

Simulation Study for the Comparison of Power Flow Models for a Line Distribution Network with Stochastic Load Demands

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Abstract: We use simulation to compare different power flow models in the process of charging electric vehicles (EVs) by considering their random arrivals, their stochastic demand for energy at charging stations, and the characteristics of the electricity distribution network. We assume the distribution network is a line with charging stations located on it. We consider the *Distflow* and the *Linearized Distflow* power flow models and we assume that EVs arrive at the network with an exponential rate, have an exponential charging requirement, and that voltage drops on the distribution network stay under control. We provide extensive numerical results investigating the effect of using different power flow models on the performance of the network.

1 INTRODUCTION


In recent years, the growing electricity consumption, the active adoption of renewable generation, and the energy transition result in congestion in the electricity network. On one side, more companies use electricity for their production, more houses are heated with heat pumps and more people drive electric cars. On the other side, companies and citizens are generating more and more electricity from wind and sun, which they mostly feed back to the electricity network. This causes network capacity problems, or in other words, congestion. This is illustrated in (Hoogsteen et al., 2017), where the impact of the energy transition on a real electricity network is evaluated. Here, the authors showed that charging a small number of EVs is enough to cause a blackout in a neighborhood. Therefore, it is imperative to study the performance of the network under different power flow models, since these models are used for the design and control of the network.

Irrespective of the power flow model, it is important to respect the physical laws of the network and its constraints. In an electricity network, an important constraint is the requirement of keeping voltage losses, or in other words, the voltage drop, on a cable

in the network under control. These voltage losses are caused by the physical properties of the cables in the network. Keeping the voltage losses under control ensures that every user in the network receives safe and reliable power at a voltage that is within some standard range, which varies from one country to another (Kersting, 2018). For example, according to Dutch law, the voltage drop in a distribution network, a small part of the electricity network, is not allowed to be more than 4.5% (van Westering and Helleendoorn, 2020).

In this paper, we consider the stochastic process of charging EVs in a neighborhood such that the voltage drop in the distribution network stays under control. In this process, the stochasticity comes from random arrivals (at parking lots with charging stations) and charging requirements of EVs, while the power allocated to each EV in the network depends on the number of EVs and the corresponding location of EVs that are currently charging in the network. We model this process as a queue, with EVs representing *jobs*, charging stations classified as *servers*, and the service being delivered as the power supplied to EVs, constrained by physical laws and network constraints. The particular queuing model that we employ can be seen as falling under a general class of queuing networks called *bandwidth-sharing networks*. To model the physical laws and network constraints, we use two

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approximations of the alternating current (AC) power flow equations (Molzahn and Hiskens, 2019), i.e.; we study the *Distflow* and a linearized version of the *Distflow* model, by ignoring some power losses, that is called the *Linearized Distflow* (Baran and Wu, 1989b; Baran and Wu, 1989a).

In this simulation study, we assess the accuracy and effectiveness of the *Linearized Distflow* model compared to the *Distflow* model. Our goal is to compare the different power flow models on the performance of the stochastic process of EV charging, by the mean number of EVs and the mean charging time of each EV in the network. Furthermore, we gain insights into the behavior of the network by including variability in the distribution of the arrival rates to different parking lots.

The contributions we obtained from analyzing such a network are summarized as follows. We observe that the performance of the *Linearized Distflow* model is comparable to the *Distflow* model, i.e. the mean number of EVs and the mean charging time of an EV under both power flow models are similar and the relative difference between critical arrival rates (the specific arrival rates under both power flow models for which the mean number of EVs and the mean charging time of an EV grow to its maximum capacity) is below 5%. Thus, our first contribution is that we obtain more evidence that using the much simpler *Linearized Distflow* model is a valid and accurate approximation of the *Distflow* model, even if the system is highly heterogeneous. Namely, in our numerical examples, we consider cases where one station has almost all the load of the system. Even for such heterogeneous cases, the performance of the network is the same under the *Linearized Distflow* and the *Distflow* model. In other words, we do not lose much accuracy from a performance perspective by ignoring some power losses. The second contribution is rather surprising. It is very well known from queuing theory that variability in the network causes worse performance. However, what is surprising in this case is that the network does not perform symmetrically under the same loads. If the load of an individual parking lot is way larger than the other loads of the other parking lots, the performance of the network is different from the performance of the network if the (same) largest load is put on another parking lot.

