Workforce Modelling with Experience Accumulation

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Abstract: The purpose of this work is to investigate the population dynamics of on-the-job training in a military application, a scenario where mentees enter the system to receive training under the supervision of mentors, before becoming mentors themselves. This work builds on the two-level model which only considers the total mentee (level 1) and mentor (level 2) populations. Accumulation of experience is used to denote training progression, improving the tracking of mentees through their training. Two new models are considered: (1) A multi-stage mentee training model, which sub-divides the training into stages the mentees must progress through, and (2) A transport model, a limit case as the number of stages is increased to infinity, resulting in a continuous training medium mentees progress through. These two models are investigated by solving their respective analytical models. To verify system behaviours, analytical solutions are compared to discrete-event simulations.

1 INTRODUCTION

For military occupations, an important aspect of professional development is on-the-job training. In the Canadian context, this takes the form of a systematic progression where personnel go through various phases: they enter the system as mentees (e.g., apprentices) and receive training under the supervision of mentors (e.g., journeymen or masters), before becoming mentors themselves and productive members of the workforce (Schaffel et al., 2021). To reduce personnel costs while ensuring a necessary workforce size and readiness, careful planning and modelling must be undertaken on these training systems.

Bastian and Hall review approaches used in military workforce planning and modelling (Bastian and Hall, 2020). Prime examples of population dynamics modelling include differential equations (Boileau, 2012; Vincent and Okazawa, 2019), Markov decision models (Zais and Zhang, 2015; Diener, 2018; Suvorova et al., 2019) and system dynamics (Forrester, 1965) which utilizes continuous stocks and flows as well as feedback loops to examine how organizational structure, policies and decisions interact. An example of this last method was implemented to model mentee-mentor dynamics for pilots (Séguin, 2015). Analysis of the composition of workforces as well near-term projections of occupational requirements also relied on statistical methods (Bryce and Henderson, 2020; Okazawa, 2020). Yet, the most common approach is based on using discrete-event simulations to track the progress of personnel through a military training system (Novak et al., 2015; Henderson and Bryce, 2019).

The military workforce population models usually capture only the upward rank flow due to promotion, not the experience accumulation due to the mentee-mentor dynamics. A notable exception is a system dynamics model which considers trainee pilots and experienced pilots who mentor the trainees (Séguin, 2015). In most of the research, the population variables are continuous in order to generate computationally-efficient models that are easier to use on large populations. These models are less accurate when studying occupations with small populations.

This work builds on the two-level mentor-mentee model derived from the ubiquitous "predator-prey" model (Swift, 2002), which considers the total mentee and mentor populations over time (Schaffel et al., 2021). However, in this mentor-mentee model, individual mentees cannot be distinguished from one another in the total pool of mentees and thus a mentee with minimal experience or time training as a mentee may upgrade at any time to a mentor. Therefore, some mentees may become mentors after completing only a small amount of on-the-job-training, which is not

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realistic. Furthermore, the training progression of individual mentees is unclear as they form an indistinguishable pool from one another. In a realistic scenario, mentees would accumulate experience through their training. Modelling this experience accumulation process allows for the tracking of the training of each mentee in the system. Therefore, we introduce a new experience accumulation mechanism to resolve the issues encountered with the previous two-level model.

In this paper, two new models are devised based on the two-level model by dividing the total mentee population into multiple stages to bin the training progression of mentees. First, we introduce a multi-stage mentee training model that bins the mentee pool into a generalized discrete number of stages, forming a system of multiple ordinary differential equations (ODEs). Second, we devise a "transport" model, which is the result of increasing the number of stages to infinity, forming a partial differential equation (PDE). To verify the system dynamics of these models, a discrete-event simulation (DES) of each model is compared to their corresponding analytical versions.

The remainder of this report is organized as follows. Section 2 gives an overview of the previously introduced two-level model (Schaffel et al., 2021). Next, Section 3 and Section 4 describe the multi-stage and transport models respectively. In both instances, a benchmark scenario is used to examine the analytical solutions and corresponding DES's are set up to verify the dynamics of each model. Section 5 compares the analytical solutions of the three models discussed. Finally, Section 6 concludes the paper.

2 TWO-LEVEL MODEL

In this section, the two-level mentor-mentee model (Schaffel et al., 2021) is re-introduced. This model considers the mentee and mentor populations to each be a single pool of the corresponding type of person. The previously described saturation effects (Schaffel et al., 2021) are not considered, and the training capacity is assumed to be infinite. The analytical equations for this model are described in Section 2.1. In Section 2.2, a DES of the model is set up. Finally in Section 2.3, the model is investigated with a benchmark scenario, which will be used throughout the paper.

