Trajectory-Based Dynamic Boundary Map Labeling

Ming-Hsien Wu and Hsu-Chun Yen[a]
Dept. of Electrical Engineering, National Taiwan University, Taiwan

Keywords: Boundary Labeling, Dynamic Map Labeling.

Abstract: Traditional map labeling focuses on placing labels on a static map to help the reader gain a better understanding of the content of the map. As the content of a dynamic map changes as time progresses, traditional static map labeling algorithms usually cannot be applied to dynamic maps directly. In this paper, we consider the design of algorithms for trajectory-based dynamic boundary labeling, in which non-overlapping labels, connecting to points on the map through straight-line leaders, are placed on one side of a viewing window which moves or rotates along a trajectory. The goal is to maximize the total visible time of all labels during the course of the navigation. To avoid visual disruptions, the effect of flickering is also taken into account in our design. Even though the problem can be formulated using mathematical optimization, heuristic strategies are also incorporated in the design to reduce the running time to make the solutions more practical in real-world applications. Finally, experimental results are used to illustrate the effectiveness of our design.

1 INTRODUCTION

In applications such as geographic information systems, navigation systems, and information visualization, map labeling is crucial for the user to recognize the labels which provide extra information useful for understanding a map better. The traditional map labeling problem, known to be NP-complete, finds the maximum number of non-overlapping labels (possibly using leaders if necessary), drawn as rectangles associated with features to be labeled in a (static) map. A feasible solution of map labeling should not have overlapping labels and/or leaders.

Among various labeling styles, Bekos et al. (Bekos et al., 2007) first introduced the so-called *boundary labeling*, in which all the labels are attached to the boundary of a rectangular view R, and the points to be labeled in R are connected to the labels by leaders which can be straight-lines or rectilinear line segments. In this model, a common objective is to arrange the labels and leaders to avoid crossings if at all possible, while minimizing the total length of leaders or the number of bends.

To cope with an increasingly wider utilization of mobile devices and navigation systems, dynamic map labeling has received increasing attention in recent years. In contrast to static maps, dynamic maps can be moved or rotated by the user over the time. Therefore, static map labeling algorithms cannot be applied to labeling dynamic maps directly. When a user wants to find the trajectory to the destination with a navigation system, he or she inputs the destination, then the navigation system outputs the trajectory to the user. The user recognizes the location and the trajectory by the labels of the map and moves along the trajectory to explore more details of the map. The screen's view can move, rotate while the user navigates along the trajectory of the map. Hence, a good labeling strategy can help the navigation system better serve the user. Prior work regarding algorithmic design of dynamic labeling can be found, for example, in the following literature (Nöllenburg et al., 2010; Gemsa et al., 2016a; Gemsa et al., 2016b; Gemsa et al., 2013; Barth et al., 2016; Haunert and Hermes, 2014; Fekete and Plaisant, 1999; Heinsohn et al., 2014; Fink et al., 2012).

Figure 1: A map and a trajectory.

Fig. 1 shows a map and a trajectory (colored in blue) along which the user navigates. As a viewing

[a] https://orcid.org/0000-0002-1764-1950

Wu, M. and Yen, H.
Trajectory-Based Dynamic Boundary Map Labeling.
DOI: 10.5220/0011629400003417
In *Proceedings of the 18th International Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications (VISIGRAPP 2023) - Volume 3: IVAPP*, pages 142-149
ISBN: 978-989-758-634-7; ISSN: 2184-4321

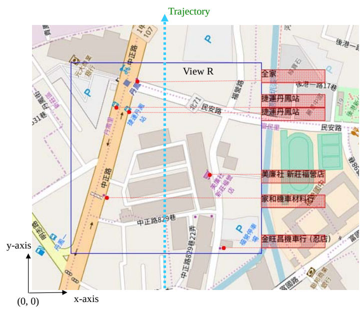


Figure 2: A viewing window with labels placed along the boundary of the window. Sites are colored in red on the map.

