

# Synchronizing Vehicle Routing and Photo-Voltaic Production

Alejandro Olivas Gonzalez, Alain Quilliot and H el ene Toussaint  
*Labex IMOB3, LIMOS Laboratory, UCA/CNRS, C ezeaux Campus, Clermont-Ferrand, France*

Keywords: Routing, Scheduling, Bilevel Optimization, Machine Learning.

Abstract: We deal here with the routing and scheduling of electric vehicles in charge of performing internal logistic tasks inside some protected area. Some vehicles are provided in energy by a local photo-voltaic facility with limited production/storage capacities and time dependent production rates. In order to avoid importing energy from outside (notion of *self consumption*) one must synchronize energy consumption and production while minimizing both production and routing costs. Because of the complexity of resulting bi-level optimization model, we handle it while shortcutting the production scheduling level with the help of surrogate estimators, whose values are computed through a pricing mechanism and machine learning devices. According to this purpose we design, implement and test several algorithms and get an evaluation of the potentiality of such surrogate component based approaches. This work was carried on in partnership with national power company EDF, in the context of the PGM0 program.

## 1 INTRODUCTION

The notion of multi-level decisional (Caprara, 2014; Chen, 2013; Colson, 2005; Dempe, 2015) model is mainly related to situations when decision is shared between several players, independent from each other or tied together by some hierarchical or collaborative link. Then, solving such a model aims at providing a best scenario in case all the players accept to submit themselves to a common authority (centralized paradigm), or, if it is not the case (collaborative paradigm), at helping them into the search for a compromise (Kleinert, 2021). Standard approaches involve decomposition schemes, which may be hierarchical (Benders decomposition, Stackelberg equilibrium,...) or transversal (Lagrangean relaxation). Still in both case some major difficulties remain: They are related to the sensitivity issue, which means the way one may retrieve information from the different levels in order to make them interact, and to the collaborative issue, which may impose the players to deal with incomplete information. It comes that a trend, boosted by the rise of machine learning technology (Krystow, 2018; Wojtuziak, 2012), is to bypass some levels of the global model and replace them by surrogate estimators, likely to approximate the constraints and costs induced by the decisions taken at those levels.

It is this point of view which we adopt here while dealing with the joint management of local photovoltaic energy production by a PV-facility (Luthander, 2015; Smart Together, 2016) and its consumption by a fleet of electric vehicles in charge of logistic tasks inside a restricted area. This problem arose in the context of the activities of IMOB3 (*Innovative Mobility*) Labex in Clermont-Ferrand, which conducts research on both autonomous electric vehicles and solar energy, and of the national PGM0 program promoted by power company EDF. The fact is that both market deregulation and emergent technologies currently induce the rise of local renewable energy producers (factories, farms and even individual householders) who simultaneously remain consumers (Balbiyad, 2019; Deb, 2018; Grimes, 2008) and so make self-consumption become an issue. A key operational feature of self-consumption management happens to be the need for synchronization between strongly time-dependent energy production and its consumption.

So we consider here, on one side, a production manager who runs a PV (*Photo-Voltaic*)-Plant, which not only distributes energy between end-users (electric vehicles) but also buys and sells energy on the market. On the other side, we consider a fleet manager who schedules and routes electric vehicles in such a way that they efficiently achieve a set of internal logistic tasks. Both interact through

*recharging transactions*: In order to avoid that the vehicles waste time while waiting for their battery to be recharged, the PV-Plant relies on a set of identical batteries, so that the vehicles only need to move toward the PV-Plant and switch batteries in order to be recharged. This *plug out/in* operation is instantaneous. But limited storage and recharge capacities impose both players to carefully synchronize the strongly time-dependent energy production and its consumption. This synchronization requirement makes resulting bi-level decision problem complex, even if we restrict ourselves to the centralized paradigm. Though many searchers have recently showed interest into the decisional problems which may be related to the management of renewable energy, they most often focused either on production control and scheduling (Adulyasak, 2012 ; Irani, 2003, Erdelic, 2019) or on the issues related to power consumption (Albrecht, 2013; Koc, 2019), without dealing with this synchronization issue (Bendali, 2021; Trotta, 2022). Our goal here is to address it by shortcutting the part of the process related to production management and handling the vehicle routing master level while using a surrogate formulation of the cost and constraints related to production. We try two approaches: the first one is based upon a parametric pricing mechanism; the second involves a convolutional neural network. We suppose, for the sake of simplicity, that our system behaves in a deterministic way.

The paper is organized as follows. We first (Section 2) introduce the *PV\_Prod\_VRP* problem. We set it according to the MILP (*Mixed Integer Linear Programming*) framework and discuss different formulations. Next (Section 3), we present the 3-step algorithmic framework for the handling of *PV\_Prod\_VRP* through the use of surrogate estimators. In Section 3.1 we provide the details of a *Branch and Cut* algorithm which performs (Step 1) the computation a collection of *elementary trips*; In Section 3.2 we specify the surrogate estimators of the production cost which we use in order to schedule the *elementary trips* (Step 2): the first one relies on a parametric pricing mechanism while the second one involves a convolutional neural network. Section 4 is devoted to numerical experiments.

