Integrated Production and Energy Supply Planning on an Industrial Site by Mixed-Integer Linear Programming

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Abstract: Industrial production sites are nowadays faced with two major concerns: the need to reduce the environmental impact of their processes and the economic difficulties caused by the rising energy prices. Both challenges may be partially tackled by powering industrial processes with electricity generated on-site from renewable sources. However, the volatility of the electricity prices and the intermittence of the locally generated renewable energy sources result in the need to solve an integrated industrial production and energy supply planning problem. This work investigates a single-machine multi-product lot-sizing problem for an industrial process powered by both grid electricity and on-site renewable energy. We propose a new extension of the Proportional Lot-sizing and Scheduling Problem relying on a two-level structure for time discretization. The first level is related to the product demand satisfaction, the second one is used for both production and energy supply planning. The proposed extension is compared to a previously published extension of the General Lot-sizing and Scheduling Problem dealing with a similar problem. Our preliminary numerical results show that in most cases, our model provides a production plan of the same cost, but with a significantly reduced computational effort.

1 INTRODUCTION

Industrial companies are increasingly under pressure to mitigate the CO2 and pollution emissions linked to the manufacturing of industrial products. They are also confronted with a sharp rise of the price of conventional energy sources (gas, grid electricity...) so that the availability and affordability of energy is becoming a critical parameter in manufacturing. One way to deal with these two challenges consists in powering industrial processes with electricity generated on-site from renewable sources.

In an industrial production plant, the energy management system is usually limited to two elements: the energy bought from an outside supplier on the national market and the energy consumed by the manufacturing processes. As conversion devices (e.g. wind turbines, photovoltaic panels) able to produce electric power from renewable energy sources are becoming less expensive, it is now possible for an industrial plant to build its own environmental friendly energy system and to use this 'green' electricity to power its processes. For example, more than 30000 solar panels have been installed on the rooftop of Bentley's factory in Crewe as well as on the roof of its carport, which provide up to two third of the factory's energy demand with a power generation capacity of 7.7kW. In California, the brewery factory Anheuser-Busch's Budweiser installed 6500 solar panels as well as wind turbines, which provide 30% of the factory's electricity use. Batteries may also be installed to store electricity. However, the intermittence of renewable energy sources (wind, sun...) makes it impossible to fully replace gas or grid electricity by on-site generated electricity to power an industrial process: both types of energy should thus be used in combination. Moreover, the time-of-use pricing scheme widely used by electricity providers means that it is necessary to accurately track the timing and quantity of grid electricity bought from these providers. In this circumstance, making sure that energy supply and consumption are balanced at all time is challenging. Thus, an integrated energy supply and industrial production planning problem needs to be solved.

With all these challenges mentioned above, energy-efficient production planning has attracted great research attention in the last decades (Terbrack et al., 2021; Gao et al., 2020). Some works consider that all the needed energy comes from electricity bought from the main electric grid at a time-varying price and introduce energy consumption costs in the objective function of the production planning model:

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see e.g. (Johannes et al., 2019). Other works study the case where energy can be obtained from both the main electric grid and a non-adjustable on-site electricity generation system based on renewable energy sources. Most of these studies focus on the short-term scheduling of a job shop (Golpîra et al., 2018) or a flow shop (Biel et al., 2018; Wang et al., 2020). However, lot-sizing in the presence of intermittent renewable energy sources has been rarely studied in the literature. (Rodoplu et al., 2019) addressed a single-item multi-machine lot-sizing problem. Chance constraints were introduced to deal with the fluctuation of renewable energy sources. A lot-sizing and scheduling problem was investigated in (Wichmann et al., 2019), with consideration of renewable energy resources and energy storage system.

The present work focuses on a short-term production planning problem, namely a multi-item single-machine lot-sizing problem, and seeks to integrate the energy supply planning into the problem modeling. Basically, a lot-sizing problem is a production planning problem in which we seek to find the optimal trade-off between the startup and inventory holding costs. Startup costs are fixed costs occurring every time the machine's status changes in order to process of a new type of product, while inventory holding costs represent the costs of keeping finished products in inventory between the time they are produced and the time they are used to satisfy the customers' demand. In our case, the energy supply system comprises three main elements: on-site power generation devices producing a time-varying amount of green electricity from renewable energy sources, an on-site energy storage system and the main electric grid with which electricity may be traded at a time-varying price. See Figure 1 for a graphical representation of the studied integrated energy supply and industrial production system.