2.1 Analytical Model

The two-level model for an infinite training capacity is defined as (Schaffel et al., 2021),:

$$\dot{x} = a - bx \tag{1}$$

$$\dot{y} = bx - cy \tag{2}$$

where x and y denote the mentee and mentor populations respectively, a is the intake rate of mentees into the system, b is the upgrade rate of mentees to mentors, and c is the attrition rate of mentors. A mentee will take an average of $1/b$ time units to upgrade to a mentor, and a mentor will stay in the system for an average of $1/c$ time units before leaving. The intake, upgrade, and attrition rates are assumed to be constant over time. For the model, the rate of change for the mentee population ($\dot{x}$) is equal to the intake rate of mentees (a) minus the upgrade rate of mentees to mentors (bx), while the rate of change of the mentor population ($\dot{y}$) is equal to the upgrade rate (bx) minus the attrition rate of mentors (cy). In this model, the fraction of the mentee and mentor populations chosen to be upgraded or attrited is assumed to be proportional to their respective sizes.

Solving the ODEs, the analytical solution is shown in Eqn. (3) and Eqn. (4). The mentee and mentor populations decay exponentially from their initial populations (x_0 and y_0) to the target populations (x_t and y_t) of $x_t = a/b$ and $y_t = a/c$ respectively as time (t) increases since the upgrade and attrition rates are positive and real values. The system decays to the target populations regardless of the size of the initial mentee and mentor populations.

$$x(t) = x_t + (x_0 - x_t)\, e^{-bt} \tag{3}$$

$$y(t) = y_t + \frac{b}{c-b}\,(x_0 - x_t)\, e^{-bt} \tag{4}$$

$$+ \left[y_0 - \frac{b}{c-b}\,(x_0 - y_t) \right] e^{-ct} \tag{5}$$

A drawback of this model is its inability to track the progress of every mentee in the system (Schaffel et al., 2021). The model only consists of two pools for the mentees and mentors respectively. Once a mentee is added into the mentee pool, it becomes indistinguishable from the other mentees in the system. Since mentees cannot be distinguished from one another and a constant proportion of the mentee pool is always selected to upgrade, a mentee can be expected to upgrade at any time. For example, a mentee could be selected to upgrade after being added very recently, staying in the mentee pool for a very small amount of time, or a mentee could remain in the mentee pool for a very large amount of time if it is never part of the proportion of the population selected to upgrade. Therefore, while the average training time for a mentee is $1/b$ time units, a mentee can take anywhere from zero to an infinite amount of time to upgrade, which is not realistic. Since a mentee may upgrade at any time, there is no clear training progression for the mentees to follow.

2.2 Discrete-Event Simulation

To verify the analytical equations, a DES of the two-level model was set up. A Discrete-Event Simulation permits the tracking of each mentee and mentor in the system allowing for control and observance over the simulation process and output. At each event within the simulation, defined processes may be triggered to occur for a mentee or mentor, such as an addition or attrition from the system, or an upgrade of a mentee to a mentor.

To set up the simulation, mentees and mentors were assigned a training time and a time to attrit, randomly given from exponential probability distributions with means of $1/b$ and $1/c$ respectively (Schaffel et al., 2021). A mentee would enter the system and remain as a mentee for their assigned training time. They would then upgrade to a mentor and further remain in the system for their assigned time until they leave via attrition.

Another consideration of implementing a DES is the handling of whole mentee and mentor populations instead of the continuously-variable populations present in the analytical model. This is particularly important when determining the intake procedure of mentees into the system. In the analytical model, the intake is a constant function over time (a). However, this is not possible in a DES since individual mentees (i.e., not fractions of a mentee) need to enter the system at discrete points in time. Therefore, to replicate the constant intake of the analytical model, the time interval between when mentees enter the system was randomly chosen, given by an exponential distribution with a mean of a (Schaffel et al., 2021). In the current implementation, a mentee may enter the system at any time while still maintaining the average intake rate of a mentees per time unit.

Of note, because all holding and inter-arrival times are exponentially distributed, this simulation models a continuous-time Markov chain.

2.3 Results for a Benchmark Scenario

In the chosen benchmark scenario, the mentee and mentor populations undergo a growth where their populations are doubled. Transitioning from an initial state (x_0, y_0) to a target steady state (x_t, y_t), the input parameters are fixed to the following values:

$$
\begin{aligned}
(x_0, y_0) &= (25, 100) \\
(x_t, y_t) &= (50, 200) \\
c &= 0.05
\end{aligned}
$$

The intake (a), upgrade (b) and attrition (c) rates are assumed to be fixed and not vary over time. Therefore, a and b are calculated to reach the target populations:

$$
\begin{aligned}
a &= cy_t = 10 \\
b &= \frac{a}{x_t} = 0.2
\end{aligned}
$$

From the defined values, the system will transition from initial mentee and mentor populations of 25 and 100 respectively, to target populations of 50 and 200. To reach this target, 10 mentees must enter the system on average each time unit, and each mentee will require an average of 5 time units of training before upgrading to a mentor. Finally, a mentor will stay in the system for an average of 20 time units before exiting the system.

Fig. 1 shows the statistics for the mentee (blue, bottom curve) and mentor (green, top curve) populations from the DES model for 1000 experiments. The mean of the two populations is given by the dashed line, and the shaded region gives two standard deviations of the results. The analytical solution is also plotted with the benchmark scenario. The two populations begin at their initial values of 25 and 100 respectively and decay exponentially to their final values of 50 and 200. The analytical solution and the simulated mean of the two populations lie on top of each other for all time and are within two standard deviations of the simulated results. The standard error near steady state was found to be around 0.225 and 0.45 for the mentee and mentor populations respectively. Since the discrete model is a Poisson point process, the expected standard errors are $\sqrt{c}$ and $\sqrt{b}$ for the mentee and mentor populations, which are roughly equal to the values observed. Given this, we are confident the discrete model is set up correctly and is producing accurate results.

