorial map (Lienhardt, 1991), or a generalized map (Sansone et al., 2016) often the shortest path between the obstacle/boundary and a given point is used.

Let $(\mathcal{D}, \alpha, \sigma)$ denote a two-dimensional combinatorial map (2map). There are two versions of distance transform on a 2map. One considers the edges $\alpha^*(d)$ as a unit and counts the number of edges to follow as the distance. This corresponds to the distance in graphs. The alternative considers the darts $d \in \mathcal{D}$ as a unit and the following neighbors for propagating distances:

$$\Gamma_{2map}(d) = \{\alpha(d), \sigma(d), \sigma^{-1}(d)\} \tag{8}$$

In the combinatorial map, each edge is replaced by two darts. Therefore, computing the DT for darts provides double resolution for the resulting distance map. Moreover, for every dimension $1, \ldots, n$ we receive one distance, the distance through the highest dimension n, and the (larger) geodesic distance along the bounding $i-cell$, $0 < i < n$. This characterizes more of a



(a) Initialization and Propagation of DT

(b) Contractions and Propagation of DT

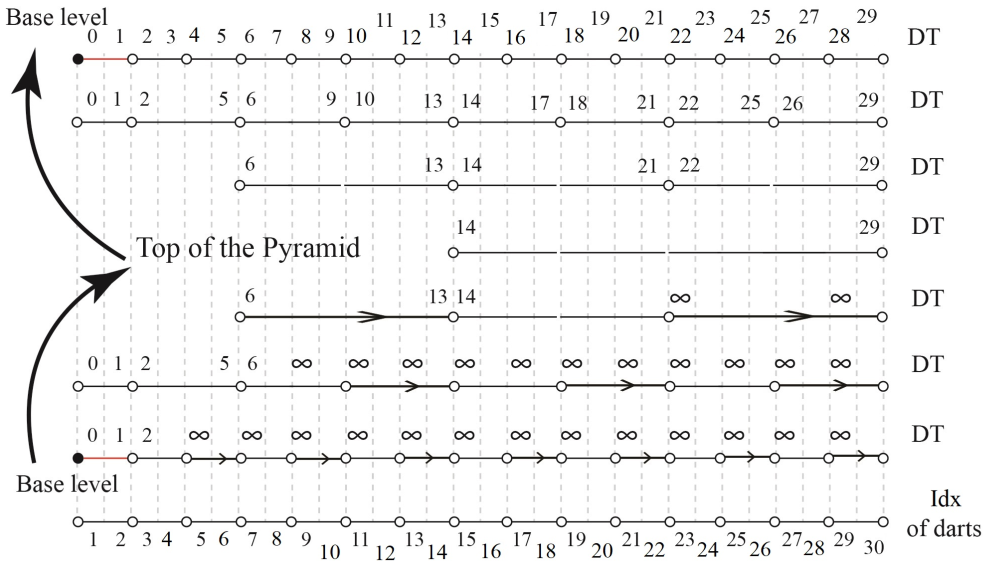
(c) Top of the pyramid

(d) Copy and Propagation of DT

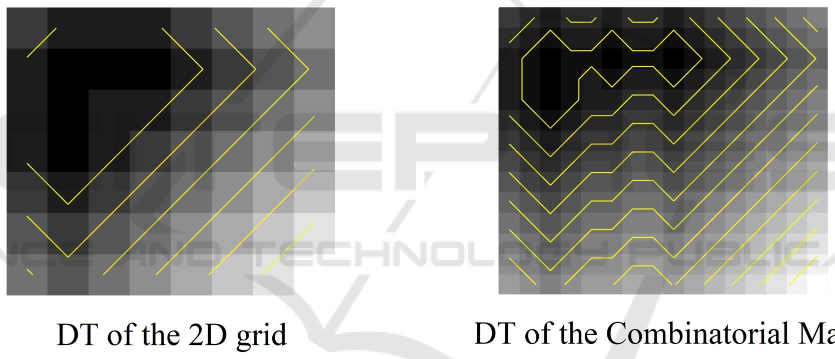
(e) Copy and Propagation of DT

(f) Base of the pyramid

Figure 3: Computing of the DT in 2D grid.



(a) Computing of the DT in 1D combinatorial map.



(b) DT in the 2D grid and in the CM of Fig. 3.

Figure 4: Computing the DT in combinatorial map.

shape than just the highest dimension. In addition, it is a sort of map-edit-distance (Combier et al., 2013) in analogy to the graph edit distance (Gao et al., 2010).