window is relatively small compared with the size of the entire map, at any point in time only a portion of the map along the trajectory can be displayed in the window. See Fig. 2. In this paper, we consider the dynamic map labeling problem with sliding labels in the framework of boundary labeling. A *site* (or *point*) on the map is connected to its associated label at a *port* through a horizontal straight-line called a *leader*. By sliding labels we mean that a port connecting to a label can slide on the side of the label. It should be noted that if two sites are too close to each other, their labels cannot be displayed simultaneously without causing overlapping. As sites enter and leave the viewing window in the process of navigating along the trajectory, our goal is to find a feasible solution achieving the maximum total visible time of labels subject to certain requirements. The first requirement is that labels do not overlap. As pointed out in Been et al. (Been et al., 2006), label "jumping" or "flickering" should be avoided in dynamic labeling, meaning that the movement of labels should be continuous in order not to cause visual disruptions. In our setting, a trajectory is decomposed into line segments as displayed in Fig. 1, in which the route consists of five segments h_1, h_2, h_3, h_4 and h_5. The angles between the five segments are $\theta_1, \theta_2, \theta_3$, and θ_4. In our strategy, the optimization procedure is separated into two modes, namely, *moving* and *rotating* modes. When navigating along a straight line segment (such as those h_1, h_2, h_3, h_4 and h_5), a moving mode is involved. At a junction between, for example h_1 and h_2, a turn of θ_1 angle is encountered during which labels may enter and leave the viewing window. Hence, an optimization procedure operating in the rotating mode is carried out before starting a new phase of a moving mode.

In the moving mode, we present the objective using a mixed integer programming (MIP) formulation with the flickering constraints. By solving the linear system, non-overlapping label placements with the maximum total visible time can be found, in spite of the fact that the running time increases substantially as the input size grows. The rotating mode is a bit more complicated as it involves non-linear components. Our algorithm involves a non-linear programming first, aiming at finding a visible region of angles between each pair of labels, and then apply a linear programming to carry out the actual label assignments. In order to reduce the running time, we also propose some heuristic algorithms, capable of finding a feasible solution for most practical applications in a reasonable amount of time. Experimental results show our approach to be promising.

The interested reader should note that the main focus of our work differs from that of (Nöllenburg et al., 2010) in the sense that (Nöllenburg et al., 2010) dealt with one-sided boundary labeling subject to continuously changing the scale of the map as well as allowing labels to be clustered into smaller groups, whereas in our setting, our study is w.r.t. a trajectory-based dynamic boundary labeling. Prior work such as (Gemsa et al., 2016a; Gemsa et al., 2016b; Gemsa et al., 2013; Barth et al., 2016) dealt with internal labeling in which labels are placed close to the sites to be labeled inside the map, as opposed to placing labels externally in our setting.

2 PRELIMINARIES

2.1 Boundary Labeling

The *boundary map labeling problem* can be formulated as follows: given a rectangular *viewing window* (or simply *view*) R and a set of points P, each point $p \in P$ inside the view R is to be assigned (if possible) to a rectangular *label* which contains the information associated with the point. Moreover, each point should use a straight line or a rectilinear line (also called a *leader*) to connect to the associated label placed on the side of R. *Boundary labeling* is to find labels' positions such that labels do not overlap with each other, and leaders cannot intersect either. The point to which a leader is attached is referred to as a *port*. A port can be either *fixed* or *sliding*. The former assigns a port to a fixed position of a label (e.g. the middle of the label), while the latter allows a port to slide along the side of a label. In general a label can be placed on one of the four sides of R. An example of a boundary labeling is depicted in Fig. 3 in which labels are placed on the right side of R. Such a labeling style is called *1-sided boundary labeling*. Throughout this paper, we consider only 1-sided boundary labeling, unless stated otherwise. As labels are placed on the right side of R, we refer to a label's upper left-hand corner as the *anchor* of the label, and a point in R is connected to a (sliding) port along the left side of the label through a horizontal straight-line. Notice that as we use horizontal straight-lines for leaders, at any point in time there might be points in R that cannot be labeled without causing overlapping. It is also clear that for a visible label, the y coordinate of the label's anchor (w.r.t. the center of R while the x and y axes parallel the horizontal and vertical sides of R, respectively) is sufficient to uniquely specify the placement of the label and well as the port position.