## 2 THE PV\_PROD\_VRP PROBLEM

We consider a fleet of  $K$  small identical electric vehicles initially located at a depot  $Depot = 0$  and which are required to perform VRP: *Vehicle Routing Problem* tours, that means to visit a set of *stations*  $J =$

$\{1, \dots, M\}$  within a time horizon  $[0, TMax]$ . Moving from station  $j$  to station  $k$  requires  $\Delta_{j,k}$  time units and an amount  $E_{j,k}$  of energy. *Recharge transactions* take place at *Depot*. In order to avoid that the vehicles wait every time they recharge, the fleet relies on a set  $B$  of identical batteries, with capacity  $C$  and *charge speed*  $CS$ , and vehicles may switch their battery every time they come back to *Depot*. This *plug out/in* operation is considered as almost instantaneous. It comes that while the vehicles are running with *active* batteries, *idle* batteries are recharged at *Depot* before being used again by the vehicles. For any battery  $b$  in  $B$ ,  $V_b$  denotes its initial load. We call *elementary trip* any VRP sub-tour that a vehicle may perform without recharging: such an *elementary trip* is a sequence  $\{Depot = j_0, j_1, \dots, j_s, j_{s+1} = Depot\}$  such that the sum  $\sum_j E_{j,Succ(j)}$  does not exceed capacity  $C$ .

In order to implement a *self-consumption* policy, *Depot* is provided with a *PV-Plant*, that means with a photovoltaic facility which assigns the batteries to the vehicles and produces its own energy that it distributes between the currently idle batteries or that it sells to the market. In case this self-produced energy is not enough, the PV-Plant can also buy energy to the market. The time space  $[0, TMax]$  being divided into small periods  $i = 1, \dots, N$ , all with same length  $p$ , we denote by  $C^R$  the *recharge* capacity, i.e the energy which may be loaded into a battery during 1 period. We also denote by  $R_i$  the expected production of the PV-Plant at period  $i$ , by  $A_i$  the energy unit purchase price at period  $i$ , and by  $B_i$  the energy unit sale price. Clearly we have, for any  $i$ ,  $A_i \geq B_i$ . Figure 1 describes the way the PV-Plant and the vehicle fleet interact.

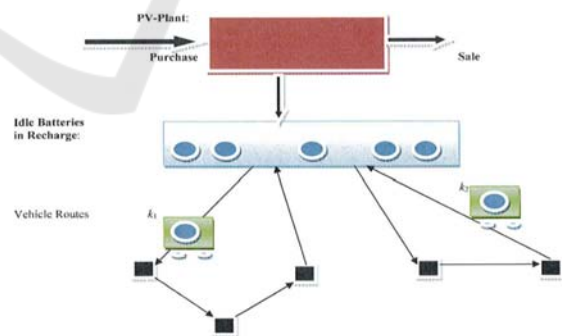


Figure 1: The Global Self-Consumption System.

Resulting *PV\_Prod\_VRP* problem comes as follows:

***PV\_Prod\_VRP* Problem:** {Simultaneously schedule the vehicle fleet and the activity of the PV-Plant, in such a way that:

- Every station is visited once;

- Every time a vehicle  $k$  comes back to *Depot*, the PV-Plant assigns it a battery loaded to make possible its next *elementary trip*;
- The global energy load of the batteries at the end of period  $N$  must be at least the same as at the beginning of the process.
- Some global cost is minimized, which combines standard VRP cost with the difference between the energy purchase cost and the profit derived from energy sales}.

In order to formalize, let us first suppose that the vehicles have been scheduled, which means that a collection  $\Pi_0$  of *elementary trips*  $\pi$  has been computed and each  $\pi$  scheduled inside a set of consecutive periods  $I(\pi)$ . We denote by  $\Sigma_0 = \{(\pi, I(\pi)), \pi \in \Pi_0\}$  the resulting set of *scheduled trips*  $\sigma = \{(\pi, I(\pi))$ . For any trip  $\pi$ , we denote by  $E(\pi)$  its energy consumption, by  $T(\pi)$  its duration, by  $S(\pi)$  its set of stations, by  $E^{Mean}(\pi)$  the quotient  $E(\pi)/Card(I(\pi))$ , and we extend those notations to *scheduled trips*  $\sigma$ . Then, *PV\_Prod* sub-problem is about the way the PV-Plant loads the batteries and assigns them to *scheduled trips*  $\sigma$ :

**ILP PV\_Prod( $\Sigma_0$ ) MILP (Mixed Integer Linear Program) Model:**

**{Compute**

- $\{0, 1\}$ -vector  $U = (U_{\sigma,b}, \sigma \in \Sigma_0, b \in B)$ :  $U_{\sigma,b} = 1$  iff  $b = b(\sigma)$  is the battery assigned to  $\sigma$ ;
- $\{0, 1\}$ -vector  $\delta = (\delta_{b,i}, b \in B, i = 1, \dots, N)$ :  $\delta_{b,i} = 1$  iff  $b$  is idle at period  $i$ ;
- $X^A = (X^A_i, i = 1, \dots, N)$ ,  $X^B = (X^B_i, i = 1, \dots, N)$ ,  $X^D = (X^D_{b,i}, b \in B, i = 1, \dots, N)$ , which respectively denote the energy bought, sold and distributed to battery  $b$ , by the PV-Plant;
- $W = (W_{b,i}, b \in B, i = 0, \dots, N)$ :  $W_{b,i}$  is the energy inside battery  $b$  at the end of period  $i$ .