3.1 Logarithmic DT in a 1D Combinatorial Map

To compute the DT in the CM, a similar algorithm to Alg. 1 can be used but with two modifications. First, the unique indices are defined for darts instead of vertices. Second, in each step, we propagate distances by α- and σ-propagation. The α-propagation of the DT is performed as follows:

$$D(d_i) = \min\{D(d_i), D(\alpha(d_i)) + |Idx(d_i) - Idx(\alpha(d_i))|\} \quad (9)$$

Note that, during the contraction of the $e = (d, \alpha(d))$, the $Idx(\sigma(d))$ of the contracted dart is updated after each contraction as follows:

$$Idx(\sigma(d)) = Idx(\alpha(d)) \quad (10)$$

The σ-propagation is performed as follows:

$$D(d_i) = \min\{D(d_i), D(\sigma(d_i)) + 1, D(\sigma^{-1}(d_i)) + 1\} \quad (11)$$

Fig. 4a shows an example of computing the DT in 1D CM. In constructing the pyramid in the bottom-up procedure, first, the α-propagation and then the σ-propagation are performed. In contrast, in the top-down procedure, they are performed the other way around. The steps of the algorithm are shown in Alg. 2 and Fig. 4b displays the finer resolution of the DT in comparison with the DT over 2D grid.

Table 1: Execution (*ms*) of proposed Logarithmic DT, MeijsterGPU and FastGPU.

Image-size	Mit. Log DT	MRI Log DT	Ran. Log DT	Ran. MeijsterGPU	Ran. FastGPU
256×256	0.0953	0.1209	0.0645	3.8524	1.7844
512×512	0.410	0.7310	0.3152	14.2005	4.2309
1024×1024	2.6308	5.1501	0.9364	25.8475	12.4668
2048×2048	4.1088	8.9506	1.8577	110.7817	44.9560

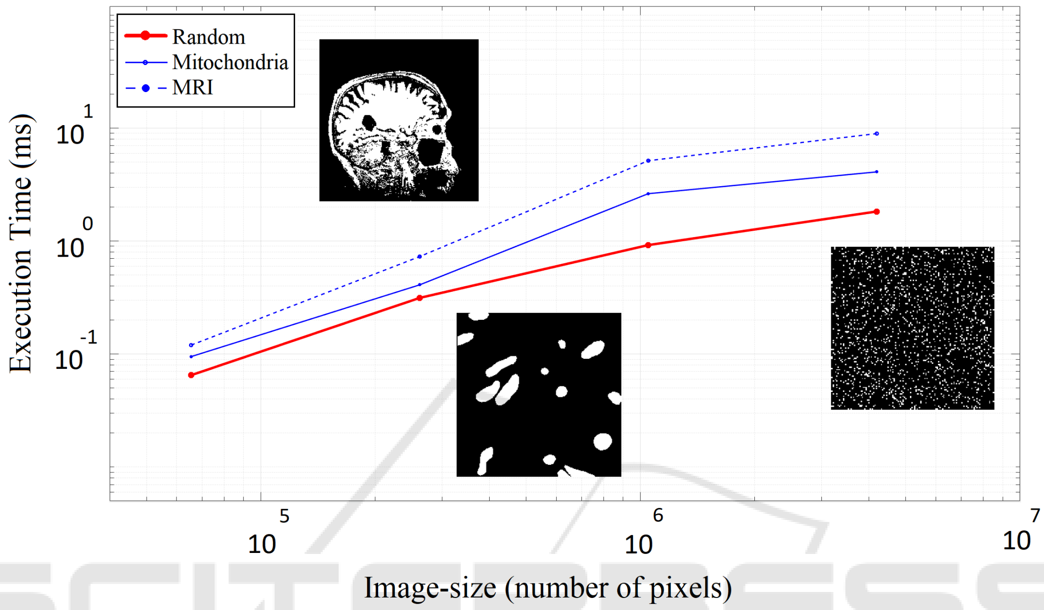


Figure 5: The proposed logarithmic DT over different images.