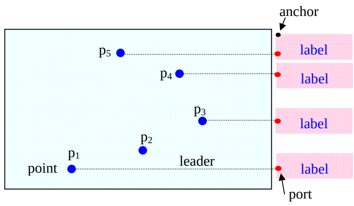


Figure 3: 1-sided boundary labeling.

2.2 Trajectory-Based 1-Sided Boundary Labeling

In a typical GIS application, a navigation system computes the optimum route on a map for the user and renders the output route to guide the user during the course of the navigation. A way to do so is through a *trajectory map* along with a view R which, moving along the trajectory route, displays the information at positions of interest to the user (referred to as *points*) residing in R using labels. In our study, the labeling scheme is based on 1-sided boundary labeling, in which each point is associated with a label (a rectangle) on the right side of view R. The user's position is fixed at the center, and view R and its labels always align to the user's view. See Fig. 2. To simplify our study, we assume the route to consist of a sequence of line segments as Fig. 1 shows. In our design, we further decompose the trajectory into *moving* and *rotating* parts. In the moving part, the user only navigates the map straight ahead (i.e., in the vertical direction of R) along the trajectory while in the rotating part, the user rotates the map's headway orientation. Fig. 1 illustrates that a trajectory is decomposed into moving parts $h_1,\ldots,h_5$ and rotating part $\theta_1,\ldots,\theta_4$. Using such a decomposition, we are able to divide the overall labeling problem into independent subproblems alternating between moving and rotating. Therefore, our strategy is to treat the moving and rotating modes separately, and propose algorithms to solve the problems individually.

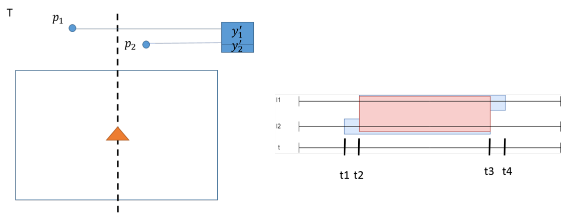


Figure 4: Timeline and block interval.

3 LABELING ALGORITHMS

3.1 The Moving Mode

We first consider the case when the view R moves in the vertical direction upwards. To give the reader a better feel for how our algorithm works, the idea of a *timeline* is used, which is illustrated in Fig. 4. For more about timelines in dynamic labeling, see also (Barth et al., 2016). While view R moves upwards in the vertical direction, points p_2 and p_1 individually are visible during intervals $[t_1,t_3]$ and $[t_2,t_4]$, respectively. During $[t_2,t_3]$, the two labels, if displayed simultaneously, overlap. The time line graph associated with the two points is shown in Fig. 4(Right). The interval $[t_2,t_3]$ is called the *block interval* between p_1 and p_2. In the two-point case, the labels of p_1 and p_2 can always be displayed without causing overlapping as long as their y coordinates are distinct, for a port can slide along the left side of a label. Things become more complicated if an additional point is included in the view. Fig. 5 shows the timeline together with the block intervals highlighted in grey for three points p_1,p_2,p_3. Suppose the y-coordinates of p_1 and p_3 is smaller than the height of a label (assuming uniform label height), then between t_3 and t_4, the three labels (say, l_1,l_2,l_3) cannot co-exist even under the sliding port model. In fact, the anchors positions of l_1,l_2,l_3 decide the max total visible time of the three labels. Given a label l, we write (l,t,on) (resp., (l,t,off)) to denote that label l is enabled (resp., disabled) to display at time t. Consider the following three label assignments:

- (l_3,t_1,on); (l_1,t_3,on); (l_3,t_4,off); (l_1,t_6,off).
  In this case, the total visible time is $(t_4-t_1)+(t_6-t_3)$.
- (l_3,t_1,on); (l_3,t_2,off); (l_2,t_2,on); (l_2,t_3,off); (l_1,t_3,on); (l_1,t_6,off).
  In this case, the total visible time is $(t_2-t_1)+(t_3-t_2)+(t_6-t_3)$.
- (l_3,t_1,on); (l_3,t_2,off); (l_2,t_2,on); (l_2,t_3,off); (l_1,t_3,on); (l_1,t_4,off); (l_2,t_4,on); (l_2,t_5,off); (l_1,t_5,on); (l_1,t_5,off).

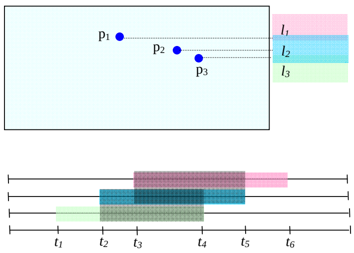


Figure 5: Block intervals with three labels.

This is not a legal assignment, as label l_2 is turned on during $[t_2,t_3],[t_4,t_5]$ but is off during $[t_3,t_4]$, which results in flickering.

Clearly the first assignment has a higher total visible time than the second.

We are now in a position to state our problem setting formally in the moving mode.

- (**Input:**) A map M with a set $P = \{p_1,...,p_n\}$ of points, and a trajectory J of a vertical line segment, a view R with the starting point of the trajectory located at the center of the bottom edge of R (see Fig. 2).
- (**Output:**) For each point $p_i \in P$ which resides in the view R at some time during the navigation, determine the time of interval (possibly empty) during which p_i is visible subject to the following constraints:
  - the label associated with point p_i is of height h_i and is placed on the right size of R through horizontal (sliding) leaders,
  - at any point in time, no two labels can overlap with each other,
  - a label cannot flick, i.e., appear at some time, disappear later, and reappear at a later time,
  - R moves upwards at a constant speed,

  such that the total visible time of all labels is maximized.

Each point p along the trajectory is associated with a time interval $[t,t']$ such that p enters and exits R at t and t', respectively. For ease of expression, we let $P = \{p_1, p_2,..., p_n\}$ be all the points whose time intervals are not empty, i.e., they appear in R at some point in time along J. We use l_i to denote the label of p_i, and let the time interval of p_i be $[t_i,t'_i]$, $1 \le i \le n$. Although R moves along J continuously, it is sufficient to formulate the problem in the discrete case in which a decision (of whether to display a label and at which position) is only made at time belonging to $T = \{t_i,t'_i | 1 \le i \le n\}$. We sort T and let the sorted (in increasing order) sequence be $r_1, r_2,..., r_m$, for some m. Let $\Psi_j = [r_j, r_{j+1}], 1 \le j \le m-1$. We define $|\Psi_j| = r_{j+1} - r_j$, i.e., the length of interval $[r_j, r_{j+1}]$. As labels are placed on the right side of R, it is sufficient to define the position of a label l_i using the y-coordinate (denoted as y'_i) of its anchor while letting the origin be at the lower left-hand corner of R at the starting point of J. We let y_i be the y-coordinate of p_i in the same coordinate system. Notice that $|y'_i - y_i| \le h_i$. For each $1 \le i \le n$, we define a variable x_{ik} of value 0 or 1 to indicate whether l_i is visible in Ψ_k or not. Now the problem can be formulated as the following MIP formulation.

$$\max \sum_{i=1}^{n} \sum_{k=1}^{m} x_{ik} \times |\Psi_k| \qquad ...... \quad \textbf{(Eq. 1)}$$

subject to

(1) $\forall i,j,k,\ x_{ik} + x_{jk} \le 1$ if $min(y'_i, y'_j) - max(y'_i - h_i, y'_j - h_j) > 0$

(2) $\forall i,r,s,t,\ x_{ir} + x_{it} \le x_{is} + 1$ if $1 \le r < s < t \le m$

(3) $\forall i,k,\ x_{ik} = 0$ if $\Psi_k \not\subseteq [t_i,t'_i]$

In the above MIP, those $y_i, t_i, t'_i, h_i, 1 \le i \le n$, are known in advance while variables fall into two classes:

(i) $y'_{i \in \mathbb{R}}, 1 \le i \le n$

(ii) $x_{ij} \in \{0,1\}, 1 \le i \le n, 1 \le j \le m$.