The structure of the paper is as follows. In Section 2, we provide a literature review on work that has been done on stability for EV-charging and specifically in the context of a queuing network, and a comparison between the *Linearized Distflow* model and the *Distflow* power flow model. In Section 3, we provide a detailed model description. In particular, we

introduce the queuing network, the constraints and assumptions of the electrical distribution network, and the power flow models we consider. In Section 4, we present several numerical experiments showing the accuracy and effectiveness of the *Linearized Distflow* model and the effect of including variability in the distribution of the arrival rates to the performance of the network.

2 LITERATURE REVIEW

First, we discuss the literature on stability for EV-charging and stability in queuing networks for EV-charging which relate to the network capacity problems that we mentioned earlier. Second, we provide literature that compares the *Distflow* model and the *Linearized Distflow* model.

The term stability, from a queuing perspective, means the stability of the queuing model and is defined as the positive recurrence of a Markov process. Informally speaking, stability means the ability of all queues to complete the service of all jobs, without the number of outstanding building up infinitely. In our simulations, we consider a finite number of parking spaces at all parking lots, which implies that the queuing model is always stable. Therefore, we investigate the performance of the network in terms of the mean number of EVs and the mean charging time of an EV in the network but still discuss the literature on stability results.

Stability Results for EV-Charging. The literature on stability results for EV charging is limited to numerical experiments. An early paper on stability analysis in EV-charging is by Huang et al (Huang et al., 2013). The authors present a new quasi-Monte Carlo stability analysis method to assess the dynamic effects of plug-in electric vehicles in power systems. They conclude that improvements in stability control are worth further study since the number of EVs is growing. Other simulation studies are conducted to obtain stability conditions. In (de Hoog et al., 2014), the authors explore the constraint of requiring a minimum voltage to charge EVs throughout a network and demonstrate that the physical locations of individual demand and generation of energy play a significant role in determining whether voltages throughout the network remain within required limits or not. Similarly, in (Zhang et al., 2016), the impact of charging EVs on the voltage stability of the distribution network is simulated and analyzed. The simulation results show that the voltage stability is related to the individual loads, total load in the network, and physi-

cal properties of the network. In (Ul-Haq et al., 2015), simulations are performed on another test network. For this network, different charging strategies are implemented and it is shown that for some scenarios this can cause significant voltage instability. Last, in (Deb et al., 2018), the authors perform a numerical study on a specific test system, where they investigate the impact of a single EV charging station on the voltage stability, power losses, and economic losses of the distribution network. Here, it is also observed that the location of the EV charging station is important in the smooth operation of the grid.

Stability Conditions for Queuing Networks. The literature of the stability of queuing networks for EV charging is very limited. It has been studied in (Carvalho et al., 2015). Here, the authors find by simulation that there is a threshold on the arrival rates of EVs, such that if the actual arrival rate is greater than this threshold, some cars have to wait for increasingly long times to fully charge. The first analytical study is (Christianen et al., 2022), where the authors compare these thresholds on the arrival rates under different power flow models and compute the difference between these rates explicitly as the number of parking lots grows to infinity.

Comparison of the Linearized Distflow and Distflow Model. The practical use of the Linearized Distflow compared to the Distflow model is based on the assumption that power losses on cables are typically small. It has been shown experimentally that this only introduces a small relative error, on the order of 1% (Farivar et al., 2013). However, these small relative errors may be exaggerated when used in a complex stochastic process and this is what we examine in this paper. Multiple other numerical studies have been conducted to verify the accuracy and effectiveness of the Linearized Distflow model (Baran and Wu, 1989b; Wang et al., 2014a; Chen et al., 2016; Tan et al., 2013; Yuan et al., 2016; Wang et al., 2014b; Yeh et al., 2012; Li et al., 2019; Cao et al., 2019).

