Distributionally Robust Optimization of Adaptive Cruise Control Under Uncertainty

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Keywords: Adaptive Cruise Control, Distributionally Robust Optimization, Stochastic Optimization, Autonomous Vehicle.

Abstract: Due to the recent advances in intelligent and connected vehicles, Adaptive Cruise Control (ACC) has become a key functionality of advanced driver-assistant systems (ADAS) to enhance comfort and safety. The evaluation of ACC's efficiency and safety is also crucial for the industry to prove the reliability of its products. In our paper, we propose a distributional robust optimization-based ACC reference generation model to produce the optimal commands facing the uncertainty of sensors. By taking into account the uncertainty set with knowledge of the first and second moments, the original optimization problem with chance constraints can be simplified and solved more efficiently. Numerical experiments in a driving simulator illustrate that the robustness of the results is largely increased by minimizing the risks of violation of safety constraints.

1 INTRODUCTION

Over the past decade, there has been a marked increase in the development of connected and intelligent driving technologies, which has garnered significant interest from both industrial and academic researchers. This results in various technological breakthroughs and commercial solutions. The development of intelligent applications in tandem with new generations of cars (connected and semi-autonomous) has changed the paradigm of driving when ensuring and guaranteeing the safety of drivers on different types of roads. Advanced Driver Assistance System (ADAS) is one of the promising applications that has been integrated into smart vehicles in order to improve driving comfort and safety. One of the most fundamental aspects of ADAS is Adaptive Cruise Control (ACC), which has been investigated extensively by researchers and engineers.

(Levine and Athans, 1966) is one of the oldest articles invoking ACC and describing its objectives. The authors of this reference claimed that the objective of ACC is to maintain a safe distance from a leading vehicle by adjusting the speed and acceleration. In the situation in which there is no leading vehicle, it should also maintain the desired speed set by the driver or system. Nowadays, and with the evolving of connected objects deployment, ACC relies on two essential components represented respectively by sensors to collect environmental information and a controller to set inter-vehicle spacing based on the collected information. Hence, an ACC-enabled vehicle drives at a preset speed (following a given trajectory) until detecting a leading vehicle. Next to that, it switches the vehicle to the distance regulation mode when activating its ACC controller. This former will provide the necessary safety distance and send other commands to controllers.

In response to the need for improved driving comfort and safety, researchers have put forth numerous approaches to design adaptive cruise control (ACC) systems that emulate the behavior of expert human drivers. Therefore, ACC systems are designed considering key factors such as energy consumption, collision avoidance, etc. Then, it becomes imperative to validate the proposed ACC systems thoroughly and assess their performance before deploying them in the market.

Nevertheless, the validation and evaluation process comes with some drawbacks, such as real-world road tests, which are expensive, time-consuming,

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Zhang, S., Hadji, M. and Lisser, A.
Distributionally Robust Optimization of Adaptive Cruise Control Under Uncertainty.
DOI: 10.5220/0011670800003396
In *Proceedings of the 12th International Conference on Operations Research and Enterprise Systems (ICORES 2023)*, pages 71-81
ISBN: 978-989-758-627-9; ISSN: 2184-4372

and can only cover a limited number of driving scenarios. Thus, we conduct the validation process in a simulator that can generate scenarios automatically based on our needs. This validation forms part of the functional testing of advanced driver-assistance systems (ADAS) in order to provide feedback on the system's performance. This feedback may include information on avoided potential collisions, the rationality of the system's decision-making, and the identification of any potential flaws. The goal of this testing is to ensure that the ADAS functions properly and can provide a useful aid to human drivers.

In our work, we propose a reference generator that is used to evaluate the performance of an ACC system. This reference generator generates commands that serve as a benchmark for the ACC system's output. We then compared the reference commands generated using different frameworks. Our results show that the trajectory generated using the DRO approach was more robust compared to the other frameworks. Figure 1 shows the ACC validation process with the generated benchmark.

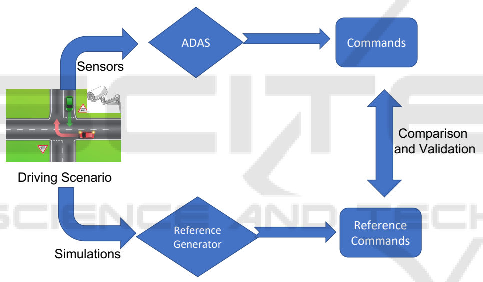

Figure 1: Illustration of how the ADAS validation process occurs.

There are plenty of approaches (with advantages and drawbacks) to cope with the problem of generating reference trajectories using, for example, sampling-based methods, graph-based methods, and optimization-based approaches. Our goal is to improve the comfort and safety of autonomous vehicle by using optimization-based approaches to identify rational values for acceleration, speed, and relative position. The optimization-based approaches allow us to tailor the different criteria of the objective function and facilitate the constraints description and formulation according to different driving scenarios. Hence, in this paper, we propose an optimization-based model for the generation of ACC reference commands.

Usually, an ACC system relies on various types of sensors, including cameras, lidar, radars, etc. The performance of each considered sensor is influenced by certain factors like its maintenance and the environment in which it is used (see (Rasshofer et al., 2011) for more details). Hence, collected data from these sensors is subject to inherent inaccuracies that must be taken into account when computations are performed. Under the assumption that the first and second moments of sensor errors are known, either in full or in part, we propose a distributionally robust optimization (DRO) model with chance constraints to handle and integrate this uncertainty.