**Objective Function:** Minimize

$$\sum_i A_i X^A_i - \sum_i B_i X^B_i.$$

**Constraints:**

- For any  $b, i$ ,  $W_{b,i} \leq C$   
and  $X^D_{b,i} \leq C^R \cdot \delta_{b,i}$ ; (R1)
- For any  $b$ ,  $W_{b,0} = V_b$ ; (R2)
- $\sum_b W_{b,N} \geq \sum_b V_b$ ; (R3)
- For any  $i$ ,  
 $R_i + X^A_i = X^B_i + \sum_b X^D_{b,i}$ ; (R4)
- For any  $b \in B, i$ :  
 $(1 - \delta_{b,i}) = \sum_{\sigma, s.t. i \in I(\sigma)} U_{\sigma,b} \leq 1$ ; (P1)
- For any  $\sigma \in \Sigma_0, \sum_b U_{\sigma,b} = 1$ ; (P2)

- For any  $b \in B, i$ :  $W_{b,i} = W_{b,i-1} + X^D_{b,i} - \sum_{\sigma, s.t. i \in I(\sigma)} E^{Mean}(\sigma) \cdot U_{\sigma,b}$ . (P3)}

**Explanation:** (R1) means that we charge a battery  $b$  only if it is idle. (R3) imposes the batteries to be globally loaded with at least as much energy at the end of the whole process as at the beginning. (R4) tells the way energy is distributed between sale, purchase and battery loading. (P1) means that  $b$  is active at period  $i$  only if has been assigned to a unique *scheduled trip*  $\sigma$ , active at period  $i$ . (P2) says that any *scheduled trip*  $\sigma$  is assigned a unique battery  $b$ . (P3) describes the evolution of a battery  $b$  from a period  $i - 1$  to next period  $i$ .

We may now formalize our global *PV\_Prod\_VRP* problem. The VRP decision makes our  $K$  vehicles visit at least once any station  $j$  in  $\{1, \dots, M\}$  and means a collection  $\Sigma_0$  of *scheduled trips*  $\sigma = (\tau, I(\tau))$  such that:

- For any  $i$ ,  $Card(\{\sigma \in \Sigma_0, s.t. i \in I(\sigma)\}) \leq K$ . (S1)
- For any  $j$ ,  $Card(\{\sigma \in \Sigma_0, s.t. j \in S(\sigma)\}) \geq 1$ . (S2)

If we consider as standard VRP cost of  $\Sigma_0$  the global riding time  $\sum_{\sigma} T(\sigma)$  (*Driver Cost*) then, a *time versus money* coefficient  $\alpha$  being given, our *PV\_Prod\_VRP* problem comes as follows:

**PV\_Prod\_VRP Problem:** {Compute a collection  $\Sigma_0$  of *scheduled trips*, such that (S1, S2) hold and which minimizes the sum  $\alpha \cdot (\sum_{\sigma \in \Sigma_0} T(\sigma)) + Val\_PV\_Prod(\Sigma_0)$ , where  $Val\_PV\_Prod(\Sigma_0)$  is the optimal  $PV\_Prod(\Sigma_0)$  value}.

Denoting by  $\Sigma$  the set of all possible *scheduled trips* allows us to propose the following MILP formulation of *PV\_Prod\_VRP*:

**PV\_Prod\_VRP MILP Formulation:**

**{Compute:**

- $\{0, 1\}$ -vector  $Z = (Z_{\sigma}, \sigma \in \Sigma)$ :  $Z_{\sigma} = 1$  means that we select *scheduled trip*  $\sigma$ ;
- $X^A, X^B, X^D, W, U$  and  $\delta$  as in above *PV\_Prod* MILP model;

**Objective Function:**

Minimize  $\sum_i A_i X^A_i - \sum_i B_i X^B_i + \alpha \cdot \sum_{\sigma} T(\sigma) \cdot Z_{\sigma}$ ;

**Constraints:**

- (R1, ..., R4, P1, ..., P3) of *PV\_Prod*;
- For any  $i = 1, \dots, N$ ,  $\sum_{\sigma, s.t. i \in I(\sigma)} Z_{\sigma} \leq K$ ; (S1)
- For any  $j = 1, \dots, M$ ,  $\sum_{\sigma, s.t. j \in S(\sigma)} Z_{\sigma} \geq 1$ . (S2)
- For any  $\sigma \in \Sigma, \sum_b U_{\sigma,b} = Z_{\sigma}$ ; (S3)

**Explanation:** (S3) means that any selected *scheduled trip* must be assigned a battery.

**An Example:** Let us consider a customer set  $X = \{A, B, C, D, E\}$  and 2 vehicles  $v_1$  and  $v_2$ , which follow routes  $\Gamma_1$  and  $\Gamma_2$  as in figure 3 below: Numbers above

every arc respectively represent the time and the energy required in order to traverse the arc.

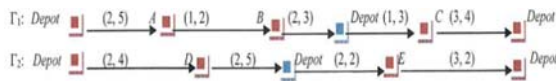


Figure 2: Routes  $\Gamma_1$  and  $\Gamma_2$ .

We also suppose that: we are provided with 2 identical batteries  $b_1$  and  $b_2$ , both with capacity  $C = 12$  and with initial loads respectively equal to 7 and 6; the time space is divided into 10 periods, all with duration equal to 2; *recharge* capacity  $C^R$  is equal to 3 and production data come as in table 1 below:

Table 1: PV Prices and Production Coefficients.

$i$	1	2	3	4	5	6	7	8	9	10
$A_i$	2	3	7	7	3	2	6	7	4	2
$B_i$	1	2	4	4	1	1	3	3	2	1
$R_i$	4	4	3	5	2	6	4	4	4	5

Then we derive a feasible solution from  $\Gamma_1$  and  $\Gamma_2$

- $\Gamma_1$  starts at time 4 with battery  $b_2$ , comes back to *Depot* at time 9, waits in *Depot* until time 14 and starts until time 18 with battery  $b_1$ ;
- $\Gamma_2$  starts at time 2 with battery  $b_1$ , comes back to *Depot* at time 6, waits in *Depot* until time 12 and starts again with battery  $b_2$  until time 16.
- Vectors  $X^A$ ,  $X^B$  and  $X^D$  are given by table 2 below:

Table 2: Solution Values.