3 MULTI-STAGE MENTEE TRAINING MODEL

In Section 2.1, it was noted that there is a lack of training progression in the two-level model since the mentees form an indistinguishable pool and may upgrade at any time. In an attempt to resolve this issue,

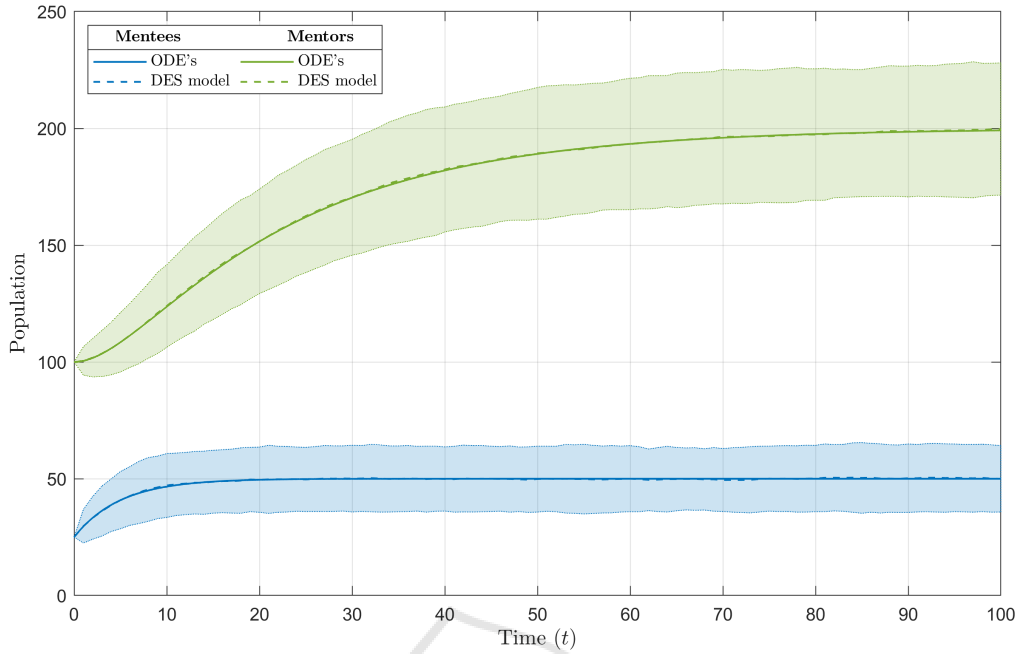


Figure 1: Analytical and DES implementations of the two-level model over time with the benchmark scenario.

the model was modified to split the mentee progression into multiple stages, which a mentee must pass through to upgrade to a mentor. In this section, the multi-stage mentee training model is introduced and discussed. In Section 3.1, the analytical equations for the multi-stage model are introduced, and in Section 3.2, a DES implementation of the model is set up. The results with the benchmark scenario are then investigated in Section 3.3 for two scenarios: A five-stage system and a 25-stage system.

3.1 Analytical Model

The system of equations for the multi-stage mentee training model is shown as follows:

$$\begin{aligned} \dot{x_1} &= a - bnx_1 \\ \dot{x_2} &= bn(x_1 - x_2) \\ &\vdots \\ \dot{x_n} &= bn(x_{n-1} - x_n) \\ \dot{y} &= bnx_n - cy \end{aligned} \tag{6}$$

where n denotes the number of mentee stages before upgrading to a mentor; and $x_1, x_2, \cdots, x_n$ denote the mentee populations at each training stage over time. In practical terms, the number of stages is generally determined experimentally with a higher number allowing us to model mentee training stages with increasing granularity. The upgrade rate for each mentee stage is proportional to the number of stages in the system (nb) so that the total training time through all the stages averages to $1/b$. Like for the two-level model, the fraction of the mentee population chosen to upgrade from each stage is assumed to be proportional to its total size. The mentor equation remains the same as for the two-level model where its rate of change is given by the difference in the mentors attrited and the mentees upgraded from the final stage. Finally, we will assume that the initial mentee populations are evenly distributed between the stages. Therefore, each stage will have an initial population of x_0/n.