1.1 Our Contribution

According to the aforementioned details, we propose in this paper two DRO approaches to formulate a new ACC reference generator. Indeed, we formulate both a deterministic optimization model (Zhang et al., 2022) and twos DRO model which address the challenge of sensor error uncertainty when the first and second moments of the error distribution are partially known. Our optimizations are based on quadratic programming (QP) to determine the most appropriate command to optimize the distance between two vehicles while jointly satisfying all the problem constraints at the same time. We provide a comprehensive comparison of the obtained results using our generated driving data, which simulates realistic driving scenarios. We also emphasize the added value of the DRO model compared to the deterministic formulation.

The remaining of this paper is organized as follows. Section 2 addresses mainstream ACC algorithms that can be found in the literature. We describe and discuss a new ACC validation model in Section 3. Section 4 is dedicated to assessing the performance of the proposed new solution and its comparison to other existing models. We conclude the paper in Section 5.

2 RELATED WORK

Research on ACC has been conducted from a number of perspectives to ensure its effectiveness and safety in complex real-life driving scenarios, including its design, implementation, and validation.

A large number of ACC systems use optimal control methods to achieve the desired outcome (Chehardoli, 2020; Jiang et al., 2020; Zhu et al., 2018). In recent years, model predictive control (MPC) has also gained popularity due to its receding horizon approach (Takahama and Akasaka, 2018; Li et al., 2010; Naus et al., 2010). Other features have also been investigated, including driver behavior modeling(Varotto et al., 2020), string stability(Khound et al., 2021), and collision avoidance(Magdici and Althoff, 2017).

As a crucial part of autonomous driving, validation of the functionality of the ADAS is also a very critical task. It applies not only to the ACC but also to other modules that require evaluation. A unified test architecture and validation process are presented in (Lattarulo et al., 2017), which presents a comprehensive framework for evaluating path planning and control algorithms. Similar works can also be found in (Lattarulo et al., 2018) and (Alnaser et al., 2019).

In addition to evaluating the overall testing framework, it is essential to carefully examine the individual functionality such as adaptive cruise control (ACC) system. An experimental platform for evaluating and demonstrating an optimization-based ACC controller is presented in (Mehra et al., 2015), while (Djoudi et al., 2020) presents a simulation-based tool chain for generating reference data and analysing test results. A number of other insightful studies concerning the testing and validation of ACC can be found in (Schmied et al., 2015; Shakouri et al., 2015). In this paper, a DRO model is proposed for ACC reference generation that takes into account uncertainties with partial information. The results of the DRO model can then be incorporated into the validation process. To the best of our knowledge, no prior work has been conducted on the application of DRO to ACC reference generation.

DRO was first introduced by Scarf (Scarf, 1958) in 1958 and widely applied in risk management. The main requirement of this framework is to define a set of probability measures, namely the uncertainty set, that include the true stochastic model for the problem. Furthermore, the problem is typically optimized using a worst-case analysis over the choice of a distribution from this set. This framework has attracted considerable attention due to the advancement of optimization techniques, and it has a wide range of applications across a variety of fields nowadays (Rahimian and Mehrotra, 2019). The uncertainty set is normally constructed using historical data and is known as a data-driven uncertainty set (Delage and Ye, 2010; Mohajerin Esfahani and Kuhn, 2018; Miao et al., 2021). By analyzing sensor errors in the ACC system, we can apply the DRO framework to improve the performance of our ACC reference generator.

3 PROBLEM FORMULATION

3.1 Overview

In the following, we rely on the usage of the provided mathematical formulation of (Zhang et al., 2022) and will address driving scenarios.

The objective of this section is to describe how we model the ACC driving scenario and formulate the related optimization problem. Note that the typical ACC driving scenario involves two cars driving simultaneously in one lane, namely the **ego** car and the **target** car. It is the ego car that is equipped with an ACC system, whereas the target car is the leading vehicle positioned in front. Figure 2 illustrates the driving scenario, as well as the states of two cars at the moment t_i. The purpose of the ACC reference generation is to generate a sequence of acceleration commands, that is, the decision variables in our optimization problem. The ego car's ACC system is designed to remain at a distance from the target car while taking into account a variety of factors, such as vehicle dynamics, driving comfort, and traffic regulations.

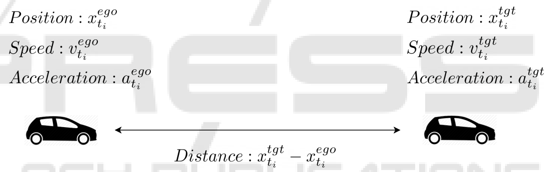

Figure 2: ACC driving scenario at the moment t_i.

Suppose that the total duration of a driving scenario is Ts composed of n sampling time dt, i.e. $T = ndt$ with a corresponding timestamp $[t_0, t_1, ...t_i, \dots \ t_n]$ where $t_{i+1} = t_i + dt$, $\forall i \in \{0, 1, \dots \ n-1\}$. At each moment t_i, the ACC of the ego car uses sensors to gather information from the target car and generates the acceleration commands. In the following, we list the parameters and the decision variables used in our model.

The input parameters are given by the ego car sensors, and the decision variables represent the ACC optimal commands. The parameters of the ego car are the initial position $x_{t_0}^{ego}$, and the initial velocity $v_{t_0}^{ego}$ whilst the parameters of the target car are composed of the position vector $X_T^{tgt} = (x_{t_1}^{tgt}, x_{t_2}^{tgt}, \dots \ x_{t_n}^{tgt})^T$, the velocity vector $V_T^{tgt} = (v_{t_0}^{tgt}, v_{t_1}^{tgt}, \dots \ v_{t_{n-1}}^{tgt})^T$ and the acceleration vector $A_T^{tgt} = (a_{t_0}^{tgt}, a_{t_1}^{tgt}, \dots \ a_{t_{n-1}}^{tgt})^T$ in the whole driving scenario. The decision variable is the ACC ego car acceleration commands vector $A_T^{ego} = (a_{t_0}^{ego}, a_{t_1}^{ego}, \dots \ a_{t_{n-1}}^{ego})^T$.