3 MODEL DESCRIPTION AND FORMULATION

This section describes the main components of the EV-charging model, i.e.; we describe the characteristics of the queuing, the distribution network, and the power flow models.

3.1 Queuing Model of EV-Charging

We use a queuing model to study the process of charging EVs in a distribution network. EVs referred to as *jobs* require service. This service is delivered by charging stations, referred to as *servers* and the service being delivered is the power supplied to EVs. At all parking lots, there is one charging station with $K > 0$ parking spaces, and each parking space has its own EV charger.

Thus, in the queuing system, we consider N single-server queues, each having its own arrival stream of jobs. Denote by $\mathbf{X}(t) = (X_1(t), \dots, X_N(t))$ the vector giving the number of jobs at each queue at time t . At all parking lots, all EVs arrive independently according to Poisson processes with rate $\lambda_i, i = 1, \dots, N$ and have independent service requirements which are $Exp(1)$ random variables. If all parking spaces are occupied, a newly arriving EV does not enter the system but is assumed to leave immediately.

At each queue, all jobs are served simultaneously and start service immediately (there is no queuing). Furthermore, each job receives an equal fraction of the service capacity, i.e. the charging rate, allocated to a queue. Denote by $\tilde{\mathbf{p}}(t) = (\tilde{p}_1(t), \dots, \tilde{p}_N(t))$ the vector of service capacities allocated to each queue at time t . From now on, for simplicity, we drop the dependence on time t from the notation. For example, we write X_j and \tilde{p}_j instead of $X_j(t)$ and $\tilde{p}_j(t)$.

Service capacities are state-dependent and subject to changes to the current vector $\mathbf{X} = (X_1, \dots, X_N)$ of number of jobs. For each state of the system, i.e. a given number of EVs charging at each parking lot, we assume that the charging rates $\tilde{\mathbf{p}}$ are the unique solution of the optimization problem

$$\max_{\tilde{\mathbf{p}}} \sum_j X_j \log \left(\frac{\tilde{p}_j}{X_j} \right), \quad (1)$$

which are called proportional fair allocations. For the optimization problem, the feasible region can take many forms and depends heavily on the power flow model that is used. In Section 3.3, we discuss the feasible regions for both power flow models in more detail.

We can then represent the number of electric vehicles charging at every station as an N -dimensional continuous-time Markov process. The evolution of the queue at node j is given by

$$X_j(t) \rightarrow X_j(t) + 1 \text{ at rate } \lambda_j,$$

and

$$X_j(t) \rightarrow X_j(t) - 1 \text{ at rate } \tilde{p}_j.$$

3.2 Distribution Network Model

As distribution networks are typically radial (Aveklouris et al., 2019), we model the distribution network as a directed graph $\mathcal{G} = (I, \mathcal{E})$, where we denote by $I = \{0, \dots, N\}$ the set of nodes and by \mathcal{E} its set of directed edges, assuming that node 0 is the root node. We assume that \mathcal{G} has a line topology. Each edge $\varepsilon_{j-1,j} \in \mathcal{E}$ represents a line connecting nodes $j-1$ and j where node j is further away from the root node than node $j-1$. Each edge $\varepsilon_{j-1,j} \in \mathcal{E}$ is characterized by the impedance $z = r + ix$, where $r, x \geq 0$ denote the resistance and reactance along the lines, respectively. We make the following natural assumption, given that $r \gg x$ in distribution networks (Khatod et al., 2006; Tonso et al., 2005).

Assumption 3.1. All edges have the same resistance value $r > 0$ and reactance value $x = 0$.