Algorithm 2: Computing DT in the 1D Combinatorial Map.

```
    Input: $CM = (D, \alpha, \sigma)$
    Initialization: $DT = \infty,\ \forall d \in D$
3:  $DT(\sigma^*(d)) = 0,\ \forall d \in E_1$
    $\alpha$- and $\sigma$-propagation to adjacent neighbors
    Selecting the CKs  (Bottom-up traversing)
6:  While ($DT = \infty$ in the current level)
        Contracting the edges with two unknown DT
        $\alpha$- and $\sigma$-propagation to adjacent neighbors
9:  end (Top of the Pyramid)
    For ($j = L$ downto $1$)  (Top-down traversing)
        Imitate the DT from $L \rightarrow L-1$
12:     $\sigma$- and $\alpha$-propagation to adjacent neighbors
    end
```

4 COMPARISONS AND RESULTS

To highlight the advantages of the proposed logarithmic algorithm, we compare the execution times with two CUDA-based Implementations: MeijsterGPU and FastGPU in (de Assis Zampirolli and Filipe, 2017). Simulations use MATLAB software employing CPU with AMD Ryzen 7 2700X, 3.7GHz, and NVIDIA GeForce GTX 2080 TI that run over three different categories of images: Random, Mitochondria, and MRI. Table. 1 displays the outcome of the implementations. The first column shows the image size. The next three columns show the execution times (ms) of the proposed logarithmic DT (Log DT) in the three different classes of images. The last two columns show the execution time by the other two methods. Fig. 5 compares the results of the logarithmic algorithm between the different classes. Since the Random images contain smaller foreground objects than the other classes, they are executed faster. In Fig. 6 the logarithmic method is compared to MeijsterGPU and FastGPU methods. The logarithmic DT is not only significantly faster than the other ones but also has much higher performance dealing with larger images. Note that all operations and processes in the proposed algorithms are local and independent. Therefore, each available thread of the GPU in the shared memory is dedicated to each local process. The bottleneck of the algorithms is the capacity of the shared memory. Therefore, having sufficient independent processing elements the algorithms are fully parallel with logarithmic complexity.

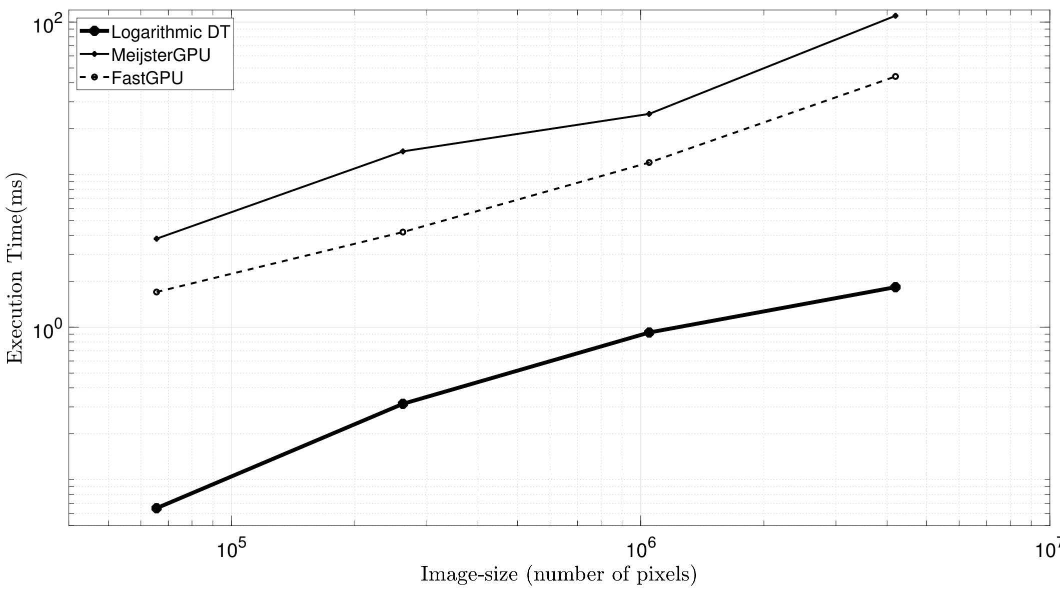


Figure 6: Comparison of the proposed algorithm with MeijsterGPU and FastGPU (de Assis Zampirolli and Filipe, 2017).

5 CONCLUSION

Distance transform (DT) computes how far inside a shape a point is located. In this paper, we study how the distances can be calculated in a discrete domain like a pixel grid and in combinatorial maps. Using the irregular pyramid we proposed a new algorithm that computes the DT in the logarithmic parallel complexity. Moreover, by defining the DT over the combinatorial maps, the smoother DT is calculated with double precision. Using the dart ordering, we proposed the logarithmic algorithm for computing the 1D combinatorial maps. However, the algorithm can be extended to higher n-dimensions. Finally, the practical results show that the algorithm with parallel logarithmic complexity notably decreases the execution time and makes it beneficial in particular for large images.

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