Notice that the values of y'_i decide the timeline as well as those Ψ_j. In Eq. 1, (1) is to ensure that overlapping labels are not assigned to the same Ψ_k; (2) is a condition to avoid flickering; (3) is obvious as label l_i associated with point p_i can only be displayed during $[t_i,t'_i]$.

In the moving mode, because view R only moves up along the trajectory, it is desirable to find clusters of points between which their label assignments can be done independently. The idea is quite simple. If the y-distance between two points p_i and p_j is larger than $h_i + h_j$, their labels have no way of overlapping regardless where their anchors are located. By searching through the entire set of points along the trajectory, it is easy to identify clusters each of which can be dealt with separately in the optimization procedure. In our experiment, the above MIP is solved using the gradient descent approach, and the algorithm is named *Algorithm 1*.

To further reduce the running time, a greedy heuristic can be applied to get rid of type (i) variables in Eq. 1. The first step is to sort the $p_i, 1 \le i \le n$, according to their y-coordinates. For each cluster, we start by assigning the lower left-hand corner of the label associated with the top-most point in the sorted list of the cluster to the y-coordinate of the point, and the anchor of the label of the lowest point to its y-coordinate. By doing so, a maximum amount of space is made available to accommodate the labels of the remaining points. Iteratively, the top-most and the bottom-most points are chosen from the list whose anchors have not yet been assigned, and the anchor of the top-most (resp., bottom-most) point is assigned in a way to minimize the overlapping with those labels that have been assigned above (resp., below) the point. The above heuristic algorithm of anchor assignment takes $O(n)$ time, even though it may still result in label overlapping. The next step is to use the MIP in Eq. 1 to find the best assignment with the anchor positions of labels given. Such a heuristic algorithm is called *Algorithm 2*.

3.2 The Rotating Mode

We now consider label assignments in the rotating mode, in which points may rotate in view R when the trajectory is turning at a junction between two segments. While the points are rotating, the existing labels may move up or down which in turn may result in overlapping. It is also possible to encounter new points entering the view during the course of the rotation. Like in the moving mode, our objective is to find the maximum visible time while points are rotating. Our problem setting in the rotating mode is stated below.

- (**Input:**) A map M with a set P of points, a view R with its center located at c, and an angle of rotation θ_r.
- (**Output:**) For each point $p \in P$ which resides in the view R at some time during the course of R rotating θ_r degree, determine the time of interval (possibly empty) during which p is visible subject to the following constraints:
  - the label associated with point p_i is of height h_i and is placed on the right size of R through horizontal (sliding) leaders,
  - at any point in time, no two labels can overlap with each other,
  - a label cannot flick, i.e., appear at some time, disappear later, and reappear at a later time,
  - R rotates θ_r degree at a constant angular speed around c,

  such that the total visible time of all labels is maximized.

In the rotating scenario, the view R rotates at a constant angular speed w.r.t. the center of R until the turning angle (i.e., those θ in Fig. 1) is reached. During the rotating mode, points to be labeled could be either those already in existence (labeled) at the end of the previous moving mode, or those that enter the view during the course of the rotation. The mathematical formulation to optimize the total visible time is more complicated in comparison with the moving mode, as it involves nonlinear components. As a result, we apply a two-stage approach to yield solutions for the problem. We let θ_r be the angle that the map turns (assuming that the view is always aligned to the user's view) in order to enter the subsequent moving mode. We assume that all the points involved in the rotating mode are w.r.t. the coordinate system in which the origin $(0,0)$ is at the center of the view R, and the x and y axes parallel the horizontal and vertical edges of R, respectively, at the point right before the rotation begins (i.e., at the end of the previous moving mode). A point p_i on the map is also recorded as (r_i, θ_i) w.r.t. the polar coordinate system. In order to keep a leader a straight horizontal line segment, a label moves up or down (depending on whether the rotation is counter-clockwise or clockwise) along the side of the view R. That is, the anchor of a label moves (w.r.t. the above mentioned coordinate system) in the process of the rotation.