Furthermore, let $\tilde{s}_j = \tilde{p}_j + i\tilde{q}_j$ be the complex power consumption at node j . Here, \tilde{p}_j and \tilde{q}_j denote the active and reactive power consumption at node j , respectively. By convention, a positive active (reactive) power term corresponds to consuming active (reactive) power. Since EVs can only consume active power (Carvalho et al., 2015), it is natural to make the following assumption.

Assumption 3.2. The active power \tilde{p}_j is non-negative and the reactive power \tilde{q}_j is zero at all charging stations $j \in I$.

Let \tilde{V}_j denote the voltage at node j . Given Assumptions 3.1 and 3.2, the voltages at each node j , \tilde{V}_j , can be chosen to have zero imaginary components (Carvalho et al., 2015; Aveklouris et al., 2019). For each $\varepsilon_{j-1,j} \in \mathcal{E}$, let $I_{j-1,j}$ be the complex current and $\tilde{S}_{j-1,j} = \tilde{P}_{j-1,j} + i\tilde{Q}_{j-1,j}$ be the complex power flowing from node $j-1$ to j . Here, $\tilde{P}_{j-1,j}$ and $\tilde{Q}_{j-1,j}$ denote the active and reactive power flowing from node $j-1$ to j . The model is illustrated in Figure 1.

Voltage Drop Constraint. The distribution network constraints, that is in our case only the voltage drop constraint, represent the feasible region of (1) and are described by a set \mathcal{C} . The set \mathcal{C} is contained in an N -dimensional vector space and represents feasible power allocations. In our setting, a power allocation is feasible if the maximal voltage drop; i.e., the relative difference between the base voltage \tilde{V}_0 and the minimal voltage in all buses between the root node and any other node is bounded by some value $\Delta \in (0, \frac{1}{2}]$. Thus, the distribution network constraints can be described as

$$\mathcal{C} := \left\{ \tilde{\mathbf{p}} : \frac{\tilde{V}_0 - \min_{1 \leq j \leq N} \tilde{V}_j}{\tilde{V}_0} \leq \Delta \right\}, \quad (2)$$

where $0 < \Delta \leq \frac{1}{2}$. In Section 3.3, we give more concrete definitions of the constraint set \mathcal{C} for each power flow model.

3.3 Power Flow Models

We introduce two commonly used models to represent the power flow that is valid for radial systems; i.e., systems where all charging stations have only one (and the same) source of supply. They are called the *Distflow* and *Linearized Distflow model* (Low, 2014; Baran and Wu, 1989b). Both models are valid when the underlying network topology is a tree, which is the case in this paper (as we consider a line topology).

Given the impedance r , the voltage at the root node \tilde{V}_0 and the power consumptions $\tilde{p}_j, j = 1, \dots, N$, both power flow models satisfy three relations. First, we have power balance at each node:

$$\tilde{S}_{j-1,j} - r|I_{j-1,j}|^2 = \tilde{s}_j + \tilde{S}_{j,j+1}, \quad j \in I \setminus \{0\}. \quad (3)$$

Here, on the one hand, the quantity $r|I_{j-1,j}|^2$ represents line loss so that $\tilde{S}_{j-1,j} - r|I_{j-1,j}|^2$ is the receiving-end complex power at node j from node $j-1$. On the other hand, the delivering-end complex power is the sum of the consumed power at node j and the complex power flowing from node j to node $j+1$. Second, by Ohm's law, we have for each edge,

$$\tilde{V}_{j-1} - \tilde{V}_j = rI_{j-1,j}, \quad \varepsilon_{j-1,j} \in \mathcal{E}, \quad (4)$$

and third, due to the definition of complex power, we have for each edge,

$$\tilde{S}_{j-1,j} = \tilde{V}_{j-1}\bar{I}_{j-1,j}, \quad \varepsilon_{j-1,j} \in \mathcal{E}. \quad (5)$$