The solution of the multi-stage analytical model is as follows:

$$\mathbf{x}(t) = e^{-nbt} \sum_{j=1}^{n} \mathbf{c_{1j}} \sum_{i=1}^{j} \frac{t^{j-i}}{(j-i)!} \mathbf{v_{1i}} + e^{-ct} \mathbf{c_2} \mathbf{v_2} + \mathbf{x_p} \tag{7}$$

The solution, $\mathbf{x}(t)$, is a column vector containing the populations over time of the individual mentee stages and the mentor population:

$$\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \\ y(t) \end{pmatrix} \tag{8}$$

The solution contains one degenerate eigenvalue, $-nb$, with a multiplicity of n, a second distinct eigenvalue, $-c$, and a constant particular solution, $\mathbf{x_p}$. The particular solution is the following column vector denoting the steady state values of each population:

$$\mathbf{x_p} = \begin{pmatrix} \frac{a}{nb} \\ \vdots \\ \frac{a}{nb} \\ \frac{a}{c} \end{pmatrix} \tag{9}$$

For the generalized eigenvectors of the degenerate eigenvalue, $\mathbf{v_{1i}}$, of length $n+1$, its elements $(v_{1i,1}, v_{1i,2}, \cdots, v_{1i,n+1})$ are all zero except for the following:

$$v_{1i,n-i+1} = \frac{c-nb}{(nb)^i}$$
$$v_{1i,n+1} = \frac{1}{(nb-c)^{i-1}},$$

The eigenvector corresponding to the distinct eigenvalue of length $n+1$ (the mentor solution) has the following form:

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \tag{10}$$

Finally, the constants in the solution have the following values:

$$\mathbf{c_{1j}} = \left(\frac{x_0 - x_t}{n}\right) \frac{(nb)^j}{c-nb} \tag{11}$$

$$\mathbf{c_2} = y_0 - y_t - \sum_{j=1}^{n} \frac{\mathbf{c_{1j}}}{(nb-c)^{j-1}} \tag{12}$$

3.2 Discrete-Event Simulation

To set up a DES implementation of the multi-stage mentee training model, the two-level discrete model from Section 2.2 was modified so that the training for a mentee was repeated n times to replicate the progression through each stage. For each repetition, a mentee was assigned a training time given randomly by an exponential probability distribution with a modified mean of $1/nb$. A mentee would enter the system and remain in the first stage for their first assigned training period before upgrading to the second stage. This process would repeat n times, simulating a progression through each stage, until the mentee upgraded to a mentor. Even with this modification, the overall simulation remains markovian.

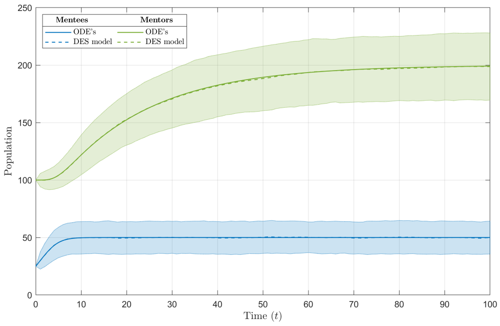
One consideration faced with implementing a DES was the handling of whole mentee populations to determine the initial number of mentees at each stage. For example, in a system with 10 mentee stages and 25 initial mentees, each stage would need to be populated with 2.5 mentees to have an even distribution. This is possible in the analytical model as the mentee and mentor populations are continuous; however, this cannot be done in a discrete implementation. To solve this issue, the number of initial mentees at each stage was randomly drawn from a discrete uniform distribution with values between 1 to n. Therefore, given enough experiments, the average distribution of initial mentees should be even across all of the stages.

3.3 Results for the Benchmark Scenario

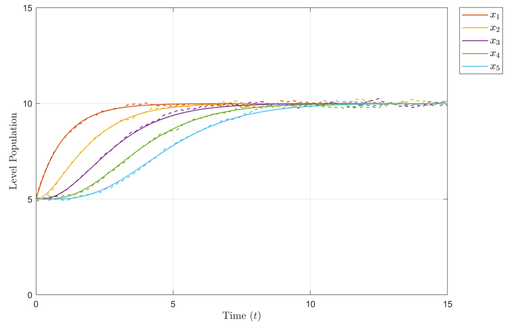
Fig. 2 shows the statistics for systems with 5 and 25 stages over time with the benchmark scenario. In both cases, their respective DES simulations were run for 1000 experiments. Fig. 2a and Fig. 2c show their total mentee and mentor populations over time. The mean of the two populations is given by the dashed line, and the shaded region gives two standard deviations of the results. Fig. 2b and Fig. 2d show the population of each stage over time with the dashed line indicating the mean. For all plots, their respective analytical solutions are also plotted as solid lines in the same colour.

From Fig. 2a, the mentee and mentor populations are set initially at 25 and 100 respectively, and decay exponentially to their target populations of 50 and 200. Compared to the two-level solution given in Fig. 1, the mentee population for five training stages decays quicker to steady state. Furthermore, the lower number of mentees upgraded each time causes the mentor population to spend more time near its initial population before beginning its exponential decay. From Fig. 2b, each stage begins at its initial population of five mentees, then decays exponentially to the same steady state populations of 10. It was observed that each stage acts on the behaviour of the previous stage: Stage 1 decays first, then Stage 2, up to Stage 5 which decays to steady state last. This was interpreted as a progression of mentees through the training stages: Mentees entering the system enter at stage 1 first, undergo training then upgrade to stage 2, which gives an increase in the population of the second stage. These mentees undergo training and upgrade to stage 3, giving a population increase in stage 3. This process continues up the stages and into the mentor population as mentees progress through the system.