$i$	1	2	3	4	5	6	7	8	9	10
$X^A_i$	1	0	0	0	1	0	0	0	0	1
$X^B_i$	0	1	3	2	0	0	1	4	2	0
$X^D_{b1,i}$	2	*	*	3	3	3	3	*	*	3
$X^D_{b2,i}$	3	3	*	*	*	3	*	*	2	3

Time versus money coefficient  $\alpha$  is equal to 2. Related cost is:  $2 \cdot 17$  (Vehicle Time Cost) + 5 (Energy Purchase) - 43 (Energy Sale) = - 4. We earn 38 money units, under a riding time of 17.

**A Short Discussion: Variants of the PV\_Prod Model.** In practice, production configurations may be more complex. Let us mention here two variants:

**First variant:** Batteries remain identical, but may be used in order to store energy and next sell it.

According to this hypothesis, a battery  $b$  may receive energy at period  $i$  and sell it at period  $i' > i$ . The *Recharge* model must be updated through the introduction of an additional vector  $Y^B = (Y^B_{b,i}, b \in B, i = 1, \dots, N)$ , which means the amount of energy sold at any period  $i$  by battery  $b$ .

**Second variant:** A fixed storage unit is added to the PV-Plant, which may be used either for sale or for battery feeding when energy is scarce or expensive.

According to this hypothesis, such a *Buffer* battery  $BUFF$ , with storage capacity  $C^{BUFF}$ , initial load  $V^{BUFF}$ , recharge capacity  $C^R_{BUFF}$  and discharge capacity  $C^D_{BUFF}$ , induces the introduction of the following variables into the *Recharge* model:

- $X^{BUFF}_i$  = energy sent from the PV-Plant to the *Buffer* battery at  $i$ ;
- $X^{B_{BUFF}}_i$  = energy sold by the *Buffer* battery at  $i$ ;
- $Y^{BUFF}_{b,i}$  = energy sent by the *Buffer* battery to battery  $b$  at  $i$ ;
- $W^{BUFF}_i$  = energy inside the *Buffer* battery at the end of  $i$ .

### 3 PV\_PROD\_VRP HANDLING: SURROGATE COMPONENTS

*PV\_PROD\_VRP* is a complex model, with 3 discrete decision levels, respectively related to *elementary trips*, *scheduled trips*, and batteries. Its MILP formulation is hardly practicable and would badly fit true life contexts, where decisions related to respectively vehicles and the PV-Plant are likely to depend on distinct players. Instead, we propose 2 approaches:

**First approach:** Solving above *PV\_Prod\_VRP* MILP model, while restricting ourselves to a pre-computed set  $\Sigma$  of *scheduled trips*.

**Second Approach:** Solving the *PV\_Prod\_VRP* bi-level model, while partially short-cutting the slave *PV\_Prod* level through the introduction of surrogate constraints and criteria.

It is this second approach which we follow here.

#### 3.1 Shortcutting the PV\_Prod Level

Our purpose here is to compute the *scheduled trip* set  $\Sigma_0$  without explicitly involving the PV-Plant. But, while the schedule  $\sigma \rightarrow I(\sigma)$  must take into account prices  $A_i$ ,  $B_i$  and production rates  $R_i$ , we can only say that a well-fitted collection  $\Pi_0$  requires small amounts of both time and energy. So we should split the routing part of the problem (computing the *elementary trips*) from its scheduling part (scheduling those *elementary trips*). This leads us to the following parametric *VRP\_Surrogate* process:



**VRP\_Surrogate Parametric Algorithm:** Initialize flexible scaling parameter  $\gamma$ ; Not *Stop*; Current best solution  $Best\_Sol$  is undefined;

While Not *Stop* do

**1<sup>st</sup> step:** Compute an elementary trip collection  $\Pi_0$  which minimizes  $\alpha \cdot \sum_{\pi \in \Pi_0} T(\pi) + \gamma \cdot \sum_{\pi \in \Pi_0} E(\pi)$ ,

**2<sup>nd</sup> step:** Turn  $\Pi_0$  into a *scheduled trip* collection  $\Sigma_0$ , i.e compute intervals  $I(\pi)$ ,  $\pi \in \Pi_0$  in a way which meets some surrogate constraints (SURRE) and which minimizes some surrogate cost  $\Phi(\pi \rightarrow I(\pi))$ ;

**3<sup>rd</sup> step:** Update  $\gamma$  and *Stop*; Solve  $PV\_Prod(\Sigma_0)$  and update  $Best\_Sol$ ;

### 3.2 Step 1: Branch and Cut

Trying exact methods leads to set an ILP model. For any subset  $A \subseteq J = \{1, \dots, M\}$ , we set  $\delta^+(A) = \{\text{arcs } (j, k) \text{ such that } j \notin A \text{ and } k \in A\}$  and  $Cl(A) = \{\text{arcs } (j, k) \text{ s.t at least } j \text{ or } k \text{ is in } A\}$ . We get the following *Elementary\_Trip* ILP model, which involves a specific SNS: *Strong No Sub-Tour* Constraint:

**Elementary\_Trip ILP model:**

{Compute a (0, 1)-valued vector  $Z = (Z_{j,k}, j, k = 0, \dots, M)$  in such a way that :

- For any  $j$ ,  $\sum_k Z_{j,k} = \sum_k Z_{k,j} = 1$ ; (S2)
- For any subset  $A$  of  $\{1, \dots, M\}$ , (SNS)  
 $C \cdot (\sum_{(j,k) \in \delta^+(A)} Z_{j,k}) \geq \sum_{(j,k) \in Cl(A)} E_{j,k} \cdot Z_{j,k}$
- Minimize  $\alpha \cdot (\sum_{j,k} Z_{j,k} \cdot T_{j,k}) + \gamma \cdot (\sum_{j,k} Z_{j,k} \cdot E_{j,k})$ .