However, for a line topology, the distribution network constraints in (2), using (3)–(5), can be rewritten for both power flow models (Christianen et al., 2022). Following the approach in (Christianen et al., 2022), the distribution network constraints under the Distflow model \mathcal{C}^D reduce to

$$\mathcal{C}^D := \left\{ \tilde{\mathbf{p}} : \tilde{V}_0^D \leq \frac{1}{1-\Delta} \right\}, \quad (6)$$

where \tilde{V}_0^D can be found recursively by

$$\tilde{V}_{N-1}^D = 1 + r\tilde{p}_N, \quad (7)$$

$$\tilde{V}_{j-1}^D = 2\tilde{V}_j^D - \tilde{V}_{j+1}^D + \frac{r\tilde{p}_j}{\tilde{V}_j^D}, \quad j = 1, \dots, N-1. \quad (8)$$

In the Linearized Distflow model, it is assumed that the active power losses $r|I_{j-1,j}|^2$ are much smaller than the active power flows $\tilde{P}_{j-1,j}$. In other words, the Linearized Distflow model neglects the loss terms associated with the squared current magnitudes $|I_{j-1,j}|^2$. In that case, the distribution network constraints under the Linearized Distflow \mathcal{C}^{LD} reduce to

$$\mathcal{C}^{LD} := \left\{ \tilde{\mathbf{p}} : 2r \sum_{j=1}^N \sum_{k=j}^N \tilde{p}_k \leq \frac{\Delta(2-\Delta)}{(1-\Delta)^2} \right\}. \quad (9)$$

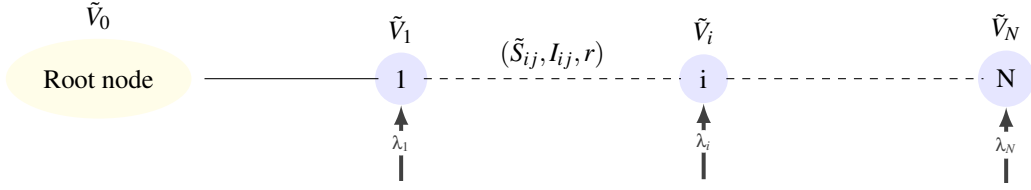


Figure 1: Line network with N charging stations and arriving vehicles at rate λ_i , $i \in \{1, \dots, N\}$.

4 NUMERICAL STUDY

In the previous section, we discussed our model for the EV-charging process. Here, we obtain general insights into the performance of the model by simulation on a large range of parameter settings. Moreover, this allows us to compare the behavior of the model under the Distflow model and the Linearized Distflow model. We vary the total arrival rate to the network and the distribution of the total arrival rate to different parking lots. We focus on the effect of the mean number of EVs in the network and the mean charging time of an EV, possibly at each parking lot.

4.1 Critical Arrival Rate

To control the network, we observe that there is a *critical* arrival rate λ_c (≈ 0.18) if the arrival rate to each parking lot is assumed to be the same. At every parking lot, the mean number of EVs and the mean charging time of an EV grow to maximum capacity as soon as the actual arrival rate is greater than the critical arrival rate. See Figure 2, where the mean number of EVs and the mean charging time of an EV are plotted versus the individual arrival rate to each parking lot. We observe the mean number of EVs and the mean charging time of an EV for both power flow models at each parking lot. We fix the number of parking lots at $N = 2$, the resistance for each cable at $r = 0.1$, the maximal capacity at each parking lot at $K = 100$ and the parameter to control the voltage drop at $\Delta = 0.05$ for the Distflow model (dashed) and the Linearized Distflow model (solid) at parking lot 1 (blue) and at parking lot 2 (red). Although the total arrival rate is varied, the solid curves are close to the dashed lines for the two parking lots. Moreover, the solid curves are below the dashed curves. This is to be expected; in (Christianen et al., 2022), the authors observed that the Linearized Distflow power flow model allows for too optimistic arrival rates since the Linearized Distflow model overestimates the voltages in comparison with the voltages given by the Distflow model. From this, we observe that the allocated power to each parking lot is higher under the Linearized Distflow than the allocated power under the Distflow model. Higher

allocated power means faster charging. Hence, EVs leave the parking lots faster and the mean number of EVs charging is lower.

