Nearest Neighbours and XAI Based Approach for Soft Labelling

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Abstract: Hard label assignment is a challenging task in case of epistemic uncertainty. This work initially converts the hard labels of evidential instances into probabilistic labels based on *k*-nearest neighbours. Neighbours are identified in a way that at least half of them must belong to the hard label of the corresponding evidential instance. The probabilistic label of a decision query is computed by combining the probabilistic labels of the nearest neighbours using Dempster's combination rule. Synthetic data is considered to verify the probabilistic labels over hard labels by varying the number of samples, number of neighbours and the overlapping degree between the classes. It is observed that the performance of the method mainly depends on the overlapping degree between classes. Probabilistic labels are intuitive compared to hard labels in case of high overlapping region. Moreover, few publicly available datasets are also considered to verify the performance of probabilistic labels on boundary instances. The proposed method achieves an accuracy of 90.44% and 98.24% on breast dataset trained with 10% and 90% of data respectively. Therefore the proposed method is sample efficient, calibrated, and interpretable.

1 INTRODUCTION

Machine learning (ML) algorithms learn patterns from training data for assigning labels to prediction queries. However, availability of less number of evidential instances (training samples), and uncertainty in their decision alternatives (classes) may reduce the performance of the ML algorithms. If the ML model is not interpretable, then it may not be able to provide explanations for decision alternatives. These two characteristics of Explainable Artificial Intelligence (XAI) – performance and explainability – motivates to find a learning methodology which is sample efficient and interpretable to provide explainable outcomes.

A model is said to be accurate when it is able to differentiate among disjoint classes. Most of the learning paradigms in literature may not be able to produce accurate models because of uncertainty in evidential instances (Kavya and Christopher, 2022). If the learning model is not accurate, then assigning a hard label to unknown decision query is quite difficult and challenging task.

Hard label, also known as crisp label, can be encoded as one-hot vector; this indicates that the instance strictly belongs to a particular class. But the strict encoding is difficult when there is uncertainty in the instances. The difficulty with this kind of hard labelling can be handled in two ways: first is to gather additional instances which supports learning algorithms in differentiating disjoint classes for generating models, and second is to convert the hard labels to probabilistic labels. In domains like healthcare, gathering additional information which is having less uncertainty may not be possible all the time. A probabilistic label or soft label can be encoded as a belief structure in which each disjoint class is assigned with a membership degree. Hard label is a special case of probabilistic label when the membership degrees of all the classes other than one particular class are equal to zero. Probabilistic labelling is useful in cases where the overlapping region among disjoint classes is high. Most of the existing datasets for developing an accurate classification model are hard labelled irrespective of the uncertainty in evidential data.

The existing learning algorithms like *k*-nearest neighbours, support vector machine, decision trees, neural networks, and ensemble approaches which are trained on the evidential samples of hard labels are assigning hard labels to decision queries though these models are not 100% confident. In such less confident scenarios, these learning algorithms would have followed probabilistic labelling, but none of them does

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because they do not know the conversion of hard label to probabilistic label. Therefore, this work mainly focuses on for generating accurate models when the evidential data is limited and uncertain.

The proposed probabilistic labelling method initially converts the hard labelled evidential data into probabilistic labelled evidential data by identifying neighbours. The proposed method imposes a condition while identifying neighbours of an evidential instance; condition is that at least half of the neighbours must belong to the hard label of the corresponding evidential instance. This condition assures that the membership degree corresponds to the hard label of an evidential instance will never be less that 0.5. When a new decision query arrives, the proposed method identifies the nearest evidential instances, and combines their probabilistic labels using Dempster's combination rule. This rule results in a belief structure and it is considered as the probabilistic label of that decision query. The main contributions of this work are as follows:

1. A relabelling method is proposed based on the k-nearest neighbours algorithm for converting hard labels of evidential instances to probabilistic labels.
2. A decision-making model is proposed based on the k-nearest neighbours algorithm and Dempster's combination rule for assigning probabilistic labels to decision queries.