The explanation of above (SNS) constraint comes with the statement below:

**Theorem 1:**  $\{0, 1\}$  vector  $Z$  meets (S2, SNS) iff arcs  $(j, k)$  such that  $Z_{j,k} = 1$  define a collection  $\Pi_0$  of sub-tours  $\pi_1, \dots, \pi_S$ , with  $S = \sum_k Z_{0,k} =$  such that:

- For every  $s = 1, \dots, S$ ,  $\pi_s$  starts from Depot = 0 and ends into Depot, and requires an energy amount no more than  $C$ ;
- Every station  $j$  is visited exactly once by collection  $\Pi_0$ .

Constraints SNS may be separated in polynomial time through a max flow (min cut) procedure.

**Sketch of the Proof:** Constraints (SNS) imply that  $Z$  gives rise to a collection  $\tau$  of sub-tours  $\tau_0, \dots, \tau_S$ , which globally involve exactly once any station  $j$ , and which all contain Depot = 0 (*No Sub-Tour*). If some tour  $\tau_s$  spends more energy than capacity  $C$ , then a subset  $A$  of  $\{0, \dots, M+1\}$  exists which makes  $Z$  violate (SNS). We deduce the first part of our statement. As for the second part, we see that, some vector  $Z$  (integral or rational) being given, separating (SNS)

means searching for  $B = \{0, 1, \dots, M\} - A$ , such that:  

$$\sum_{j,k \in B} Z_{j,k} \cdot E_{j,k} + C \cdot (\sum_{(j,k) \in \delta^+(B)} Z_{j,k}) < \Lambda$$

$$= \sum_{j,k} Z_{j,k} \cdot E_{j,k} \quad (*)$$

In order to do it, we construct a network  $G^{Aux}$ , whose node set is  $\{0, 1, \dots, M+1\}$  and whose arc set  $U^{Aux}$  may be written  $U^{Aux} = U \cup Copy(U)$  with:

- $U = \{(j, k), j, k = 0, \dots, M\}$ , such that  $Z_{j,k} \neq 0$ : provided with a capacity  $w_u = Z_{j,k} \cdot (C - E_{j,k})$ ;
- With any arc  $e = (j, k)$  in  $U$ , we associate an arc  $u = Copy(e) = (j, M+1)$  provided with a capacity  $w_u = Z_{j,k} \cdot E_{j,k}$ . Then arc set  $Copy(U)$  is the set of all arcs  $Copy(e)$ ,  $e \in U$ .

But searching for  $B$  such that (\*) means searching for  $B \subseteq \{0, 1, \dots, M\}$ , such that:

$$\sum_{u \in U^{Aux}, \text{ s.t } (origin(u) \in B) \wedge (destination(u) \notin B)} w_u < \Lambda.$$

It is known that, in case  $B$  exists, it may be retrieved through the standard *Min Cut* algorithm.  $\square$

Theorem 1 and related proof provide us with an efficient *separation* procedure which opens the way to the implementation of a *Branch and Cut* process.

### 3.3 Step 2: Surrogate Components

In order to enhance  $PV\_Prod(\Sigma_0)$  feasibility, we impose the following surrogate necessary (but not sufficient) constraints:

- For any period  $i$ ,  $Card(\{\sigma \in \Sigma_0 \text{ such that } i \in I(\sigma)\}) \leq K$ . (S1)
- For any  $i_0 = 1, \dots, N$ :  $C^R \cdot (\sum_{i \leq i_0-1} n(\Sigma_0, i)) + \sum_b V_b \geq \sum_{\sigma \text{ s.t } Start(\sigma) \leq i_0} E(\sigma)$ ,

where  $n(\Sigma_0, i)$  is the number of *scheduled trips*  $\sigma$  idle at period  $i$ , and  $Start(\sigma)$  is the starting period of  $\sigma$ , and which means that we must be able to feed the batteries in such a way that the trips becomes possible. (SURRE2)

Then, in order to make possible the use of any surrogate estimator  $\Phi(\pi \rightarrow I(\pi))$ , we implement *Step 2* while relying on a non deterministic local search heuristic *Scheduled\_Trip*( $\Pi_0, \Phi$ ). So, what remains to be done is to discuss estimator  $\Pi$ .

#### 3.3.1 Defining $\Phi(\Sigma_0)$ According to a Pricing Mechanism

The idea here is that the cost of a schedule ( $\pi \rightarrow I(\pi)$ ) is determined by the distribution of resulting values  $n(\Sigma_0, i)$ , which means, for any period  $i$ , the number of batteries which are available for recharge at  $i$ . Let us denote by  $E = (\sum_{\pi \in \Pi_0} E(\pi))$  the global charge which has to be loaded into the batteries and by  $I = (\sum_{\pi \in \Pi_0} \lceil T(\pi)/p \rceil)$  the number of periods required in order to perform all trips of  $\Pi_0$ . The point is that if we suppose

that all batteries receive a same charge  $E^{Mean} = E/I$  at every period when they are idle, then we get the cost of the production process through the following formula:  $Cost = I \cdot Q^{Stand}_{i,n}(\Sigma_0, i)$ , where *standard price*  $Q^{Stand}_{i,n}$  for the recharge of  $n$  idle batteries at period  $i$  is given by:

$$Q^{Stand}_{i,n} = A_i \cdot (n \cdot E^{Mean} - R_i) \text{ if } n \cdot E^{Mean} \geq R_i$$

$$\text{and } Q^{Stand}_{i,n} = B_i \cdot (n \cdot E^{Mean} - R_i) \text{ else.}$$