This statement is reinforced by Figure 3, where the relative difference in the mean number of EVs in the network in percentages between the Distflow model and the Linearized Distflow model is plotted versus the total arrival rate to the parking lots. It is still assumed that $N = 2$, $r = 0.1$, $K = 100$ and $\Delta = 0.05$ for all of these lines. For the blue curve, we have equal arrival rates for all parking lots. From Figure 3, it is apparent that the relative difference in the mean number of EVs between both power flow models is below 5% for almost all total arrival rates to the network. However, when the total arrival rate is close to two times the critical arrival rate λ_c (≈ 0.36), we have already seen in Figure 2 that the mean number of cars for both power flow models seem to reach their maximum capacity and that this happens slightly earlier for the Distflow model than for the Linearized Distflow model. This causes the high relative difference in the mean number of EVs for both power flow models.

4.2 Variability of the Distribution of the Total Arrival Rate

The previous section brings us naturally to the discussion of adding variability to the distribution of the total arrival rate. Instead of assuming equal arrival rates for all parking lots, we vary the fraction of EVs that arrive at each parking lot for a wide range of the total arrival rate to the network.

For all combinations of total arrival rates and fractions of EVs that arrive at each parking lot, the heat map of the mean number of cars has an interesting structure. In Figure 4, we show the total mean number of EVs in the network as a function of the total arrival rate and the fractions of EVs that arrive at each parking lot. As we see in Figure 4, the mean number of EVs has a non-symmetric structure. When we increase the fraction of EVs that arrive at parking lot 1 (and thus decrease the fraction of EVs that arrive at parking lot 2) from the situation of equal arrival

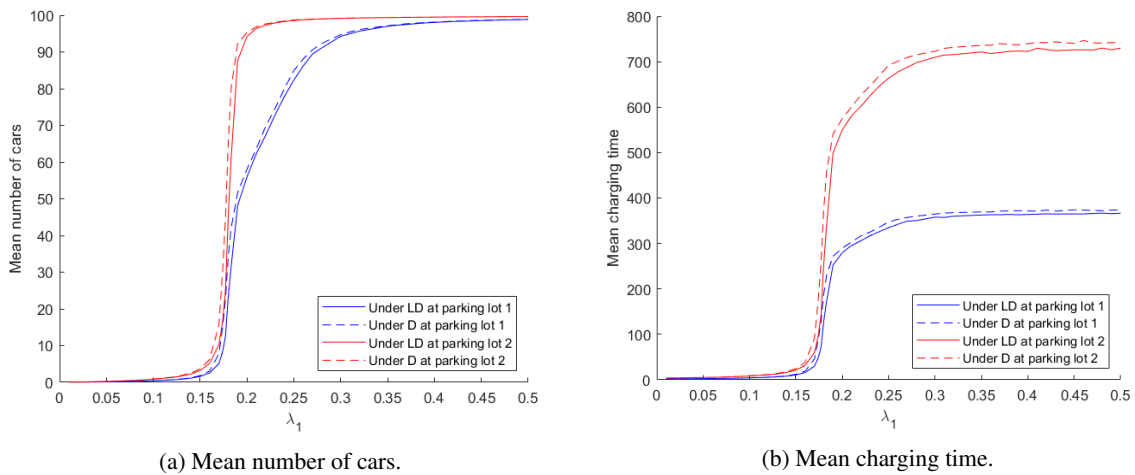


Figure 2: Performance measures vs. the individual arrival rate per parking lot for the Distflow model (dashed) and the Linearized Distflow model (solid) at parking lot 1 (blue) and at parking lot 2 (red).