2 FOUNDATIONS AND RELATED WORKS

The objective of learning algorithms for solving classification problems is to find a function, $f : X \rightarrow Y$, where $X \in \mathcal{R}^n$ and $Y \in \{c_1, c_2, ..., c_m\}$, which minimises the error between original label and predicted label. Dataset (D) consists of samples where each sample $(x \in D)$ is a combination of n attribute-value pairs and a original label $(c_i \in Y)$. P number of instances in D are considered to train the classification model, and the remaining instances in D are considered to test the performance of the developed classification model.

2.1 Probabilistic Labels

In general, original label c_i of an instance $x \in D$ can be encoded as a vector of length m where i^{th} position is equal to one and all the remaining positions are equal to zero. For example, if the original label of x is encoded as $f(x) = [0,0,1,0,0]$, then it can be interpreted as x belongs to c_3 (Vega et al., 2021). Assigning entire probability to one single class is undesirable especially in healthcare domain where the information contains uncertainty.

In contrast, probabilistic labels share the probability among disjoint classes. A probabilistic label can be encoded as a vector of probabilities where each probability is the membership degree of the corresponding class. For example, if the probabilistic label of x is encoded as $f(x) = [0.2, 0.3, 0.1, 0.2, 0.2]$, then it can be interpreted as the decision-making model is 30% confident that x belongs to c_2. This vector serves as a knowledge to decision-maker for choosing an optimal decision.

In literature, there are two different types of probabilistic labels, namely, instance-wise and group-wise. Former is having independent probabilistic label for each instance, whereas, latter is having one common probabilistic label for group of instances. This work considers instance-wise probabilistic labels which means that that each instance is having a label of probability values corresponding to decision alternatives.

2.2 *K*-nearest Neighbours

Let q be the decision query to which a label (l) needs to be assigned by k-Nearest Neighbours (k-NN) classification model (M) (Christopher, 2019). M applies distance measures like Euclidean, Manhattan, and others to identify the set of nearest evidential instances, $NN = x_1, x_2, ..., x_k \in D$ for q. The labels of the evidential instances in NN are considered for assigning l to q. The choice of the distance measure, optimal number of neighbours, and the strategy for finding nearest neighbours are the three main factors which can impact the performance of k-NN.

2.3 Dempster Shafer Theory

Dempster Shafer theory considers the powerset of decision alternatives to represent uncertainty, and a combination operation which satisfies both associative and commutative properties to combine belief structures (Dempster, 2008). Let Θ be the Frame of Discernment (FoD) which consists of a finite non-empty set of mutually exclusive or disjoint class labels in a dataset, D. The powerset of Θ is represented as

$$\wp(\Theta) = \{\{\emptyset\}, \{\theta_1\}, \{\theta_2\}, ..., \{\theta_1, \theta_2\}, ..., \Theta\}$$

Let $y(x_i)$ and $y(x_j)$ are the probabilistic labels associated with x_i and x_j instances. Dempster's combination

rule is applied to combine x_i and x_j as

$$r_{x(2),\theta} = \begin{cases} 0 & \text{If } \theta = \emptyset \\ \frac{1}{1-k} \sum\limits_{A \cap B = \theta} y(x_i, A) y(x_j, B) & \text{If } \theta \in \wp(\Theta) \end{cases} \quad (1)$$

$$\text{Normalisation factor, } k = \sum_{A \cap B = \emptyset} y(x_i, A) y(x_j, B)$$

where, $r_{x(n),\theta}$ is the membership degree for θ in the resultant probabilistic label obtained by combining the probabilistic labels of n instances.

2.4 Related Works

Membership degrees in a probabilistic label can be either subjective or objective. Nguyen et al. follows subjective approach, and got membership degrees directly from domain experts (Nguyen et al., 2011). Classification model trained on these subjective probabilistic labels are more accurate compared to the models trained on hard labels. However, it may not be possible for human experts to provide reliable membership degrees all the time.