Given the decision variable and the initial state of the ego car, we can derive the velocity and the position of the ego car by the equations of motion. The ego car velocity $v^{ego}_{t_{i+1}}$ at time t_{i+1} is given by the velocity at the previous sample time $v^{ego}_{t_i}$ and the acceleration $a^{ego}_{t_i}$:

$$v^{ego}_{t_{i+1}} = v^{ego}_{t_i} + a^{ego}_{t_i} dt. \tag{1}$$

The velocity for the whole driving scenario can be written in matrix form as

$$\begin{aligned} V^{ego}_T &= \begin{pmatrix} v^{ego}_{t_0} \\ \vdots \\ v^{ego}_{t_i} \\ \vdots \\ v^{ego}_{t_{n-1}} \end{pmatrix} = \begin{pmatrix} v^{ego}_{t_0} \\ \vdots \\ v^{ego}_{t_0} + \sum_{k=0}^{k=i-1} a^{ego}_{t_k} dt \\ \vdots \\ v^{ego}_{t_0} + \sum_{k=0}^{k=n-2} a^{ego}_{t_k} dt \end{pmatrix} \\ &= dt\,\mathcal{K}_n A^{ego}_T + v^{ego}_{t_0} \mathbb{1}_n, \end{aligned} \tag{2}$$

where $\mathcal{K}_n \in \mathbb{R}^{n\times n}$ and $\mathbb{1}_n \in \mathbb{R}^{n\times 1}$

$$\mathcal{K}_n = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & 0 & \dots & 0 & 0 \\ & \vdots & & \ddots & & \vdots \\ 1 & 1 & 1 & \dots & 0 & 0 \\ 1 & 1 & 1 & \dots & 1 & 0 \end{pmatrix} \tag{3}$$

$$\mathbb{1}_n = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}. \tag{4}$$

Similarly, the ego car position at time t_{i+1} is given by

$$x^{ego}_{t_{i+1}} = x^{ego}_{t_i} + v^{ego}_{t_i} dt + \frac{1}{2} a^{ego}_{t_i} dt^2. \tag{5}$$

The corresponding matrix format for all time steps is

$$\begin{aligned} X^{ego}_T &= \begin{pmatrix} x^{ego}_{t_1} \\ \vdots \\ x^{ego}_{t_i} \\ \vdots \\ x^{ego}_{t_n} \end{pmatrix} = \\ &\begin{pmatrix} x^{ego}_{t_0} + v^{ego}_{t_0} dt + \frac{1}{2} a^{ego}_{t_0} dt^2 \\ \vdots \\ x^{ego}_{t_0} + \sum_{k=0}^{k=i-1} v^{ego}_{t_k} dt + \frac{1}{2} \sum_{k=0}^{k=i-1} a^{ego}_{t_k} dt^2 \\ \vdots \\ x^{ego}_{t_0} + \sum_{k=0}^{k=n-1} v^{ego}_{t_k} dt + \frac{1}{2} \sum_{k=0}^{k=n-1} a^{ego}_{t_k} dt^2 \end{pmatrix} \\ &= dt\,\mathcal{M}_n V^{ego}_T + \frac{1}{2} dt^2 \mathcal{M}_n A^{ego}_T + x^{ego}_{t_0} \mathbb{1}_n, \end{aligned} \tag{6}$$

where $\mathcal{M}_n \in \mathbb{R}^{n\times n}$,

$$\mathcal{M}_n = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ 1 & 1 & 1 & \dots & 0 \\ & \vdots & & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{pmatrix}. \tag{7}$$

We use Equation (2) to rewrite Equation (6) in terms of the initial position, the initial velocity and the acceleration vector, i.e.,

$$\begin{aligned} X^{ego}_T &= dt\,\mathcal{M}_n V^{ego}_T + \frac{1}{2} dt^2 \mathcal{M}_n A^{ego}_T + x^{ego}_{t_0} \mathbb{1}_n \\ &= dt\,\mathcal{M}_n (dt\,\mathcal{K}_n A^{ego}_T + v^{ego}_{t_0} \mathbb{1}_n) \\ &\quad + \frac{1}{2} dt^2 \mathcal{M}_n A^{ego}_T + x^{ego}_{t_0} \mathbb{1}_n \\ &= dt^2 (\mathcal{B}_n + \frac{1}{2} \mathcal{M}_n) A^{ego}_T + v^{ego}_{t_0} dt\, \mathcal{C}_n \\ &\quad + x^{ego}_{t_0} \mathbb{1}_n, \end{aligned} \tag{8}$$

where $\mathcal{B}_n = \mathcal{M}_n \cdot \mathcal{K}_n \in \mathbb{R}^{n\times n}$ and $\mathcal{C}_n = \mathcal{M}_n \cdot \mathbb{1}_n \in \mathbb{R}^{n\times 1}$.

These parameters are summarized in Table 1.

In the following, we use the position and the velocity vector of the ego car to formulate our optimization problem.