Clearly, the energy amount loaded into an idle battery  $b$  at period  $i$  may differ from  $E^{Mean}$ . Still, above reasoning suggests us to express the surrogate cost  $\Phi(\pi \rightarrow I(\pi)) = \Phi(\Sigma_0)$  involved into the *VRP\_Surrogate* algorithm as a sum  $\sum_i Q_{i,n}(\Sigma_0, i)$ , where  $Q_{i,n}$  is the estimation of the cost induced by  $n$  batteries in recharge (idle) at period  $i$ . Besides, we notice that:

- If  $n(\Sigma_0, i) \cdot E^{Mean} \geq R_i$ , then  $Q_{i,n}$  should increase with  $A_i$ ;
- If  $n(\Sigma_0, i) \cdot E^{Mean} \leq R_i$ , then  $Q_{i,n}$  should decrease as  $B_i$  increases.

This leads us to set:

- $A^{Mean} = \text{mean value } A_i, i = 1, \dots, N; B^{Mean} = \text{mean value } B, i = 1, \dots, N;$
- $Q_{i,n} = Q^{Stand}_{i,n} \cdot (1 + \rho_1 \cdot (A_i - A^{Mean}))$  if  $n(\Sigma_0, i) \cdot E^{Mean}(\Sigma_0) \geq R_i$ , and else  $Q_{i,n} = Q^{Stand}_{i,n} \cdot (1 + \rho_2 \cdot (B_i - B^{Mean}))$ ,  $\rho_1, \rho_2$  being non negative flexible parameters.

### 3.3.2 Computing $\Phi(\Sigma_0)$ Through a Neural Network

Instead of relying on energy price coefficients  $Q_{i,n}$ , we use a neural network  $N\_Energy$  in order to evaluate a scheduled trip collection  $\Sigma_0$ .  $N\_Energy$  is implemented with the help of the *TensorFlow* open software and trained with a large number (4000) of  $PV\_Prod(\Sigma_0)$  instances. It is designed as a *convolutional* neural network. Such a network, whose main purpose is to be adaptable to inputs with flexible sizes, usually works in 2 (or more) steps: In the first step, a same standard *perceptron* *CM* called *convolutional mask*, is applied to fixed size neighbors of the components of the input vector  $IN = (IN_m, m \in M)$ , and yields an output vector  $OUT = (OUT_m, m \in M)$ ; In the next step, a *pooling* mechanism is applied to  $Out$ , in order to compact it into the fixed size input of another perceptron  $N\_Pool$  which computes the final output. In the present case this final output is a number  $\Theta$  between 0 and 1, whose semantics are that it should be possible to express the optimal value  $VAL\_PV\_Prod(\Sigma_0)$  of  $PV\_Prod(\Sigma_0)$  as a sum  $Val\_Min + \Theta \cdot (Val\_Max - Val\_Min)$ , where  $Val\_Max$  and  $Val\_Min$  are respectively a lower bound and an

upper bound of  $VAL\_PV\_Prod(\Sigma_0)$ . More precisely, the main components of  $N\_Energy$  are (see fig. 3):

**Input layer:** For any input  $\Sigma_0, A, B, R, V$  of the  $PV\_Prod$  problem, we homogenize it as a  $(N+1) \cdot 7$  vector  $IN$ , with  $IN[i] = (A^*_i, B^*_i, R^*_i, \mu^*_i, Q_i, C^*, C^{A*})$ , with:

- $NA^*_i = A_i/A^{Mean}; B^*_i = B_i/A^{Mean}$ , where  $A^{Mean} = \text{Mean value of the coefficients } A_i, i = 1, \dots, N;$
- $\mu^*_i = \sum_{\sigma, s, t, i \in I(\sigma)} E^{Mean}(\sigma); \mu^*_0 = 0;$
- $R^*_i = R_i/R^{Mean}; \mu^*_i = \mu_i/R^{Mean}$ , where  $R^{Mean} = \text{Mean value of coefficients } R_i; R^*_0 = (\sum_b V_b)/R^{Mean};$
- $Q_i = n(\Sigma_0, i)/\text{Card}(B);$
- $C^* = C/R^{Mean}; C^{R*} = C^R/R^{Mean}.$

**Convolutional mask:** *CM* works on any sub-vector  $IN^*_i = (IN[i], \dots, IN[i+4])$ , which means an input with 35 input arcs. It contains 3 inner layers, respectively with sizes 8, 4 and 2, and ends into an output layer, with 1 input value  $OUT_i$ . All 322 synaptic arcs are allowed, together with standard biased sigmoid activation functions whose derivative value in 0 is equal to  $1/2$ .

**The pooling mechanism:** works by merging consecutive values  $OUT_i$  into a single value, in such a way that we get an intermediate vector  $AUX$ , with 13 entries, all with values between 0 and 1.

**Final Perceptron  $N\_Pool$ :** Once the *pooling* mechanism has been applied, we handle resulting 13 dimensional vector  $AUX$  with a perceptron  $N\_Pool$ , with input layer with size 13, intermediate layers with size 6 and 3, and a final layer with size 1.