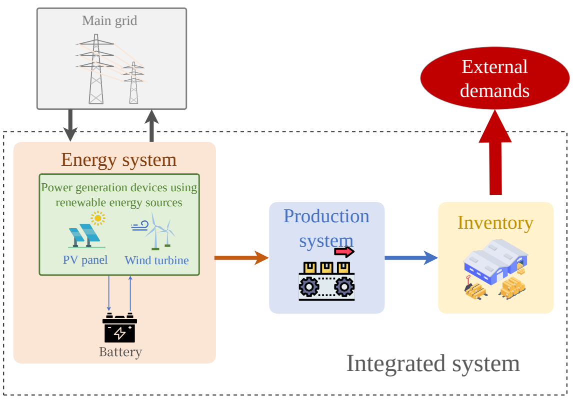

Figure 1: Integrated energy supply and industrial production system.

One modelling difficulty here comes from the fact that the time discretization needed to manage product demand satisfaction, production planning and energy supply may significantly differ. Indeed, following the terminology introduced by (Copil et al., 2017), we have to handle two exogenous time structures, i.e. two sets of points in time at which externally given events, that are defined by the data of the model, are considered. The first time structure is imposed by the timing of the external demand and usually uses rather large time buckets (typically days or weeks). The second one is imposed by the discrete time grid used to track the availability of the on-site generated electricity and the varying price of grid electricity. This time grid usually uses much smaller time buckets (typically hours or 10-minutes intervals). In between these two exogenous time structures, to plan production, we have to handle a third endogenous time structure representing the points in time at which internal events are captured by decision variables.

As mentioned above, the problem of determining optimal production lot sizes under intermittent renewable energy has been scarcely studied in the literature. A noticeable exception can be found in (Wichmann et al., 2019). This paper namely investigates an integrated industrial production and energy supply planning problem for a manufacturing site powered by a decentralized power system using renewable energy sources. Their problem modeling relies on a three-level time structure: a coarse time discretization to track the customers' demands, a fine energy-oriented time discretization to track the variations in the energy price and availability, and another flexible production-oriented fine discretization to determine the production plan. The resulting model can be seen as an extension of the General Lot-sizing and Scheduling Problem (GLSP) introduced by (Fleischmann and Meyr, 1997) and takes the form of a large-size mixed-integer linear program (MILP) formulation involving many big-M constraints. Consequently, numerical difficulties arise when trying to solve medium-size instances of the problem with a mathematical programming solver.

This is why we propose in this paper a new model for the integrated energy supply and industrial production planning problem. This model relies on a two-level time structure. This one consists in a coarse time discretization to track the demand satisfaction and in a single fine time discretization based on the exogenous time structure imposed by the energy-related input data. This fine time discretization is used both to track the variations in the energy price and availability and to plan the industrial production. The use of a fixed time grid involving short planning periods to determine the production lot sizes leads to the formulation of a 'small bucket' lot-sizing problem, i.e. to the formulation of a lot-sizing problem in which there are some restrictions on the number of item types that may be produced in each planning period. More precisely, we consider here the Proportional Lot-sizing and Scheduling Problem (PLSP) in which at most two different types of product may be produced in each period: see (Drexl and Haase, 1995). The main advantage of the proposed modeling approach is that it leads to an MILP formulation much smaller that the one obtained with the model proposed in (Wichmann et al., 2019).

Our contributions are thus twofold. First, we propose a new extension of the PLSP in order to simultaneously plan the industrial production and the energy supply in a manufacturing plant powered, at least partially, by a decentralised energy system using renewable energy sources. Second, we carry out numerical experiments to compare the proposed model with the GLSP-based model previously published by (Wichmann et al., 2019). Our numerical results show that in most cases, our model provides a production plan of the same cost than the one provided by the model introduced by (Wichmann et al., 2019) but with a significantly reduced computational effort.

The paper is organized as follows. Section 2 provides a detailed description of the optimization problem under study. Section 3 presents the proposed PLSP-based model based on a two-level time structure. Section 4 discusses the results of the computational experiments conducted on medium-size randomly generated instances. Conclusion and research perspectives are given in Section 5.