3.2 Mathematical Modeling

In the sequel, we outline how the generation of the ACC reference can be viewed as an optimization problem.

$$\min_{A^{ego}_T} \quad ||Q A^{ego}_T + P|| \tag{9}$$

$$\text{s.t.} \quad dt^2 (\mathcal{B}_n + \frac{1}{2} \mathcal{M}_n) A^{ego}_T \leq X^{tgt}_T - v^{ego}_{t_0} dt\, \mathcal{C}_n - (x^{ego}_{t_0} + d_s) \mathbb{1}_n, \tag{10}$$

$$-(v_{max} + v^{ego}_{t_0}) \mathbb{1}_n \leq dt\,\mathcal{K}_n A^{ego}_T \leq (v_{max} - v^{ego}_{t_0}) \mathbb{1}_n, \tag{11}$$

$$-a_{max} \mathbb{1}_n \leq A^{ego}_T \leq a_{max} \mathbb{1}_n, \tag{12}$$

$$-j_{max} dt \mathbb{1}_n \leq \mathcal{D}_n A^{ego}_T \leq j_{max} dt \mathbb{1}_n. \tag{13}$$

The following part explains in detail how we derive the objective function (9) and how constraints (10, 11, 12, 13) are developed.

The objective of ACC is to maintain a safe distance between the ego car and the target car. In order to calculate the reference distance between the ego car and the target car, we define two terms: the inter-vehicle time tc (e.g., 3 seconds), which gives the ego car enough time to brake and avoid a collision with the target car, and the standstill distance δS to ensure there is always enough room between the two adjacent cars.

Table 1: Summary of used parameters and variables in our formulations.

	Symbols	**Physical Meaning**	**Relationship**
Target Car	A_T^{tgt}	Acceleration profile during simulation	$A_T^{tgt} = (a_{t_0}^{tgt}, a_{t_1}^{tgt}, \dots a_{t_{n-1}}^{tgt})^T$
	V_T^{tgt}	Speed profile during simulation	$V_T^{tgt} = (v_{t_0}^{tgt}, v_{t_1}^{tgt}, \dots v_{t_{n-1}}^{tgt})^T$
	X_T^{tgt}	Position profile during simulation	$X_T^{tgt} = (x_{t_1}^{tgt}, x_{t_2}^{tgt}, \dots x_{t_n}^{tgt})^T$
Ego Car	A_T^{ego}	Acceleration profile during simulation	$A_T^{ego} = (a_{t_0}^{ego}, a_{t_1}^{ego}, \dots a_{t_{n-1}}^{ego})^T$
	V_T^{ego}	Speed profile during simulation	$V_T^{ego} = dt\,\mathcal{K}_n A_T^{ego} + v_{t_0}^{ego}\mathbb{1}_n$
	X_T^{ego}	Position profile during simulation	$dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} + v_{t_0}^{ego}dt\,\mathcal{C}_n + x_{t_0}^{ego}\mathbb{1}_n$
	J_T^{ego}	Jerk profile during simulation	$\mathcal{D}_n A_T^{ego}$

At each moment t_k, the reference distance of ACC in platoons is defined by

$$d_{t_k}^{ref} = (v_{t_{k-1}}^{ego} - v_{t_{k-1}}^{tgt})tc + \frac{1}{2}(a_{t_{k-1}}^{ego} - a_{t_{k-1}}^{tgt})tc^2 + \delta S. \tag{14}$$

So the reference distance vector in the whole driving scenario is :

$$\begin{aligned} D_T^{ref} &= tc(dt\,\mathcal{K}_n A_T^{ego} + v_{t_0}^{ego}\mathbb{1}_n - V_T^{tgt}) \\ &\quad + \frac{1}{2}tc^2(A_T^{ego} - A_T^{tgt}) + \delta S\mathbb{1}_n \\ &= (dt \cdot tc\,\mathcal{K}_n + \frac{1}{2}tc^2 I)A_T^{ego} - tcV_T^{tgt} \\ &\quad - \frac{1}{2}tc^2 A_T^{tgt} + (\delta S + v_{t_0}^{ego}tc)\mathbb{1}_n. \end{aligned} \tag{15}$$

Moreover, the current distance between the ego car and target car is

$$\begin{aligned} D_T^{vehicle} &= X_T^{tgt} - X_T^{ego} \\ &= X_T^{tgt} - [dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} \\ &\quad + v_{t_0}^{ego}dt\,\mathcal{C}_n + x_{t_0}^{ego}\mathbb{1}_n]. \end{aligned} \tag{16}$$

By combining (16) and (15), we obtain the objective function (9):

$$\begin{aligned} &\min_{A_T^{ego}} ||D_T^{vehicle} - D_T^{ref}|| \\ &= \min_{A_T^{ego}} ||X_T^{tgt} - [dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} \\ &\quad + v_{t_0}^{ego}dt\,\mathcal{C}_n + x_{t_0}^{ego}\mathbb{1}_n] - [(dt \cdot tc\,\mathcal{K}_n + \frac{1}{2}tc^2 I)A_T^{ego} \\ &\quad - tcV_T^{tgt} - \frac{1}{2}tc^2 A_T^{tgt} + (\delta S + v_{t_0}^{ego}tc)\mathbb{1}_n]|| \\ &= \min_{A_T^{ego}} ||-(dt^2\mathcal{B}_n + \frac{1}{2}dt^2\mathcal{M}_n + dt \cdot tc\,\mathcal{K}_n \\ &\quad + \frac{1}{2}tc^2 I)A_T^{ego} + X_T^{tgt} + tcV_T^{tgt} + \frac{1}{2}tc^2 A_T^{tgt} - \delta S\mathbb{1}_n \\ &\quad - v_{t_0}^{ego}tc\mathbb{1}_n - x_{t_0}^{ego}\mathbb{1}_n - v_{t_0}^{ego}dt\,\mathcal{C}_n|| \\ &= \min_{A_T^{ego}} ||QA_T^{ego} + P||, \end{aligned} \tag{17}$$

where $Q = -(dt^2\mathcal{B}_n + \frac{1}{2}dt^2\mathcal{M}_n + dt \cdot tc\,\mathcal{K}_n + \frac{1}{2}tc^2 I)$, $P = X_T^{tgt} + tcV_T^{tgt} + \frac{1}{2}tc^2 A_T^{tgt} - \delta S\mathbb{1}_n - v_{t_0}^{ego}tc\mathbb{1}_n - x_{t_0}^{ego}\mathbb{1}_n - v_{t_0}^{ego}dt\,\mathcal{C}_n$ and $||\cdot||$ is Euclidean norm.