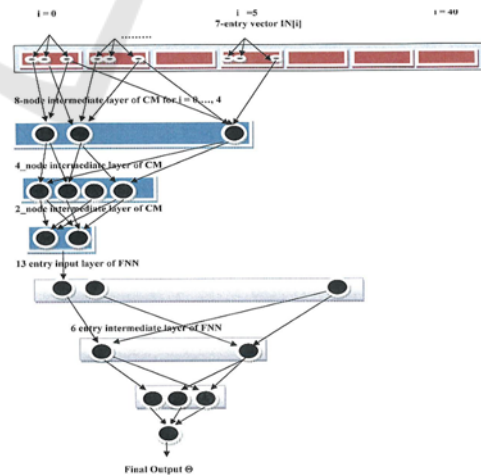


Figure 3: The Neural Network  $N\_Energy$ .

This network is complete in the sense that all 99 synaptic arcs are allowed, together with standard biased sigmoid activation functions.

At the very end, we must learn 421 synaptic coefficients. Figure 6 above shows the global structure of  $N\_Energy$ .

## 4 NUMERICAL EXPERIMENTS

**Technical Context:** We use a processor IntelCore i56700@3.20 GHz, with 16 Go RAM, together with a C++ compiler and libraries CPLEX12 (for ILP models) and TensorFlow/Keras.

**Instances:** As for the PV side, we generate 2 integers  $Q$  and  $N = Q.N_0$ ,  $N = 10, \dots, 40$ ,  $Q = 2, \dots, 5$  and split the period set into  $Q$  intervals, corresponding to different qualities (mean values  $MR_q$  of the production rate) of the weather and different level of prices (means values  $MA_q$  and  $MB_q$ ) on the market. Related production rates  $R_i$  are randomly generated with uniform law inside interval  $[MR_q/2, 3MR_q/2]$ . By the same way prices  $A_i$  and  $B_i$  are randomly generated with uniform law inside respectively intervals  $[MA_q/2, 3MA_q/2]$  and  $[MB_q/2, 3MB_q/2]$ . Proceeding this way provides us with realistic instances..

As for the vehicle part, we generate  $M$  (between 10 and 400) stations together as points with integral coordinates in a 2D square, and derive  $\Delta$ ,  $E$  values according to the Euclidean and Manhattan distances. Then we fix a target number  $S$  of *elementary trips* together with their expected length (number of periods)  $L$ . We derive both the capacity  $C$  and the duration  $p$  of a period. We set  $B = \mu.S.L/N$ , where  $\mu$  is a control parameter, and, for any  $b = 1, \dots, B$ , we generate  $V_b$  between  $C/3$  and  $C$ . In order to make the PV production match the demand from the vehicles, we update the  $R_i$  by doing in such a way that  $\sum_i R_i = H.S.C$ ,  $H$  being a control parameter with value between 0.5 and 2. Finally we generate the key parameter  $C^R$  in such a way that batteries may globally receive at least  $\beta.S.C$  energy units during the whole process,  $\beta$  being a control parameter with value between 1.5 and 4.

Table 3: Characteristics of the Instances.

Inst.	$N$	$M$	$S$	$Q$	$L$	$\mu$	$\alpha$	$\beta$	$H$
1	20	40	10	3	4	2	1	2	0.5
2	20	70	15	4	5	3	0.5	3	1
3	20	100	20	5	6	4	0.2	4	2
4	30	50	10	3	4	2	1	2	0.5
5	30	80	20	4	6	3	0.5	3	1
6	30	120	30	5	8	4	0.2	4	2
7	40	100	20	4	5	2	1	3	1
8	40	200	40	6	10	4	0.5	4	2
9	50	150	20	4	5	2	1	3	1
10	50	300	40	6	10	4	0.5	4	2

**Outputs.** For every instance:

- We apply the CPLEX12 library (Table 4) to the general  $PV\_Prod\_VRP$  MILP model, with  $30.S$  *scheduled trips* ( $Card(\Sigma) = 30.S$ ). Then we get (in less than 1 CPU h), a lower bound  $LB\_G$ , an upper bound  $UB\_G$ , CPU time  $T\_G$ , and the value  $Relax$  of the rational relaxation at the root.
- We solve (Table 5) the *Elementary\_Trip* ILP, with  $\gamma = A^{Mean}/2$ , while using the branch and cut algorithm of Section 3.2.2 We get lower and upper bounds  $LB\_SI$  and  $UB\_SI$ , CPU time  $T\_SI$  and the number  $C\_SI$  of SNS cuts generated during the process. We also provide the upper bound  $UB\_W$  obtained without the SNS constraints.
- We apply (Table 6) the global  $VRP\_Surrogate$  resolution scheme while relying on the pricing mechanism and while setting  $\gamma = A^{Mean}/2$  and  $Q_{i,n} = Q^{Stand}_{i,n}$  for any  $i, n$ . We denote by  $W\_Price$  resulting  $PV\_Prod\_VRP$  value. We do the same (Table 6) with 8 combinations ( $\gamma, \rho_1, \rho_2$ ) and denote by  $W\_Price\_8$  resulting value.
- We apply (Table 6) the global  $VRP\_Surrogate$  resolution scheme while relying on machine learning and denote by  $W\_ML$  related value.

Those results may be summarized as follows:

Table 4: Behavior of the  $PV\_Prod\_VRP$  MILP model.

Inst.	$LB\_G$	$UB\_G$	$T\_G$	$Relax$
1	735.3	735.3	628.4	344.5
2	951.3	951.3	265.3	312.0
3	709.2	999.2	159.7	413.5
4	322.1	508.2	3600	208.7
5	969.0	969.0	1819.3	611.7
6	1153.4	1486.3	3600	764.5
7	1972.7	3065.5	3600	1652.3
8	3386.9	4211.8	3600	3008.2
9	4923.0	9594.3	3600	4021.2
10	5956.6	8560.3	3600	5365.4

**Comments:** As expected, the global ILP model is in trouble, even on small instances. Still, further results (Table 7) will make appear that, at least for small  $M$ , upper bound  $UB\_G$  looks close to optimality.