Consider two points p_i and p_j whose labels are separated initially. Suppose when the map rotates θ_{ij} degree clockwise, overlapping begins, and later when the rotating angle reaches θ'_{ij} the two labels separate again. In view of the above, $\theta'_{ij} - \theta_{ij}$ can be regarded as the overlapping region of angle between p_i and p_j, during which the two labels cannot co-exist. The first stage of our optimization procedure in the rotating mode is to find out the minimum total pairwise overlapping regions of angle. More precisely,

$$\min \quad \sum_{i=1}^{n} \sum_{j=1}^{n} (\theta'_{ij} - \theta_{ij}) \quad \ldots\ldots \quad \text{(Eq. 2)}$$

$$\begin{aligned}
&\textit{subject to}\\
&r_i \sin(\theta_i + \theta_{ij}) - r_j \sin(\theta_j + \theta_{ij}) = h'_{ij}\\
&r_i \sin(\theta_i + \theta'_{ij}) - r_j \sin(\theta_j + \theta'_{ij})\\
&\qquad = h'_{ij} - (h_i + h_j)\\
&h'_{ij} = (h_i - (y_i^* - r_i \sin(\theta_i + \theta_{ij})) +\\
&\qquad (y_j^* - r_j \sin(\theta_j + \theta_{ij}))\\
&r_i \sin(\theta_i + \theta_{ij}) \leqslant y_i^* \leqslant r_i \sin(\theta_i + \theta_{ij}) + h_i\\
&r_j \sin(\theta_j + \theta_{ij}) \leqslant y_j^* \leqslant r_j \sin(\theta_j + \theta_{ij}) + h_j\\
&0 \leqslant h'_{ij} \leqslant (h_i + h_j)\\
&0 \leqslant \theta_{ij} \leqslant \theta_r\\
&0 \leqslant \theta'_{ij} \leqslant \theta_r
\end{aligned}$$

In the above, (r_i, θ_i) (resp., (r_j, θ_j)) is the polar coordinate of point p_i (resp., p_j); y_i^* (resp., y_j^*), a variable, is the y-coordinate of the anchor position of p_i (resp., p_j) w.r.t. the viewing coordinate system (i.e., $(0,0)$ is at the center of R, and x and y axes parallel the horizontal and vertical sides of R, resp.); θ_{ij} and θ'_{ij} are

Table 1: Moving mode.

Moving	Dataset 1			Dataset 2		
	Naïve	Alg. 1	Alg. 2	Naïve	Alg. 1	Alg. 2
Total Visible Time	0.808	0.870	0.863	0.401	0.526	0.525
Running Time (sec)	0.192	0.373	0.196	3.327	3.493	3.354

Table 2: Rotating mode.

Rotating	Dataset 1			Dataset 2		
	Naïve	Alg. 3	Alg. 4	Naïve	Alg. 3	Alg. 4
Total Visible Time	0.58	0.755	0.788	0.495	0.541	0.546
Running Time (sec)	0.938	2141.26	1.513	25.91	39012.84	26.882

also variables. Basically, h'_{ij} is the difference between the two ports.

As the above objective function considers only pairwise overlapping region of angle locally, the solution may still result in overlapping labels globally. The second stage of the optimization is to apply a similar approach of using "timeline" in the moving mode. Each pair of points p_i and p_j have their θ_{ij} and θ'_{ij} computed in the previous stage. We mark all the θ_{ij} and θ'_{ij} along a timeline, which results in a finite number of intervals like in the moving mode. Hence, a formulation like Eq. 1 can be applied to solving the labeling problem. Such a two-stage algorithm is called *Algorithm 3*.