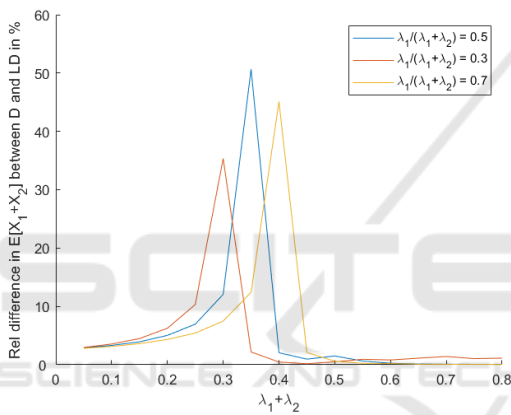


Figure 3: Relative difference in mean number of EVs between both power flow models.

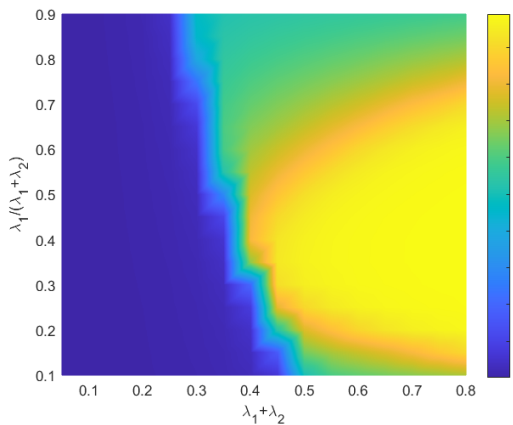


Figure 4: Mean number of cars.

rates, the total mean number of EVs in the network decreases faster than the total number of EVs in the network decreases when we increase the fraction of

EVs that arrive at parking lot 2. This is natural given the total available power that can be allocated to each parking lot. Due to the power loss on the cables, the available power that can be allocated to parking lot 1 is approximate twice the available power that can be allocated to parking lot 2. We compare the following two situations; one where we have a given number of EVs charging at parking lot 1 and no EVs charging at parking lot 2 (which corresponds to a situation where the fraction of EVs that arrive at parking lot 1 is high), and one where we have the same given number of EVs charging at parking lot 2 and no EVs charging at parking lot 1 (which corresponds to a situation where the fraction of EVs that arrive to parking lot 2 is high). Since the allocated power to parking lot 1 in the first situation is higher than the allocated power to parking lot 2 in the second situation, the mean number of EVs at parking lot 1 tends to be smaller than the number of EVs at parking lot 2. Moreover, if we consider a fixed total arrival rate, e.g., $\lambda_1 + \lambda_2 = 0.8$, the variability of the distribution of the total arrival rate has a small influence on the mean number of EVs for a wide range of the ratio of arrival rates. In Figure 5a, where the relative difference between the total mean number of EVs given any distribution of the total arrival over both parking lots and the total mean number of EVs given equal arrival rates to both parking lots is plotted, we observe that this relative difference is below 5% if the fraction of EVs that arrive at parking lot 1 range between 20% and 60%.

Another observation on the heat map of the mean number of cars is that there is a clear distinction between networks that have reached their capacity or not. In the blue region, the mean number of EVs is relatively low. However, in the green and yellow regions, the mean number of EVs is relatively high