Table 5: Behavior of the *Elementary Trip Branch/Cut*.

Inst.	$LB\_SI$	$UB\_SI$	$T\_SI$	$C\_SI$	$UB\_W$
1	884.4	884.4	1622.5	28	884.4
2	1435.3	1488.3	1 h	67	1522.5
3	1005.9	1202.9	1 h	79	1325.0
4	774.6	774.6	1254.8	503	880.3
5	1516.2	1654.2	1 h	468	1816.2
6	1886.3	1986.3	1 h	1435	2187.0
7	3891.7	4491.7	1 h	2960	4702.8
8	5020.3	5520.3	1 h	4121	Fail
9	12023.6	14077.2	1 h	4689	Fail
10	14773.0	17773.9	1 h	2867	Fail

**Comments:** *Strong no\_sub\_tour* constraints significantly increase our ability to manage the *Elementary\_Trip* problem through branch and cut. Notice that, since the *Elementary\_Trip* model tends to overestimate the energy purchase value, values *UB\_S1* are significantly larger than values *UB\_G* obtained in Table 4.

Table 6: Behavior of the Surrogate Components.

Inst.	UB_G	W_Price	W_Price8	W_ML
1	735.3	735.3	735.3	740.8
2	951.3	966.8	951.3	980.6
3	999.2	1010.3	995.0	1030.0
4	508.2	512.6.2	504.3	508.2
5	969.0	986.5	972.6	1040.2
6	1486.3	1487.0	1487.0	1512.5
7	3065.5	3025.7	3003.5	3197.3
8	4211.8	4225.8	4200.6	4354.6
9	9594.3	9397.7	9365.9	9456.1
10	8560.3	8508.9	8475.1	

**Comments:** Solving *PV\_Prod\_VRP* while relying on the parametric pricing mechanism often behaves better than the *PV\_Prod\_VRP* MILP. As for the the machine learning oriented approach, the gap between our best *PV\_Prod\_VRP* value and *W\_ML\_ILP* is in average around 4%, with a peak at 7%.

## 5 CONCLUSIONS

We dealt here with synchronization between consumption and production. We shortcut the production sub-problem and replaced it by a parametric surrogate sub-problem. But since in true life solar energy production forecasting involves a uncertainty, going further with machine learning could help us in managing related risk of failure.

## ACKNOWLEDGEMENTS

We thank both Labex IMOBS3 and PGMO program for funding this research.

## REFERENCES

Adulyasak Y., Cordeau J.F., Jans R. (2015).The production routing problem: formulations and solution algorithms. *Computers & Operations Research*, 55 , p 141-152.  
 Albrecht A., Pudney P. (2013). Pickup and delivery with a solar-recharged vehicle. *Ph.D. thesis Australian Society for Operations Research* .

Bendali F., Mole Kamga E., Mailfert J., Quilliot A., Toussaint H. (2021): Synchronizing Energy Production and Routing; *RAIRO-OR*, 55 (4), pp. 2141-2163.  
 Caprara A., Carvalho M., Lodi A., Woeninger G.J (2014). A study on the bilevel knapsack problem; *SIAM Journal on Optimization* 24 (2), p 823-838.  
 Balbiyad S. (2019). Collective self consumption: computing the optimal energy distribution coefficients under local energy management; *Report ENSTA/EDF OSIRIS*.  
 Chen L., Zhang G. (2013). Approximation algorithms for a bi-level Knapsack problem; *TCS* 497, p 1-12.  
 Colson B., Marcotte P., Savard G. (2005). Bi-level programming: A survey; *4OR* Vol 3 (2), p 87-107.  
 Deb S., Tammi K., Kalita K., Mahanta P. (2018). Impact of electric vehicle charging station load on distribution network; *Energies* 11 (1), p 178-185.  
 Dempe S., Kalashnikov V., Perez-Valdez G., Kalashikova N. (2015).: *Bi-level Programming Theory*, Springer.  
 Erdelic T., Caric T. (2019). A survey on the electric vehicle routing problem: Variants and solutions. *Journal of Advanced Transportation*.  
 Grimes S., Varghese O., Ranjan S. (2008). Light, water, hydrogen: The solar generation of hydrogen by water photoelectrolysis. *Springer-Verlag US*.  
 Irani S., Pruhs K. (2003). Algorithmic problems in power management. *SIGACT News*, 36, 2, p 63-76.  
 Kleinert T., Labbé M., Ijubic I., Schmidt M. (2021). A survey on MILP in bilevel optimization. *EURO Journal on Computational Optimization* 9, 21 p.  
 Koc C., Jabali O., Mendoza J., G.Laporte G. (2019). The electric vehicle routing problem with shared charging stations. *ITOR*, 26 , p 1211-1243.  
 Krzyszton M. (2018). Adapative supervisor: method of reinforcement learning fault elimination by application of supervised learning. *Proc. FEDCSIS AI*, p 139-149.  
 Luthander R., Widen J., Nilsson D., Palm J. (2015). Photovoltaic self-consumption in buildings: A review; *Applied Energy* 142, p 80-94.  
 Smarter Together (2016). Reports on collective self-consumption of photo-voltaic; *Smarter Together*.  
 Trotta M., Archetti C., Feillet D., Quilliot A. (2022). A pickup and delivery problem with a fleet of electric vehicles and a local energy production unit; *Proc. Triennial Symposium TRISTAN*, 6 pages.  
 Wojtuziak G., Warden T., Herzog O. (2012). Machine learning in agent based stochastic simulation; *Computer and Mathematics with App.* 64, p 3658-3665.