Even though Eq. 2 provides a good starting point for constructing the timeline graph, it suffers from a high time complexity. To ease such a time-consuming step, a heuristic algorithm similar to the greedy strategy used in Algorithm 2 is also proposed, in which angles (w.r.t. the coordinate system associated with the initial view) of all the points involved in the rotating process are sorted, and a greedy strategy similar to that in the moving case is applied to assigning anchor positions. Incorporating the greedy heuristic in the rotating mode is called *Algorithm 4*.

4 EXPERIMENTAL RESULTS

In this section, we present the evaluation of different algorithms based on the moving and rotating models which were introduced in Section 3. Our algorithm was implemented in Python 3.5.2 running on Linux Ubuntu 16.04. And Gurobi 8 was used for the solver. Algorithms 1-4 are compared against a simple algorithm in which ports of the labels are at their middle points. Such a labeling is called the *Naïve labeling* in our comparison. Our evaluation takes the *total visible time* of labeling and the algorithm's *running time* as the performance metrics. Because we decomposed the trajectory into moving and rotating modes, we will show these experiments individually. Two datasets from OpenStreetMap were used in our experiments. In our experiments, labels are assumed to be of uniform height. We first set the label height to 20, and views R as a 300×300 square. So, the maximum number of the labels that can be displayed simultaneously is 15. Dataset 1 is a map with less than 25 points, while Dataset 2 contains more than 700 points. The total visible time (in percentage) is measured against the case when overlapping-free is not required (i.e., all the labels are displayed even when they overlap with each others).

4.1 The Moving Case

Fig. 6 shows the labeling results of Dataset 1. Notice that the Naïve algorithm can only show six labels in view R (Fig. 6b), while both Algorithm 1 (Fig. 6c) and Algorithm 2 (Fig. 6d) can show seven labels in view R. Both Algorithms 1 and 2 slide the highest label to create space for other labels. However, Table 1 shows that Algorithm 1 can obtain higher total visible time. The reason is that Algorithm 1 is based on a precise MIP formulation, whereas Algorithm 2 uses a greedy heuristic to find appropriate (which may not be optimal globally) anchors first.

4.2 The Rotating Case

We evaluate the performance, in terms of the total visible time and the running time, of different algorithms in the rotating mode. Fig. 7 shows the visual results of Dataset 1. The number of displayed labels under the Naïve algorithm is less than that using Algorithm 3 and Algorithm 4. There are only six labels in Fig. 7b. Fig. 7c and 7d show seven labels in view R. The main difference between Algorithm 3 and Algorithm 4 is the running time. Because Algorithm 3's objective function contains trigonometric functions for each pair of points, it is too complicated to minimize it. Solving the trigonometric functions is a bottleneck for the solver, and it is easily beyond the tolerance of numerical errors. As we can see in Table 2, Algorithm 3 uses much time to solve the minimization problem. In addition, it could in some cases be stuck at a saddle point of the objective function, causing the total visible time to be even less than that of Algorithm 4. The dense point set of Dataset 2 in Table 2 suggests the same behavior as Dataset 1. As we can see, Algorithms 3 and 4 have a better performance than the Naïve algorithm. However, Algorithm 3 still suffers from a high running time. As Table 2 shows, using this heuristic strategy in Algorithm 4 can reduce the running time substantially.