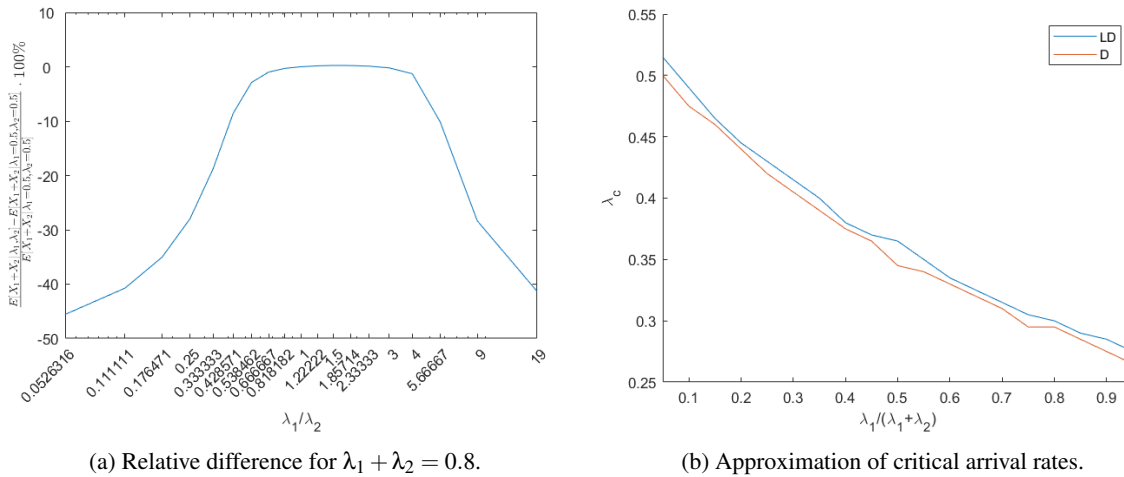


Figure 5: Observations based on the heat map of the mean number of cars.

and close to its maximum capacity, namely 200 EVs. Moreover, the green region indicates a network where the number of EVs charging at either one of the parking lots reaches its capacity. The yellow region indicates a network where the number of EVs at both parking lots reaches its capacity.

In Figure 5b, we also plot a rough approximation of the evolution of the critical arrival rates under both power flow models as we vary the fraction of EVs that arrive at parking lot 1 that we obtained as follows. As can be seen in Figure 2, there is a steep increase in the mean number of EVs and the mean charging time of an EV at a certain point. Using a fine grid search, we find the arrival rate for which the absolute difference between subsequent measurements in the mean number of cars is the largest. In Figure 5b, we observe that as the fraction of EVs that arrive at parking lot 1 increases, the critical arrival rates for both power flow models decrease. Furthermore, the critical arrival rates for the Distflow model are always below the critical arrival rates for the Linearized Distflow model; a behavior that we observed in the setting of equal arrival rates for both parking lots already.

Adding variability to the distribution of the total arrival also influences the relative difference of the total mean number of EVs in the network between both power flow models. This effect is observed in Figure 3, where we see that for a fraction of EVs that arrive at parking lot 1 that deviates from the nominal situation of equal arrival rates, the maximal relative difference between the two power flow models is lower. This has to do with parking lots reaching their maximum capacity. In the situation that we have different arrival rates to the parking lots, the mean number of EVs grows to maximum capacity only at one parking lot while in the situation of equal arrival rates, the mean number of EVs grows to maximum capacity at

both parking lots. The relative difference in the mean number of EVs for both power flow models is highest in these cases and therefore the maximal relative difference between the Linearized Distflow and the Distflow model is highest in the situation of equal arrival rates. Furthermore, also for other distributions of the total arrival rate than equal arrival rates to both parking lots (red and yellow curves), we observe that the relative difference is below 5% for almost all total arrival rates to the network, except around those values for the total arrival rate where we turn from a network with a relatively low mean number of EVs to a network with a relatively high number of EVs. For such values, we could compute the critical arrival rate for the Linearized Distflow model and adjust by a small percentage to compensate for ignoring power losses, since the critical arrival rates under both power flow models are close to each other, see Figure 5b.

In summary, the performance of both power flow models is approximately the same. Simulation results show that for a wide range of total arrival rates to the network the relative difference in the mean number of EVs between the Distflow model and the Linearized Distflow model is below 5%. Furthermore, the critical arrival rates under both power flow models are close to each other. Moreover, we can say that the variability in the distribution of the total arrival to parking lots for heavily loaded instances, as long as the fraction of EVs that arrive at parking lot 1 range between 20% and 60%, does not influence the performance of the network much, in the sense that the mean number of EVs and the mean charging time of an EV are comparable to the mean number of EVs and the mean charging time of an EV in the case where the arrival rates to all parking lots are the same, respectively.

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