Szegedy et al. follows objective approach, and used a smoothing parameter to compute membership degrees in probabilistic labels (Szegedy et al., 2016). For example, if the parameter value is 0.1 then the hard label, $[0,1]$ is converted as probabilistic label ($[0.1, 0.9]$). Accuracy of the ImageNet dataset is increased by 2% when hard labels are converted into probabilistic labels using smoothing parameter. However, Norouzi et al. argued that all the disjoint classes should not have equal membership degrees (Norouzi et al., 2016). For example, if there were 10 disjoint class, and the parameter value is equal to 0.1, then this leads to have 0.1 as the membership degree to all the classes. Arbitrary way of penalizing hard labels is not a good approach in case of more number of classes.

Hinton et al. used model distillation to assign probabilistic labels (Hinton et al., 2015). Initially, a complex model is trained to output a real-valued vector in which k^{th} value can be considered as the membership degree of k^{th} class. Then a model that is more simple is trained on the output of the complex model for prediction. Though model distillation is an effective approach, it requires huge chunks of data for training complex models.

Gayar et al. proposes a novel method for generating soft labels based on fuzzy-clustering (Gayar et al., 2006). Five publicly available datasets are considered to compare the performance of k-nearest neighbours algorithm when it is trained on hard labels and soft labels. Experimental results prove that learning soft labels is more robust compared to learning hard labels.

Vega et al. proposed an approach to use probabilistic labels for training an accurate and calibrated deep networks (Vega et al., 2021). Three classification tasks, namely, diagnosis of hip dysplasia, glaucoma, and fatty liver are considered; results prove that training with probabilistic labels increases accuracy up to 22%.

Based on the knowledge attained from aforementioned literature, it can be understood that training the models using probabilistic labels instead of hard labels, supports the development of accurate classification models. However, there are two issues that need to be be resolved: first, there is no appropriate relabelling method for converting hard labels to probabilistic labels; second, ignoring original labels may lead to unreliable probabilistic labels. This work proposes a relabelling approach which converts hard labels to probabilistic labels without ignoring the original labels.

3 PROPOSED METHOD

Framework of the proposed probabilistic labelling method based on k-nearest neighbours (k-NN) and Dempster Shafer (DS) theory consists of two phases, namely, relabelling and decision-making.

3.1 Relabelling Phase

Relabelling phase focuses on converting hard labels of evidential data into probabilistic labels by identifying nearest neighbours. Evidential data consists of instances where each instance is a combination of attribute-value pairs and a hard label. Consider an instance (x) from evidential data which needs to be relabelled. The distances from x to remaining evidential instances are computed based on Euclidean measure to identify k nearest neighbours of x. In classical k-NN algorithm, k neighbours are the instances with minimal distance from x, and can belong to any class in the evidential data. Whereas in the proposed system, at least $k/2$ neighbours are the instances with minimal distance from x, and must belong to the same class of x. The remaining $k/2$ neighbours are the instances with minimal distance from x, and can belong to any class including the class of x.

The rationale behind dividing the k neighbours into two disjoint sets is to not ignore the hard labels of evidential instances. Once the k neighbouring evidential instances are identified, the membership degree of each class in probabilistic label of x is computed using interestingness measure like confidence. Since at least 50% of the neighbours belong to the corre-

Algorithm 1: Relabelling Phase.