In addition to the objective function (9), we describe the following constraints

- Constraint (10) is the minimum distance constraint that aims to prevent vehicles collisions. It results from

$$D_T^{vehicle} \geq d_s\mathbb{1}_n. \tag{18}$$

- Constraint (11) is the maximum velocity constraint. Routes typically have a maximum velocity limit which leads to the velocity constraint. For a given speed limit v_{max}, the constraint is deduced from

$$||V_T^{ego}||_\infty \leq v_{max}. \tag{19}$$

- Constraint (12) is the maximum acceleration constraint. Car passengers' comfort is impacted by acceleration. Vehicle maneuverings like rapid acceleration or braking should be avoided. Our model proposes an acceleration limit of a_{max} based on this motivation.

$$||A_T^{ego}||_\infty \leq a_{max}. \tag{20}$$

- Constraint (13) is the maximum jerk constraint. In jerk, we measure the acceleration variances, which significantly affect the comfort level of passengers. A maximum limit j_{max} is required for this constraint.

$$||J_T^{ego}||_\infty \leq j_{max} \tag{21}$$

Since $j_{t_i} = (a_{t_i}^{ego} - a_{t_{i-1}}^{ego})/dt$, the jerk constraint can be simplified to (13) where $\mathcal{D}_n \in \mathcal{R}^{n \times n}$

$$\mathcal{D}_n = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 \\ & \vdots & & \ddots & \vdots & \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix}. \tag{22}$$

Given the form of the objective function and the constraints, our model is a convex quadratic optimization problem.

In the next section, we will discuss the uncertainty involved in ACC and how to handle it by stochastic modeling with chance constraints.

3.3 Distributionally Robust Optimization Model

3.3.1 Uncertainty Set with First and Second Moments

The model presented above is deterministic, i.e., all input parameters are known in advance. Real-life autonomous vehicle problems, however, may include different sources of noise caused by external factors, such as weather, which may affect these parameters. Model uncertainties can be addressed in a variety of ways. In our work, we apply the robust optimization framework to manage distribution-free uncertainties in models.

In the following, we model the ACC problem as a chance-constrained optimization problem. Our hypothesis is that the target car's position information $x_{t_i}^{tgt}$ contains some noise when obtained from the ego car's sensor, and we only know the first two moments of the distribution, the mean value μ_i and the variance σ_i^2, respectively. Mathematically, the target car's position $x_{t_i}^{tgt}$ is a random variable with a distribution measure $\mathcal{F}_i$ over its outcome space. Thus, the mean and variance of the target car's position is μ_T and σ_T^2 where

$$\mu_T = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} \tag{23}$$

and

$$\sigma_T = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_n \end{pmatrix} \tag{24}$$

The objective function for this stochastic optimization problem is

$$\begin{aligned} &\min_{A_T^{ego}} ||\mathbb{E}(D_T^{vehicle} - D_T^{ref})|| \\ &= \min_{A_T^{ego}} ||\mu_T + tcV_T^{tgt} + \frac{1}{2}tc^2 A_T^{tgt} - \delta S \mathbb{1}_n \\ &\quad - v_{t_0}^{ego} tc \mathbb{1}_n - x_{t_0}^{ego} \mathbb{1}_n - v_{t_0}^{ego} dt \mathcal{C}_n - [dt^2 \mathcal{B}_n \\ &\quad + \frac{1}{2} dt^2 \mathcal{M}_n + dt \cdot tc \mathcal{K}_n + \frac{1}{2} tc^2] A_T^{ego} || \\ &= \min_{A_T^{ego}} ||QA_T^{ego} + P'||, \end{aligned} \tag{25}$$

where $P' = \mu_T + tcV_T^{tgt} + \frac{1}{2}tc^2 A_T^{tgt} - \delta S \mathbb{1}_n - v_{t_0}^{ego} tc \mathbb{1}_n - x_{t_0}^{ego} \mathbb{1}_n - v_{t_0}^{ego} dt \mathcal{C}_n$.

Let $\mathcal{D}_i$ be the set of probability distributions with mean μ_i and variance σ_i^2, and it's defined as

$$\mathcal{D}_i = \left\{ \mathcal{F}_i \;\middle|\; \begin{array}{l} \mathbb{E}_{\mathcal{F}_i}[x_{t_i}^{tgt}] = \mu_i, \\ \mathbb{E}_{\mathcal{F}_i}[(x_{t_i}^{tgt} - \mu_i)^2] = \sigma_i^2 \end{array} \right\}, i = 1, \cdots, n. \tag{26}$$

The target car's position $x_{t_i}^{tgt} \sim \mathcal{F}_i$ follows a distribution$\mathcal{F}_i$ with $\mathcal{F}_i \in \mathcal{D}_i$.