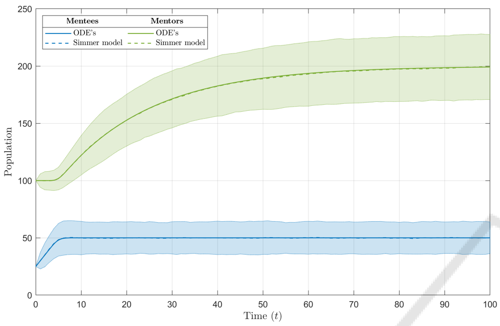
From Fig. 2d, each training stage begins at its initial population of one mentee and decays exponentially to the same steady state populations of two mentees. Similar to a system with five stages, given in Fig. 2b, each stage begins its decay in subsequent order. However, the solutions decay to steady state at a quicker rate and are much more bunched up in



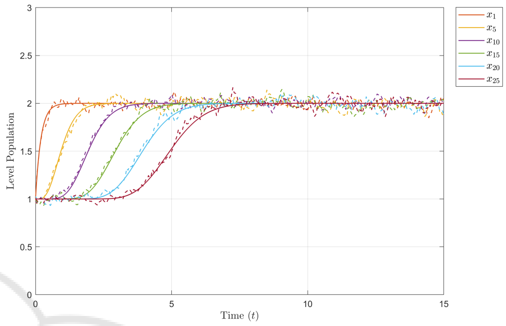
(a) Total mentee and mentor populations for five stages.



(b) Individual training stage populations for five stages.



(c) Total mentee and mentor populations for 25 stages.



(d) Individual training stage populations for selected stages for 25 stages.

Figure 2: Analytical and DES implementations of the multi-stage model over time with the benchmark scenario. Total mentee and mentor populations for (a) five stages; and (c) for 25 stages; and corresponding individual training stage populations for (b) five stages; and (d) a sub-set of 25 stages.

time. This translates to the total populations given in Fig. 2c, in which the decay to steady state for the total mentee population is also increased. The solutions are more bunched up since the average time to steady state for the total mentee population must still be $1/b$ time units.

For all plots, the analytical solutions and the simulated means of the populations lie on top of each other and are within two standard deviations of the simulated results. The standard error for the total populations near steady state was also identical to the values from the two-level model (0.225 and 0.45 for the mentee and mentor populations respectively). The standard error for the individual stages near steady state was found to be around 0.1 and 0.045 for the five- and 25- stage models respectively. These are commensurate to the standard error of the total population. Given this, we are confident the discrete models are set up correctly and are producing accurate results. Finally, it was noted that the population mean of the individual mentee stages became noisier as the number of stages is increased. This is believed to result from an amplification of the discrete effects of the model since the population size for the individual stages decreases as more stages are added.

Overall, an improvement was observed over the two-level model, in which there is now some form of progression as mentees must pass through several trackable stages. However, a similar drawback as the two-level model is still present: The same proportion of each stage is always selected to upgrade (nb), so a mentee could be selected to upgrade to the next stage at any time.

4 LIMIT CASE: TRANSPORT MODEL

From Section 3.3, it was noted that a multi-stage model showed some form of workforce progression as mentees must pass through several stages of training before upgrading to mentors. However, each stage suffered from the same issues as the two-level model, as the mentees in each stage still could not be tracked and may upgrade at any time. Therefore, in an attempt to obtain the smallest interval of training which could be tracked, the number of stages was increased to infinity, yielding a transport model. In this section, this transport model is investigated and discussed. In

Section 4.1, the analytical equations for the transport model are introduced, and in Section 4.2, a discrete implementation of the model is set up. In Section 4.3, the model is investigated with the benchmark scenario.

4.1 Analytical Model

Two key issues arose when deriving the transport model: (a) For an infinite number of stages, each stage must contain zero mentees on average to avoid a population of infinity, and (b) Each mentee requires an average upgrade time of zero time units through each stage to avoid having an infinite training time. To solve these issues, the stages were redefined a continuous medium (l) and assigned an arbitrary total length of one. For the new training regime, a mentee enters the system at $l = 0$ and begins flowing through the medium undergoing training. Once a mentee reaches $l = 1$, they upgrade to a mentor. The variable of interest was also changed to the density of mentees throughout the medium (u), which is more appropriate for a continuous medium. The mentor equation was modified to accommodate the use of density (u) instead of a population size (x): the rate of change of the mentor population is given by the difference in the mentors' attrition and the mentees exiting the medium at $l = 1$.

The system of equations for the transport model is defined as:

$$\partial_t u(l,t) = -b\partial_l u(l,t) \qquad (13)$$

$$\dot{y} = bu(1,t) - cy. \qquad (14)$$

The PDE for the mentee population was derived from the multi-stage model, Eqn. (6).

For the initial condition of the PDE (u_i), it was assumed that the initial mentee density was evenly distributed throughout the medium. For the boundary condition (u_f), the density is fixed to the ratio between the intake rate (a) and the progression rate through the medium (b) as given by:

$$u_f = \frac{a}{b}. \qquad (15)$$

The boundary condition is assumed to turn on after $t = 0$, preventing a conflict with the initial condition.