Input: Evidential data (D_{old}) with p instances, n attributes and m class labels.
User-defined parameter (k) which represents number of neighbours
Output: Relabeled evidential data (D_{new})
foreach p_i *in* D_{old} **do**
  $NN = \{\}$; // NN consists of k nearest neighbours for p_i
  $NN(p_{ii}) = \{\}$; // $NN(p_{ii})$ consists of $k/2$ nearest neighbours for p_i with c_i
  $NN(p_i) = \{\}$; // $NN(p_i)$ consists of $k/2$ nearest neighbours for p_i with any class label
  foreach p_j *in* D_{old} **do**
    $dist(p_i, p_j) = \sqrt{\sum_{a=1}^{n}(p_{ia} - p_{ja})^2}$
    if $(dist(p_i, p_j))$ *is minimal and* $(p_j \in c_i)$ *and* $(|NN(p_{ii})| < \frac{k}{2})$ **then**
      $NN(p_{ii}) = p_j$
    end
    else if $(dist(p_i, p_j))$ *is minimal and* $(|NN(p_i)| < \frac{k}{2})$ **then**
      $NN(p_i) = p_j$
    end
  end
  $NN = NN(p_{ii}) + NN(p_i)$
  foreach c_j *in* D_{old} **do**
    $PL(p_i, c_j) = \frac{|NN(c_j)|}{|NN|}$
  end
  $D_{new} = PL(p_i)$
end

sponding hard label, membership degree of the original class of x in its probabilistic label would never be less than 0.5. The probabilistic labels of all the evidential instances are computed, and a new relabelled evidential dataset is formed. The pseudocode for relabelling the evidential instances from hard to probabilistic is presented in Alg. 1.

3.2 Decision-Making Phase

Decision-making phase focuses on assigning probabilistic labels for decision queries based on the relabelled evidential data. When a decision query arrives, the proposed system identifies its k nearest evidential instances based on the Euclidean distance. The condition which was imposed on identifying k neighbours in relabelling phase is not considered in decision-making phase, because the hard label of an instance was known in relabelling phase, but the hard label of a query is not known in decision-making phase. Thus, the k neighbours of a decision query are the evidential instances with minimal distance, and can belong to any class in data.

It depends on decision-maker whether to continue with same k value or to use different k values for different phases. If there exist overlapping among disjoint classes, then increase in k value may increase the neighbours from different classes. If the evidential neighbours of a decision query belongs to different classes, then there may be no considerable difference among the membership values of different classes. Thus, it can be observed that the choice of k value has a significant impact on the membership values in a probabilistic label of a decision query when there is overlapping.

Once the nearest evidential instances for a decision query are identified based on the Euclidean distance and k value, its probabilistic label is assigned by combining the probabilistic labels of all the nearest evidential instances using Dempster's combination rule. If the probabilistic labels of nearest neighbours of a decision query are highly conflicting, then the membership values are updated from 0 to 0.0001. The pseudo code for assigning a probabilistic label to a decision query is presented in Alg. 2. The proposed method focuses more on predicting the probability of a decision query belonging to each disjoint class rather than predicting a class directly. Since

Algorithm 2: Decision-making Phase.

Input: Relabelled evidential data (D_{new}) with p instances, n attributes and m class labels.
User-defined parameter (k) which represents number of neighbours
Decision Query (x) with n attributes
Output: Probabilistic label for x ($PL(x)$)
foreach p_i *in* D_{new} **do**
  $NN(x) = \{\}$; // $NN(x)$ consists of k nearest neighbours for x
  $dist(x, p_i) = \sqrt{\sum_{a=1}^{n}(x_a - p_{ia})^2}$
  if $dist(x, p_i)$ *is minimal* **then**
    $NN(x) = p_i$
  end
end
foreach c_j *in* D_{new} **do**
  $PL(x, c_j) = r_{NN(x), c_j}$; // refer Eq.(1)
end

the probabilities are important, calibration of the proposed probabilistic labelling method needs to be measured and improved. Calibration of a model ensures that the distribution of the predicted probabilities are similar to the distribution of the observed probabilities. A model is said to be calibrated if it returns probabilities which are good estimates of the actual likelihood of a class (Vega et al., 2021).