Using Theorem 1 in (Ghaoui et al., 2003), the minimum distance constraint (18) for each moment t_i can be expressed as a chance constraint (Prékopa, 2013) with a given a threshold α, i.e.,

$$\begin{aligned} &\inf_{\mathcal{F}_i \in \mathcal{D}} \mathbb{P}_{\mathcal{F}_i}(D_{t_i}^{vehicle} \geq d_s) \geq \alpha, \forall t_i \\ &= \inf_{\mathcal{F}_i \in \mathcal{D}} \mathbb{P}_{\mathcal{F}_i}(x_{t_i}^{tgt} \geq x_{t_i}^{ego} + d_s) \geq \alpha \\ &= x_{t_i}^{ego} + d_s \leq \mu_i - \sigma_i \sqrt{\frac{\alpha}{1-\alpha}}, \end{aligned} \tag{27}$$

where the **inf** is taken with respect to all probability distributions in $\mathcal{D}_i$.

For the whole driving scenario, the minimum distance constraint in a matrix form is

$$\begin{aligned} &dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A_T^{ego} + v_{t_0}^{ego} dt \mathcal{C}_n + \\ &(x_{t_0}^{ego} + d_s)\mathbb{1}_n \leq \hat{X}_T^{tgt}, \end{aligned} \tag{28}$$

where

$$\hat{X}_T^{tgt} = \begin{pmatrix} \mu_1 - \sigma_1 \sqrt{\frac{\alpha}{1-\alpha}} \\ \mu_2 - \sigma_2 \sqrt{\frac{\alpha}{1-\alpha}} \\ \vdots \\ \mu_n - \sigma_n \sqrt{\frac{\alpha}{1-\alpha}} \end{pmatrix}. \tag{29}$$

Since only the minimum distance constraint is related to the position of the target car X_T^{tgt}, all other constraints remain unchanged.

3.3.2 Uncertainty Set with Unknown Moments

Here, we consider the case where the mean and variance of the target car's position $x^{tgt}_{t_i}$ are unknown but limited in a specific range. This DRO model could be applied to the scenario in which the sensor error is unstable.

Supposing that the target car's position $x^{tgt}_{t_i}$ is a random variable, with an outcome space (Ω_i, F_i) and a distribution measure $\mathcal{F}_i$ over the space. The mean of $x^{tgt}_{t_i}$ lies in an interval of size $2\sqrt{\gamma^i_1}$ and centered at μ_i, and the upper bound of variance of x^{tgt}_i is $\gamma^i_2\sigma^2_i$. We define the uncertainty set $\mathcal{D}_i$ as follows:

$$\mathcal{D}_i = \left\{ \mathcal{F}_i \;\middle|\; \begin{array}{l} (\mathbb{E}_{\mathcal{F}_i}[x^{tgt}_{t_i}] - \mu_i)^2 \leq \gamma^i_1, \\ \mathbb{E}_{\mathcal{F}_i}[(x^{tgt}_{t_i} - \mu_i)^2] \leq \gamma^i_2\sigma^2_i \end{array} \right\}, i = 1, \cdots, n. \tag{30}$$

The target car's position $x^{tgt}_{t_i} \sim \mathcal{F}_i$ follows a distribution $\mathcal{F}_i$ with $\mathcal{F}_i \in \mathcal{D}_i$.

As the mean position of the target car at time t_i is centered at μ_i, we maintain the same objective function as in Equation (25). By the result in (Peng et al., 2021), the minimum distance constraint can be transformed as follow

$$\begin{aligned} &\inf_{\mathcal{F}_i \in \mathcal{D}_i} \mathbb{P}_{\mathcal{F}_i}(D^{vehicle}_{t_i} \geq d_s) \geq \alpha, \ \forall t_i \\ &= \inf_{\mathcal{F}_i \in \mathcal{D}_i} \mathbb{P}_{\mathcal{F}_i}(x^{tgt}_{t_i} \geq x^{ego}_{t_i} + d_s) \geq \alpha \\ &= x^{ego}_{t_i} + d_s \leq \mu_i - \sigma_i\Big(\sqrt{\frac{\alpha}{1-\alpha}}\sqrt{\gamma^i_2} + \sqrt{\gamma^i_1}\Big), \end{aligned} \tag{31}$$

For the whole driving scenario, the minimum distance constraint in a matrix form is

$$\begin{aligned} &dt^2(\mathcal{B}_n + \frac{1}{2}\mathcal{M}_n)A^{ego}_T + v^{ego}_{t_0} dt\, \mathcal{C}_n + \\ &(x^{ego}_{t_0} + d_s)\mathbb{1}_n \leq \widetilde{X}^{tgt}_T, \end{aligned} \tag{32}$$

where

$$\widetilde{X}^{tgt}_T = \begin{pmatrix} \mu_1 - \sigma_1(\sqrt{\frac{\alpha}{1-\alpha}}\sqrt{\gamma^1_2} + \sqrt{\gamma^1_1}) \\ \mu_2 - \sigma_2(\sqrt{\frac{\alpha}{1-\alpha}}\sqrt{\gamma^2_2} + \sqrt{\gamma^2_1}) \\ \vdots \\ \mu_n - \sigma_n(\sqrt{\frac{\alpha}{1-\alpha}}\sqrt{\gamma^n_2} + \sqrt{\gamma^n_1}) \end{pmatrix}. \tag{33}$$

All other constraints remain unchanged since uncertainty is not involved.