From (Novozhilov, 2020; Ko, 2020), the solution of the transport PDE is given by the piecewise function:

$$u(l,t) = \begin{cases} u_i & \text{if } t \leq \frac{l}{b} \\ u_f & \text{if } t > \frac{l}{b}. \end{cases} \qquad (16)$$

The density across the medium begins at the initial density (u_i) and jumps up to the final density (u_f) at the contour: $l = bt$.

To determine the analytical solutions for the total mentee and mentor populations, the mentee solution given in Eqn. (16) was rewritten using a Heaviside step function:

$$u(l,t) = u_i + (u_f - u_i)H\left(t - \frac{l}{b}\right), \qquad (17)$$

where H denotes the left-continuous Heaviside step function, $H(0) = 0$. Using this representation, the total mentee population was found by integrating the mentee density across the medium, given by: Eqn. (18).

$$x(t) = u_i - b(u_f - u_i)\left[R\left(t - \frac{l}{b}\right) - t\right], \qquad (18)$$

where R denotes the ramp function. The solution for the mentor population was found by substituting Eqn. (17) into Eqn. (14) and solving the resulting ODE. The equation for the mentor population is thus given by:

$$y(t) = y_i e^{-ct} + \frac{bu_i}{c}\left(1 - e^{-ct}\right) - \frac{b}{c}e^{-ct}\left(u_f - u_i\right)\left(e^{\frac{c}{b}} - e^{ct}\right)H\left(t - \frac{1}{b}\right). \qquad (19)$$

4.2 Discrete-Event Simulation

To set up a DES implementation of the transport model, the two-level discrete model from Section 2.2 was modified so that the training time for the mentees is constant at $1/b$ instead of being drawn from an exponential distribution. This feature makes the simulation non-markovian. To mimic an even distribution of initial mentees throughout the medium (l), the initial mentees were assigned a uniformly-distributed training time between zero and $1/b$. This method replicates the analytical system since all mentees progress through the medium at a constant rate (b); therefore, the initial position of a mentee along the medium was inversely proportional to their training time.

4.3 Results for the Benchmark Scenario

To utilize the benchmark scenario with the transport model, the initial and boundary conditions were fixed to the following values:

$$u_i = 25$$

$$u_f = 50.$$

With these definitions, the total mentee population, found by integrating the mentee density over the medium, still has an initial population of 25 mentees and a final population of 50.

For the DES, the density of mentees must be calculated from the mentees present in the system over time and their respective progression through their training. Since mentees progress through the medium at a constant rate, the location of a mentee in the medium at a point in time can be inferred from their training time (initial mentees) or arrival time into the system (added mentees). To generate a surface plot of mentee densities, mentees were binned together in time and medium intervals. The bin in time was chosen to be an interval of $1/12$ time units, potentially simulating a progression through the medium once every month. The bin in the medium was chosen to be $1/60$, matching the time interval so a mentee would progress forward by one bin in the medium every time interval.

4.3.1 Surface Solution of the Mentee Population Across the Medium

Fig. 3 shows the surface plot of densities for the analytical PDE and DES with the benchmark scenario. The simulation was run for 1000 experiments. From the analytical solution, Fig. 3a and Fig. 3b, the density of mentees through the medium jumps from 25 to 50 after the contour $l = bt$. After 5 $(1/b)$ time units, the entire medium is populated at the final density.

From (Novozhilov, 2020; Ko, 2020), the transport PDE models a constant progression through the medium at a rate of b. This gives straight-line contours with a slope of b, as seen from the top-down view in Fig. 3b. Therefore, in the transport model each mentee has the same training time of $1/b$. This is different from the two-level and multi-stage models given in Section 2 and Section 3, in which the training time for mentees was exponentially distributed. This constant training time between the mentees arises because as the number of stages is increased to infinity $(n \to \infty)$, the average time a mentee spends in each stage approaches zero. For an average time of zero in each stage, there must be no variance between the training times for each mentee in the system, giving the same training time for each. Finally, the location of a mentee along the medium denotes how far along they have progressed with their training. This improves the tracking of mentees through their training as now a continuous medium is used to denote their progression.

For the simulation output, Fig. 3c and Fig. 3d, the initial density of mentees across the medium is even around 25, and the final density is even around 50. The surface plot also demonstrates nearly identical behaviour to the analytical solution, in which the spike in density up to 50 occurs along the contour: $l = bt$. From the top-down view, mentees progress through the medium at a constant rate of b, like in the analytical model. Given the similarities observed between the analytical and discrete outputs, it is believed that the two versions of the transport model are equivalent.