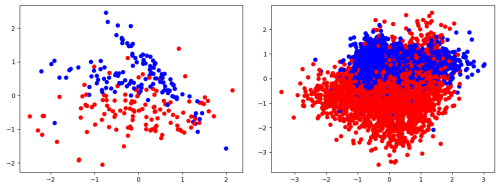
4 EXPERIMENTS AND RESULTS

This section presents the robustness of the proposed probabilistic labelling method using both synthetic data and real-world data.

4.1 Synthetic Data

Two distinct Gaussians are considered for generating synthetic data using sklearn package in python. Total number of samples, separation region between classes, and number of samples per each class are considered as three parameters to generate different datasets. The proposed method is trained and tested on all different datasets to verify its robustness. In the entire experiment on synthetic data, k value remains as 5. In each dataset, 80% of the samples are considered for training, and the remaining 20% are considered for testing the proposed method.

Case 1: Total number of samples is varied from 250 to 7000 by fixing the separation between the class as 0.5, and by maintaining the equal number of samples in each class. Figure 1 and 1 presents the synthetic data where total number of samples is 250 and 7000 respectively. In a probabilistic label, the



(a) No. of samples = 250 (b) No. of samples = 7000

Figure 1: Varying number of samples.

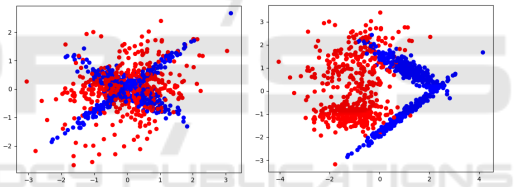
class with highest membership degree is considered as the predicted label to verify the accuracy. Table 1 presents the total number of instances along with accuracy of the proposed method and the accuracy of traditional k-NN. It can be observed from Table 1 that increasing or decreasing the total number of instances does not have significant impact on the performance of the proposed method. Moreover, the accuracy of

Table 1: Accuracy with varying number of samples.

Samples	Probabilistic labels accuracy	Hard labels accuracy
250	82	82
500	94	94
750	90	89
1000	86	86.5
2000	75	73.75
3000	86	86
4000	85.75	85.87
5000	72.6	71.8
6000	87.16	86.91
7000	81.71	81.28

the proposed method is almost similar to the accuracy of the traditional k-NN.

Case 2: The separation between the classes varies between 0 and 1 by fixing the total number of samples as 1000, and by maintaining the equal number of samples in each class. Figure 2 and 2 presents the synthetic data with 0 and 1 as the separation degree between distinct classes respectively. Table 2 presents



(a) sep_degree = 0 (b) sep_degree = 1

Figure 2: Varying separation degree.

the separation degree between classes and accuracy. It can be observed from Table 2 that the accuracy

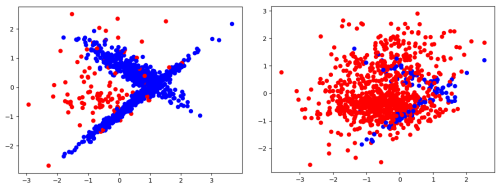
Table 2: Accuracy with sep_degrees.

Sep_deg	Acc	Sep_deg	Acc
0.1	68.5	0.2	74.5
0.3	80	0.4	84
0.5	86	0.6	90
0.7	92	0.8	92.5
0.9	95.5	1	96

increases with increase in the separation degree between the classes. Since the proposed method uses k-nearest neighbours as prototype to relabel training instances, the decrease in the separation region leads to have neighbours of different classes. If the neighbours belongs to different classes, then there may not be high difference between the membership degrees of classes in the probabilistic label. It can be mentioned

from Table 2 that the separation degree between the classes has significant impact on the accuracy of the proposed method.

Case 3: The number of samples in each class varies 0.1 to 0.9 by fixing the total number of samples as 1000, and the separation degree between the classes as 0.5. Figure 3 and 3 presents the synthetic data with different distribution of samples between distinct classes. Table 3 presents the class distribution along with accuracy of each class.