4 EXPERIMENTAL RESULTS

In numerical experiments, we randomly generate various driving scenarios in a simulator and compare the performance of the deterministic and the two DRO models on those scenarios. To compare, we evaluate the number of violated constraints in each model on given driving scenarios. During the data generation phase, different configurations of a driving scenario are applied, including the ego car's state and the target car's trajectory. The sensor error is also included in the data. Once the driving scenarios are prepared, we use a QP solver (Goldfarb and Idnani, 1983) to solve the formulated deterministic and DRO models. To conclude, we compare the number of violated constraints of the two models to demonstrate the effectiveness of the DRO model.

For the generation of an ACC driving scenario, two types of parameters are necessary: the parameters related to the environment and to the vehicles. The parameters related to the environment include the simulation configuration and vehicle regulations, such as the total scenario duration, velocity limit, collision avoidance limit, etc. Those parameters reflect the real-life driving rules and simulation setting, and therefore they are fixed during numerical experiments. The parameters related to the vehicles, such as initial position, velocity and distance, vary in each randomly generated instance due to the diversity of driving scenarios. In order to simulate driving situations realistically, the relationships between randomly generated vehicle parameters should be representative of real-world situations.

The sequel summarizes the parameters set up for numerical simulations:

- Parameters related to the environment:
  - Total duration of a scenario T: $2s$.
  - Sampling time step dt: $0.05s$.
  - Inter-vehicle time tc: $3s$.
  - Standstill distance σS: $3m$.
  - Minimum security distance d_s: $10m$.
  - Maximum velocity v_{max}: $30m/s$.
  - Maximum acceleration v_{max}: $5m/s^2$.
  - Maximum jerk j_{max}: $5m/s^3$.
  - DRO unknown moments parameter: $\gamma^i_1 = 5$ and $\gamma^i_2 = 5$.
  - Confidence level α: 0.9.
- Parameters related to the vehicles:
  - Acceleration of the target car: independent random variables following a normal distribution with mean 0 and standard deviation 2, truncated from -5 to 5.
  - Initial speed of target car and ego car: independent random variables following a normal distribution with mean 15 and standard deviation 10, truncated from 5 to 25.

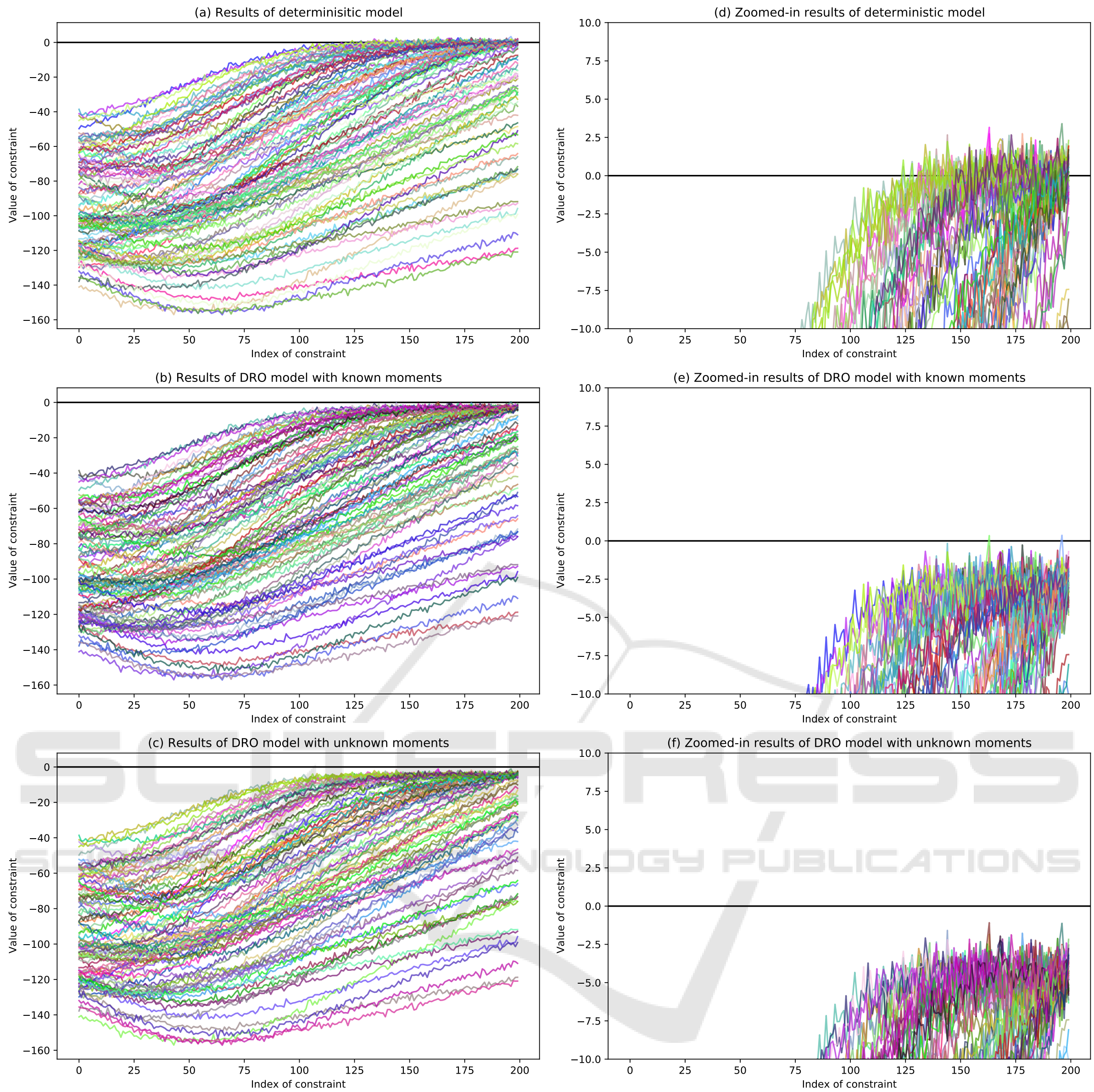

Figure 3: Constraint function values of all instances for deterministic and DRO models.