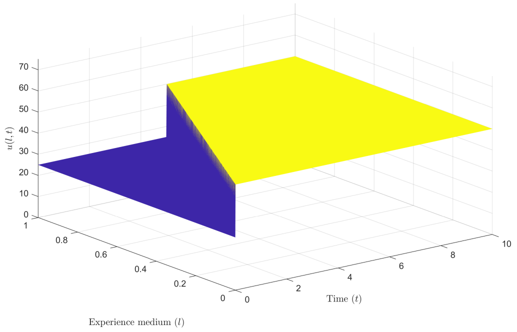
4.3.2 Solution of the Total Mentee and Mentor Populations

The solutions for the total mentee and mentor populations for the benchmark scenario are plotted in Fig. 4. The analytical solutions given by Eqn. (18) and Eqn. (19) are plotted as solid lines. The statistics of the total populations from the DES model are also plotted. The mean of the two populations is given by the dashed line, and the shaded region gives two standard deviations of the results. The mentee population linearly increases from the initial population of 25 mentees to the final population of 50. After 5 $(1/b)$ time units, all initial mentees have upgraded to mentors and the mentees now populate the system at the final density. Therefore, the mentee population reaches steady state at this time. The mentor population still undergoes its exponential decay at a rate of c. However, the decay only begins once the mentee population reaches steady state and the incoming mentors are at the final density. The analytical solution and the mean of the simulation results of the two populations lie on top of each other, are within two standard deviations of the simulated results, and their standard errors were found to be identical to the values from the previous models (0.225 and 0.45 for the mentee and mentor populations respectively). This gives further confirmation that the two versions of the transport model are equivalent and the discrete model was set up correctly.

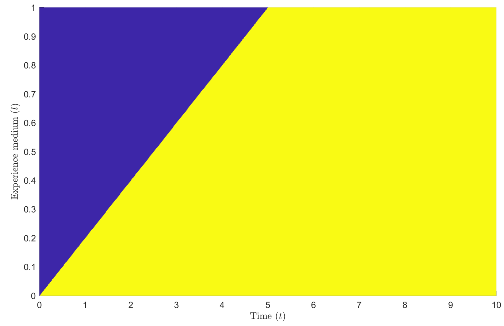
5 COMPARISON BETWEEN THE THREE MODELS

Fig. 5 shows the analytical solutions of the two-level (one mentee stage), five- and 25- mentee stage, and transport models plotted together with the benchmark scenario for comparison.

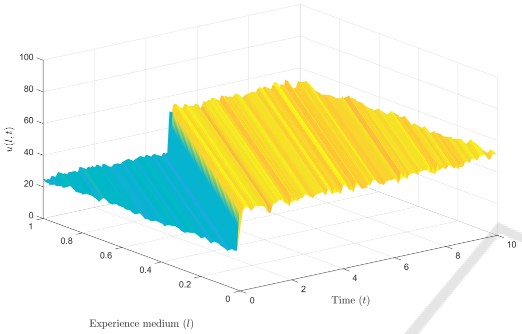
From Fig. 5a, as the number of mentee stages is increased, the decay rate of the final population increases. The transport model, shown in blue, is the limit case, in which there is an infinite decay rate and simply reaches the final population at 5 $(1/b)$ time units. From Fig. 5b, the exponential rise into the decay to steady state for the mentor population also becomes sharper as the number of stages is increased. Due to the varying sharpness in the rise, all solutions



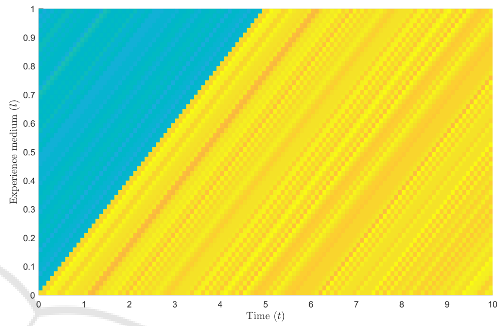
(a) 3-D view, analytical PDE.



(b) Top-down view, analytical PDE.



(c) 3-D view, discrete simulation.



(d) Top-down view, discrete simulation.

Figure 3: Surface plot of analytical and DES densities of the transport model.