2 PROBLEM DESCRIPTION

We seek to plan production over a finite horizon in an industrial plant producing J types of item. The set of items is denoted as $\mathcal{J} = \{1,2,\dots,J\}$. A first exogenously imposed time grid is introduced to track the points in time at which the external customers' demand occurs. The planning horizon is thus divided into a set $\mathcal{T} = \{1,2,\dots,T\}$ of macroperiods of fixed length l: the end of each macroperiod corresponds to a point in time at which demand may occur. Let $d_{j,t}$ denote the demand for item j to be satisfied at the end of macroperiod t. Backlogging is not allowed, i.e., all the demand should be satisfied on time. At the end of each macroperiod, the finished products remaining after the demand has been satisfied are held in inventory in the warehouse for later use. Each unit of item j held in inventory between two macroperiods incurs a unit cost h_j. The initial inventory of item j is given by $\bar{I}_{j,0}$.

We assume that the production process comprises a single production resource. This resource may be in three distinct kinds of states: idle, setup for a given item, and manufacturing a given item. The resource is idle when it is turned on but not ready for production. It may also be setup for a specific item: in this case, it is ready to produce this type of item but is not producing it. Finally, the resource is in a item-related manufacturing state when it is actually producing this specific type of item: this is however possible only if the resource is already set up for this type of item. Let k_j be the amount of time needed to produce one unit of item j on the resource. A startup is the action of changing the machine status to prepare it to produce a new type of item. A startup for item j thus occurs whenever the state of the resource is switched from any other idle, setup or manufacturing state to the setup state corresponding to this item. Let f_j be the fixed startup cost to be paid each time a startup for item j happens. All startup times are assumed to be negligible.

Energy is consumed by startups and manufacturing. The amount of energy consumed during a startup, denoted by e^f_j, depends on the product that the machine will be setup for. The energy consumed during manufacturing is proportional to the number of items produced. Let e_j denote the energy needed to produce one unit of item j. We assume that no energy is consumed when the machine is idle or when it is setup for a given product but not producing.

The energy is supplied either from the main grid or from the on-site renewable energy conversion devices. The necessity to track the time variations of the main grid energy price and of the amount of energy generated by the on-site power system leads us to consider a second exogenously imposed time grid. This one is based on planning periods sufficiently short to assume that the energy price and the green energy power generation are constant during a period. Each macroperiod $t = 1,...,T$ of length l is thus split into n energy-oriented microperiods of fixed length l^e such that $l = nl^e$. Let $\mathcal{R} = \{1,...,nT\}$ be the set of all energy-oriented microperiods and $\mathcal{R}_t = \{(t-1)n+1,...,tn\}$ be the set of energy-oriented microperiods belonging to macroperiod t. We denote by e^P_r (resp e^S_r) the purchasing (resp. selling) price of the energy traded with the main grid during microperiod r. Since the voltage of electricity used for long-distance transmission by the grid operator is not the same as the one used in the manufacturing plant, a grid transformer is needed to adapt the electricity to production use. This causes energy losses mostly due to the electrical current flowing in the coils and the magnetic field alternating. We denote by η_G the efficiency of the grid transformer. The amount of green electricity generated on-site by the local energy system during micro-period r is denoted by p_r^R.

Finally, the energy storage system consists of a battery which can store the residual electricity generated by the on-site power conversion devices and the energy bought from the main grid. Let c^B be the maximum amount of energy that can be stored in this system. The amount of energy which can be charged into (resp. discharged from) the battery during an energy-oriented microperiod is denoted by m_C (resp. m_D). Battery charging/discharging incurs some energy loss due to the internal resistance: let η_C/η_D be the charging/discharging efficiency of the battery.

The optimization problem under study here consists in building an integrated energy supply and production plan. This plan should simultaneously determine when and how many finished products to produce and when and how much energy to trade with the grid and to charge into/discharge from the battery. This plan should ensure that the demand for the finished products is satisfied at all time and that there is always enough energy to supply the manufacturing process while minimizing the total production and energy procurement costs.

3 PLSP-BASED MODEL

As will be shown by the numerical experiments to be provided in Section 4, using the GLSP-based model previously published by (Wichmann et al., 2019) to build a production and energy supply plan is possible only for small instances. Significant computational difficulties are encountered when trying to solve medium to large size instances. These difficulties mainly come from the sharp increase in the size of the MILP formulation with the number of items and horizon length. In order to overcome this, we propose a new PLSP-based model for the problem relying on the use of small buckets to plan production.