- Standard deviation of target car position σ: 1.
- Initial position of target car: random variable following a normal distribution with mean 200 and standard deviation 1.
- Speed and position of target car: random variables following normal distributions with mean calculated by an initial value and the acceleration vector, and standard deviation 1.
- Initial position of ego car: the initial position of the target car minus a random variable following a normal distribution with mean 100 and standard deviation 20, truncated from 50 to 150.

In the previous section, the optimal solution in each generated ACC driving scenario can be obtained by solving a QP problem. Several techniques exist for solving this QP problem, which can be divided into two categories: active-set methods and interior point methods. We use QP solver with Goldfarb–Idnani algorithm (Goldfarb and Idnani, 1983), which is a dual active set method, in order to obtain the optimal solution for our QP problems.

Firstly, we generate 100 random driving scenarios based on the configuration above. Then our QP solvers obtain the optimal reference of the scenarios under the deterministic model and the two DRO models, respectively. Considering the sensor error in the input data, the obtained trajectory of our ego car may violate the constraint (18) during the driving scenario. Therefore, the performance of a model can be measured by the number of times that constraint (18) is violated throughout the scenario. A more reliable model is one that produces fewer violations of constraints statistically.

According to a numerical analysis of 100 random instances solved with three models, only 50% of the results are totally feasible, which means that the constraint (18) is never violated during the scenario, when solving with the deterministic model, whereas 98% of the results are totally feasible when solving with the DRO model with known moments and 100% when solving with the DRO model with unknown moments.

Figure 3 shows a detailed analysis of constraint violations across 100 driving scenarios. Visualizations of the constraint violation value $d_s - D_T^{vehicle}\mathbb{1}_n$, adapted from constraint (18), are presented for the results of the three models. This adapted constraint value must be less than or equal to zero for the solution to be feasible (values greater than zero cause a constraint violation). The constraint value is shown in Figures 3(a), 3(b) and 3(c) for the whole scenarios, whereas a partial zoom-in is shown in Figures 3(d), 3(e) and 3(f) for easier reading. In Figure 3, each curve in its own color displays the constraint values of a driving scenario result, and the x-axis represents the time index of constraints. If the value at constraint index i exceeds 0, it means that $d_s > D_{t_i}^{vehicle}$, i.e. the constraint (18) is violated at this sampling time. It is evident from Figure 3 that the two DRO models produce significantly fewer violations than the deterministic model.

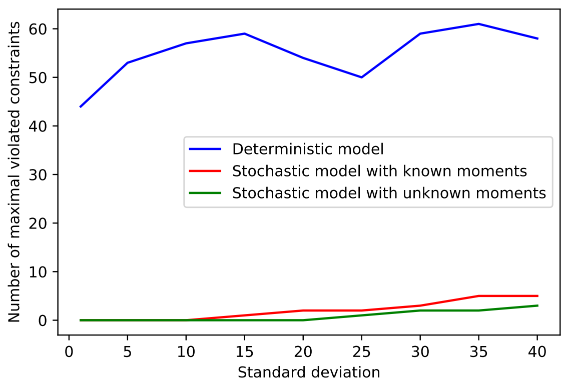

Figure 4: Maximal violated constraints under different standard deviations.

In addition to the previous experiments, we propose to evaluate the performance of the three

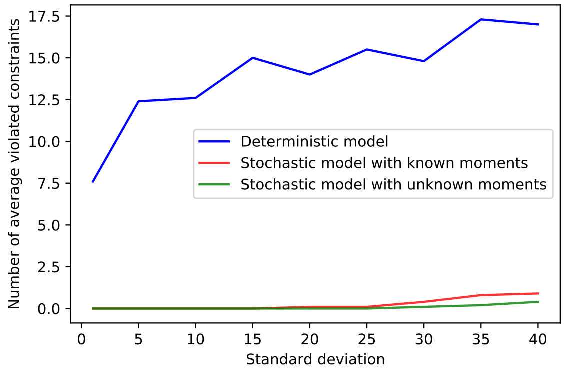

Figure 5: Average violated constraints under different standard deviations.

proposed models under different sensor precision. Keeping all other parameters unchanged, we vary the standard deviation of the target car position, which reflects the sensor's precision, from 1 to 40. The value of the standard deviation is gradually increased, and we make 100 tests for each value. Figure 4 and Figure 5 present the maximal and average number of constraint violations for each model under different standard deviations. In both cases, there are always fewer constraint violations in the two DRO models than in the deterministic model, which proves the robustness of DRO approach for uncertainty.

5 CONCLUSION AND FUTURE DIRECTIONS

The main contribution of this paper is a distributionally robust optimization-based method for ACC reference generation in driving scenarios with uncertainties. In order to satisfy the safety constraints, the reference generator takes into account the sensor errors with partial information about its distribution to produce optimal commands. The results of the numerical simulations prove the robustness of the DRO models by comparing their performances with the deterministic model under generated driving scenarios.

There are various further directions to explore for the next step. For instance, we will consider other assumptions for the sensor error and build an adapted model for real-life scenarios. Additionally, we can also apply this distributionally robust optimization model in other modules of autonomous vehicles with uncertainties in order to achieve better reliability for some crucial functionalities.

ACKNOWLEDGEMENT

This work was supported by the French government under the "France 2030” program, as part of the SystemX Technological Research Institute.

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