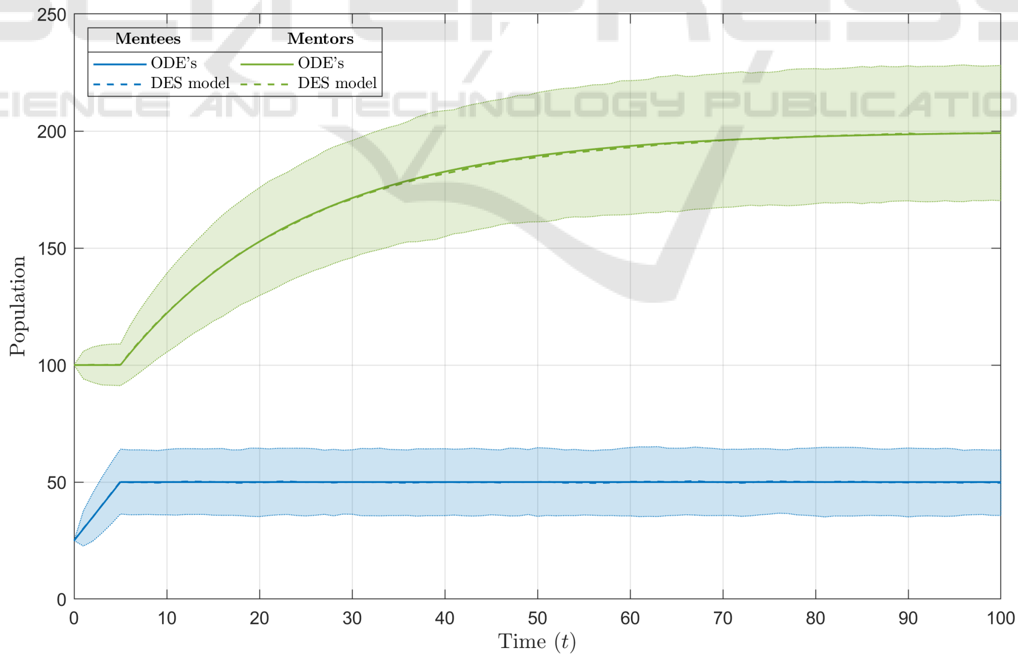
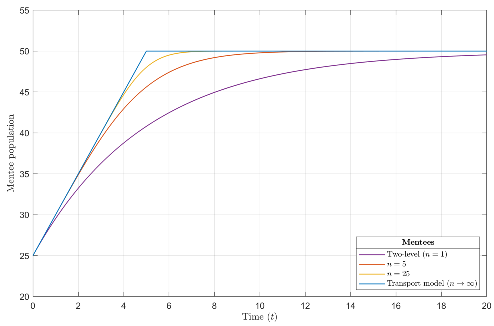
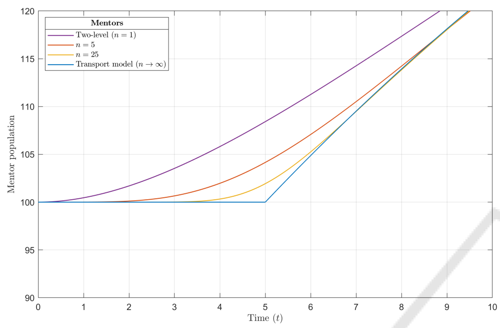


Figure 4: Mentee and mentor populations over time for analytical and DES implementations of the transport model with the benchmark scenario.

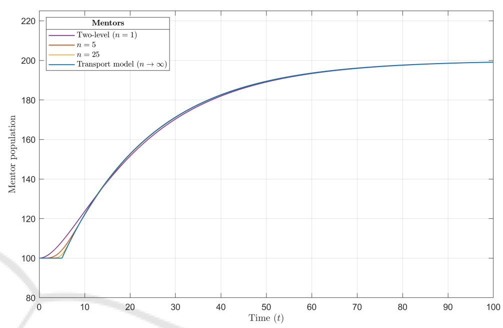
cross over each other as they decay to steady state at the final population of 200 mentors. From Fig. 5b at 20 time units, the two-level model, shown in purple, has the lowest mentor population and is the furthest from the steady state value, while the transport model has the highest population and is the closest to the



(a) Mentee population.



(b) Mentor population, zoomed in.



(c) Mentor population, full decay.

Figure 5: Comparison of the analytical solutions of the two-level (one mentee stage), multi-stage (five and 25 mentee stages), and transport models.

steady state value. Therefore, while all solutions have the same decay rate (c) and a time to steady state of infinity, if a threshold value is used to denote the system is at steady state, the transport model will always have the quickest time to steady state, and the two-level model will always be the slowest.

6 CONCLUSION

In this paper, the workforce population dynamics of on-the-job training was investigated, building upon a two-level model of the system given in (Schaffel et al., 2021). In an effort to obtain a more realistic model, two types of experiments were investigated: A multi-stage model and a transport model. The analytical solutions of these three models were investigated using a benchmark scenario in which the mentee and mentor populations were doubled. The analytical solutions and the model behaviours were verified using a DES.

A drawback of the two-level model is that the mentees are grouped in a pool and are indistinguishable from one another. A mentee could be expected to upgrade to a mentor at any time, which is not realistic and resulted in a lack of clear training progression for each mentee. For the multi-stage model, some training progression was observed as the population of the individual stages changed over time. However, each stages still suffered from the same issues as the two-level model. The transport model was the limit case of the multi-stage model as the number of stages was increased to infinity, in an attempt to track the smallest portion of the mentee population which was possible. In this system, all mentees had a constant training rate. The tracking of mentees through their training was greatly improved since their progression was now represented by a continuous medium.

The transport model is essentially a pair of queues in parallel where: (1) the mentee queue has a constant service time to simulate a constant training rate, and (2) the mentor queue has an exponentially distributed service time to model attrition. This solves several issues contained in the other two models: the mentees in the transport model are much easier to track since they will all progress through the medium at the same rate, and only depart the mentee population once the full medium is crossed, leading to a constant training time. However, the equal training time and progression for all mentees in the system may not be realistic. The mentor population is also still represented by the same equation as the two-level model, which may not be an accurate representation of a realistic scenario when combined with the PDE for the mentee population. Despite these issues, the formulation of a transport model for the training scenario provides a more flexible and less complicated means of modelling the progression of mentees to mentors than the multi-stage model. This may aid in building up and investigating more realistic workforce system implementations in the future. It is believed that a transport model could be used as a limit case for more complicated models of population dynamics allowing for a quick verification of their dynamic behaviour as well as quick-turnaround tools to help plan workforce transitions.

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