(a) Class 1 as majority (b) Class 2 as majority

Figure 3: Varying distribution of samples.

Table 3: Accuracy with class distribution.

c_1	c_2	acc_0	acc_1
0.1	0.9	55.5	1
0.2	0.8	65.1	1
0.3	0.7	65.5	99.2
0.4	0.6	69	99.1
0.5	0.5	76.6	95.8
0.6	0.4	79.5	87.1
0.7	0.3	84.1	85.2
0.8	0.2	88.9	71.7
0.9	0.1	92.6	27.2

It can be observed from Table 3 that the accuracy of a class increases with increase in the number of samples of that class. Though it is idealistic to have balanced number of samples, the performance of the proposed method does not greatly rely on the distribution of classes. Instead it depends on the closeness among the samples of same class.

Case 4: In this case, the percentage of training and testing samples varies by fixing the total number of samples as 1000, separation between the classes as 0.5, and the number of samples in each class are equal. Table 4 presents the training and testing percentage and accuracy.

It can be observed from Table 4 that accuracy increases with increase in the training data. However, the proposed method is capable enough to achieve reasonable accuracy even with less number of training instances.

Table 4: Accuracy with varying train-test-splits.

Train	Test	Accuracy
0.1	0.9	81.44
0.2	0.8	81.75
0.3	0.7	84.57
0.4	0.6	84.66
0.5	0.5	85
0.6	0.4	87.25
0.7	0.3	86.66
0.8	0.2	86
0.9	0.1	86

4.2 Real-World Data

This work considers the publicly available real-world datasets from University of California Irvine (UCI) repository.

Pima Indians Diabetes Dataset consists of 768 samples where each sample is a combination of eight continuous-valued attributes and a binary class label. Among 786 samples, 268 are predicted as diabetes and the remaining 500 are predicted as non-diabetes.

This experiment starts with partitioning the dataset two parts, namely, evidential data, and query data. 80% of the samples belongs to evidential data which are used to train the proposed method, and the remaining 20% samples are considered as queries. All the samples in the evidential data are hard labeled. The proposed method initially converts them into probabilistic labels by identifying neighbours.

When a decision query arrives, proposed method identifies its nearest evidential neighbours. If the probabilistic labels of neighbours are not having considerable difference between the membership degrees of different classes, then it leads to misclassification. There is a decision query with attribute-values as $\{a_1 : 4, a_2 : 132, a_3 : 86, a_4 : 31, a_5 : 0, a_6 : 28, a_7 : 0.419, a_8 : 63\}$ and class_0 is the original hard label. After combining the probabilistic labels of its neighbours using Dempster's combination rule, the resultant probabilistic label has 0.5614 and 0.4385 as the membership degrees for class_1 and class_0 respectively. Since the proposed method considers the class with highest membership degree to compute accuracy, this particular decision query is considered as misclassified. Change in the number of neighbours may make this particular instance to assign high membership degree for class_0. However, change in number of neighbours changes the membership degrees of remaining queries as well. Thus, classified and misclassified instances changes with change in k value. Table 5 presents the k values with corresponding accuracies. It can be observed from Table 5 that accu-

Table 5: Accuracy of diabetes dataset.

k value	Accuracy
1	61.68
3	75.32
5	75.97
7	76.62
9	81.81

racy increases with increase in the k value. If there exist overlapping between classes, then increasing k value increases the chance for neighbours from different class. This leads to decrease in accuracy. If there is no much overlapping between classes, then increasing or decreasing the k value does not have much impact on accuracy. Thus, accuracy of the proposed method depends on the overlapping region between the classes, not on k value. Overlapping depends on the uncertainty in the data; probabilistic labelling method based on Dempster's theory is one such approach to represent uncertainty in efficient manner.