(a) Starting configuration of Dataset 1

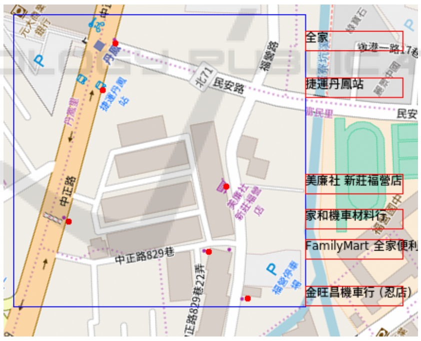
(b) Result of the Naïve algorithm

(c) Result of Algorithm 1

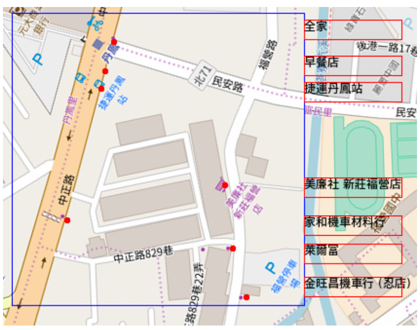
(d) Result of Algorithm 2

Figure 6: Experimental results of Dataset 1 in the moving mode.

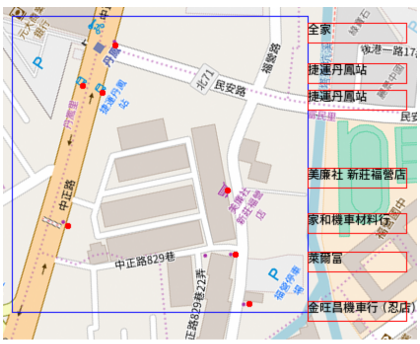
(a) Starting configuration of Dataset 1



(b) Result of the Naïve algorithm



(c) Result of Algorithm 3



(d) Result of Algorithm 4

Figure 7: Experimental results of Dataset 1 in the rotating mode.

4.3 Different Label Heights and Numbers of Points

We compare the Naïve algorithm with Algorithms 1-4 proposed in this work w.r.t. different label heights. Tables 3 and 4 show the performance w.r.t. label height ranging from 10 to 30. Height increasing results in performance degradation, suggesting that the number of labels which can be displayed simultaneously decreases due to the fact that labels tend to block more labels. As shown in the tables, Algorithms 1-4 are better than the Naïve algorithm in most cases.

Finally, we analyze the running time of Algorithms 1-4. We show Algorithms 1, 2 and 4 on different numbers of points ranging from 100 points to 500 points. Table 5 shows their running times. As Algorithm 3 needs to solve the trigonometric functions in the constraints of the objective function, the problem size that the algorithm is able to solve in a reasonable amount of time is rather limited. Table 6 shows the results of Algorithm 3 for the number of points ranging from 10 to 30. Due to its extremely high running time, Algorithm 3 is hard to be practical in real-world applications.

Table 3: Different label heights in the moving mode.

Moving	10	15	20	25	30
Naïve	0.626	0.626	0.502	0.439	0.436
Alg. 1	0.839	0.738	0.661	0.502	0.436
Alg. 2	0.837	0.711	0.661	0.502	0.436

Table 4: Different label heights in the rotating mode.

Rotating	10	15	20	25	30
Naïve	0.689	0.621	0.542	0.478	0.437
Alg. 3	0.721	0.626	0.558	0.536	0.474
Alg. 4	0.695	0.642	0.558	0.489	0.437

Table 5: Running times of Algorithms 1, 2 and 4 w.r.t. different numbers of points.

	100	200	300	400	500
Alg. 1	9.533	32.405	62.303	83.067	152.077
Alg. 2	7.237	29.127	56.037	76.406	127.237
Alg. 4	9.094	37.979	77.51	150.768	237.526

Table 6: Algorithm 3's running time w.r.t. different numbers of points.

	10	15	20	25
Running time (sec)	1522.768	6079.111	15148.168	30456.744

5 CONCLUSIONS

In this paper, we proposed various algorithms for annotating trajectory-based dynamic map in the framework of 1-sided boundary labeling. Future research directions include allowing more sophisticated operations, such as zooming, scaling, etc, to be performed during the course of the navigation, as well as relaxing the number of sides to which labels can be attached.

ACKNOWLEDGEMENTS

The second author was supported in part by National Science Council, Taiwan, ROC, under Grant MOST 109-2221-E-002-142-MY3.

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