3.1 Model Description

The main idea underlying the proposed PLSP-based model consists in aligning the production-related time discretization on the energy-related time discretization and to define the production plan using the exogenously defined energy-oriented microperiods. The use of a fixed time grid involving short planning periods to determine the production lot sizes leads to the formulation of a 'small bucket' lot-sizing problem, i.e. to the formulation of a lot-sizing problem in which there are some restrictions on the number of item types that may be produced in each planning period. More precisely, we consider here the Proportional Lot-sizing and Scheduling Problem (PLSP) in which at most two different types of item may be produced in each energy-oriented microperiod. The use of a fixed time grid (rather than a flexible one as done in the GLSP) to plan production significantly simplifies the mathematical formulation of the problem as it eliminates the need to manage the intersections between energy-oriented and production-oriented microperiods, which requires a large number of additional binary variables and big-M type constraints.

In order to formulate the problem as an MILP, we introduce the following decision variables:

- $I_{j,t}$: amount of item j held in inventory at the end of macroperiod t,
- $Y_{j,r}$: setup state of the resource at the end energy-oriented micro-period r; $Y_{j,r}=1$ if the resource is setup to produce item j at the end of r, $Y_{j,r}=0$ otherwise.
- $X_{j,r}$: startup indicator. $X_{j,r}=1$ if there is a startup for item j during r, $X_{j,r}=0$ otherwise,
- $Q_{j,r}$: quantity of item j produced in energy-oriented microperiod r.
- P_r^C/P_r^D: amount of energy charged into / discharged from the battery during energy-oriented microperiod r,
- P_r^B: state of charge of the battery at the end of energy-oriented microperiod r,
- P_r^{GP}/P_r^{GS}: amount of electricity purchased from / sold to the main grid during energy-oriented microperiod r.

3.2 MILP Formulation

Objective Function. The objective function aims at minimizing the total startup, inventory holding and energy costs over the planning horizon.

$$\min \sum_{r=1}^{R}\sum_{j=1}^{J} f_j \cdot X_{j,r} + \sum_{t=1}^{T}\sum_{j=1}^{J} h_j \cdot I_{j,t} + \sum_{r=1}^{R}(e_r^P \cdot P_r^{GP} - e_r^S \cdot P_r^{GS}). \quad (1)$$

Production-Oriented Constraints. We first focus on the set of production-oriented constraints. These constraints aim at building a feasible production plan.

$$I_{j,t} = I_{j,t-1} + \sum_{r\in\mathcal{R}_t} Q_{j,r} - d_{j,t}, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}. \quad (2)$$

$$k_j Q_{j,r} \leqslant l(Y_{j,r-1} + Y_{j,r}), \quad \forall j \in \mathcal{J}, r \in \mathcal{R}. \tag{3}$$

$$\sum_{j=1}^{J} k_j Q_{j,r} \leqslant l, \quad \forall r \in \mathcal{R}. \tag{4}$$

$$\sum_{j=1}^{J} Y_{j,r} \leqslant 1, \quad \forall r \in \mathcal{R}. \tag{5}$$

$$X_{j,r} \geqslant Y_{j,r} - Y_{j,r-1}, \quad \forall j \in \mathcal{J}, r \in \mathcal{R}. \tag{6}$$

$$X_{j,r} + Y_{j,r-1} \leqslant 1, \quad \forall j \in \mathcal{J}, r \in \mathcal{R}. \tag{7}$$

$$Y_{j,0} = 0, \quad \forall j \in \mathcal{J}. \tag{8}$$

$$I_{j,0} = \bar{I}_{j,0}, \quad \forall j \in \mathcal{J}. \tag{9}$$

$$I_{j,T} \geqslant I_{j,0}, \quad \forall j \in \mathcal{J}. \tag{10}$$

Constraints (2)-(6) define the production plan according to the assumptions used in the PLSP. Constraints (2) are the inventory balance equations. Inequalities (3) ensure that item j can be produced in microperiod r only if the machine is set up for j at the beginning or/and at the end of the period. Constraints (4) make sure that the total capacity consumed by the production taking place during each microperiod r stays below the capacity available in r. Equations (5) guarantee that the machine is set up for at most one item at the end of each microperiod. Together with Constraints (3), they ensure that at most two items may be produced during a microperiod r. The relationship between startup and setup decision variables is described by (6): if the machine is setup for item j at the end of r but was not setup for j at the end of the $r-1$, a startup for j must occur during r. (7) are basic valid inequalities that may be used to strengthen the MILP formulation: see (Belvaux and Wolsey, 2001). They indicate that if the machine was already set up for j at the end of $r-1$, then there is no need to carry out a start up for item j during r. Inversely, if a startup for product j occurs in r, it means the machine was not set up for this product in $r-1$. Constraints (8)-(10) define the initial and final conditions.