Indian Liver Patient Dataset (ILPD) consists of 416 liver disease patient records and 167 non-liver disease records. Each record is a combination of 10 continuous-valued attributes and a binary label. The proposed method initially converts the hard labels of 80% of the samples into probabilistic labels using k-nearest neighbours. The probabilistic labels of remaining 20% of the samples are computed by combining the probabilistic labels of corresponding neighbours using Dempster's combination rule. Table 6 presents the k values along with accuracies. It

Table 6: Accuracy of liver disease dataset.

k value	Accuracy
1	59.82
3	67.52
5	64.10
7	67.52
9	64.95

can be observed from Table 6 that increasing or decreasing the k value does not have significant impact on accuracy. Only the overlapping region between the class has an impact on the performance of the proposed method.

Wisconsin breast cancer dataset consists of 569 samples where each sample is a combination of 30 continuous-valued attributes and a binary class label. In this experiment, k value remains constant, and the ratio of train-test split varies. Table 7 presents the train-test-split values along with accuracies. It can be observed from Table 7 that accuracy increases with increase in the training data. However, the proposed

Table 7: Accuracy of breast cancer dataset.

Train	Test	Accuracy
0.1	0.9	90.44
0.2	0.8	92.54
0.3	0.7	93.98
0.4	0.6	94.15
0.5	0.5	95.78
0.6	0.4	96.49
0.7	0.3	96.49
0.8	0.2	96.49
0.9	0.1	98.24

method has recorded reasonable accuracy even with 10% of training samples. Thus, it can be concluded that the proposed probabilistic labelling method is sample efficient.

4.3 Comparative Analytics

This sub-section presents comparative analysis of the proposed probabilistic labelling method with other recent relevant works in literature. Most of these works use either fuzzy functions or kernel functions for assigning probability values, and Dempster's combination rule for assigning decision probabilities. Table 8 presents the performance of these models on UCI datasets.

It can be observed from Table 8 that the proposed decision-making model achieves the maximum accuracy for five out of seven datasets. Therefore, it can be concluded that the decision-making model with probabilistic labelling is superior compared to other works.

5 CONCLUSION

The proposed method initially converts hard labels of evidential instances to probabilistic labels using k-nearest neighbours. The condition that ensures that at least half of neighbours belongs to the hard label of the corresponding evidence, supports in giving priority to the original class. After relabelling evidential instances, the decision-making model assigns probabilistic labels to decision queries by combining the labels of neighbours using Dempster's combination rule. It is proven from the experimental results that proposed method is sample efficient. Moreover, the proposed method is said to be calibrated because it is able to represent the actual likelihood of classes in terms of posterior probabilities. The soft label gives an understanding to a decision-maker about different alternatives and their combinations. The degree

Table 8: Comparative Analysis.

Recent Works	Breast	IRIS	Heart	Diabetes	Liver	Hepatitis	Sonar
(Denoeux, 2008)			76.3			80.57	75.81
(Xu et al., 2013)			82.59			79.4	72.57
(Xu et al., 2014)		95.6	85			85.5	
(Xu et al., 2016)			83.7			79.88	68.26
(Liu et al., 2017)			85.56			83.85	76.02
(Qin and Xiao, 2018)	97.07	96.7	86.7			89.03	
(Jiang et al., 2019)	69.91	95.33	75.19	70.96	76.48	81.29	
(Peñafiel et al., 2020)	93.85	95.9	72.7				
(Song et al., 2021)	97.07		86.7	76.82	78.04		
(Zhu et al., 2021)	97.15	99.33	91.48			81.61	
(Ranjbar and Effati, 2022)	95.22		79.48	70.96		81.62	
Proposed Method	**98.24**	**100**	85.36	**81.81**	67.52	**89.65**	**76.19**

of overlapping among the classes is the only factor that has significant impact on the membership degrees in probabilistic labels. Since identifying neighbours for each instance is computationally complex, density models or fuzzy approaches may be considered as the prototype to convert hard labels to probabilistic labels in future.

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