Energy-Oriented Constraints. We now focus on the set of energy-oriented constraints aiming at building a feasible energy supply plan in each energy-oriented micro-period.

$$P_r^U + \frac{P_r^{GS}}{\eta_G} + \frac{P_r^C}{\eta_C} = \eta_G \cdot P_r^{GP} + p_r^R + \eta_D \cdot P_r^D, \quad \forall r \in \mathcal{R}. \tag{11}$$

$$P_r^B = P_{r-1}^B + P_r^C - P_r^D, \quad \forall r \in \mathcal{R}. \tag{12}$$

$$0 \leqslant P_r^B \leqslant c^B, \quad \forall r \in \mathcal{R}. \tag{13}$$

$$P_r^C \leqslant m_C, \quad \forall r \in \mathcal{R}. \tag{14}$$

$$P_r^D \leqslant m_D, \quad \forall r \in \mathcal{R}. \tag{15}$$

Equalities (11) ensure that the energy supply and demand is balanced within each microperiod. The energy demand consists in the energy consumed by the manufacturing process P_r^U, the energy sold to the main grid P_r^{GS}/η_G and the energy charged into the battery P_r^C/η_C. Note how, since part of the energy is lost within the grid transformer, the amount of energy needed to sell P_r^{GS} to the main grid is given by P_r^{GS}/η_G. Similarly, the actual energy needed to charge P_r^C into the battery is computed by P_r^C/η_C due to the losses during the battery charging process. The energy supply consists in the energy purchased from the grid $\eta_G \cdot P_r^{GP}$, the energy generated from the renewable resources p_r^R and the energy discharged from the battery $\eta_D \cdot P_r^D$. Note that, when we buy P_r^{GP} from the main grid, only $\eta_G \cdot P_r^{GP}$ energy is really available for the plant. The same applies for the energy discharging process. The energy balance in the battery is defined by Equations (12). The amount of energy stored in the battery at the end of microperiod r is equal to the amount stored at the beginning of r plus the amount charged into the battery during r minus the amount discharged from s. Constraints (13) impose that the amount of energy stored in the battery does not exceed its capacity. Inequalities (14) and (15) are the battery maximum charging and discharging rate constraints.

Coupling Constraints. The final set of constraints links together the decision variables related to production and the one related to energy supply.

$$P_r^U = \sum_{j=1}^{J} e_j^f \cdot X_{j,r} + \sum_{j=1}^{J} e_j \cdot Q_{j,r}, \quad \forall r \in \mathcal{R}. \tag{16}$$

Equations (16) guarantee that the energy consumption by the factory equals to the sum of the energy consumed for the startups and for the production on the machine.

Domain Definition Constraints. Finally, the definition domain of the decision variables are given by Constraints (17)- (19).

$$X_{j,r}, Y_{j,r} \in \{0,1\}, \forall j \in \mathcal{J}, r \in \mathcal{R}, \tag{17}$$

$$I_{j,t}, Q_{j,r} \geqslant 0, \forall j \in \mathcal{J}, t \in \mathcal{T}, r \in \mathcal{R} \quad (18)$$

$$P_r^C, P_r^D, P_r^{GP}, P_r^{GS}, P_r^B, P_r^U \geqslant 0, \forall r \in \mathcal{R}. \quad (19)$$

4 COMPUTATIONAL EXPERIMENTS

In this section, numerical experiments are conducted to assess the performance of the proposed PLSP-based model and compare it with the one of the GLSP-based model proposed in (Wichmann et al., 2019). The procedure used to randomly generate a set of instances is described in Subsection 4.1. Preliminary computational results are given and discussed in Subsection 4.2.

4.1 Instances

The parameter setting used to randomly generate instances is mostly based on the numerical values provided in (Wichmann et al., 2019). As this paper did not involve energy losses in the grid transformer nor during the battery charging/discharging process, we use the values provided in (Zhang et al., 2017) and (Golpîra et al., 2018) for these parameters.

The production system comprises a single machine producing a set of products on a finite planning horizon spanning several days. Each day is divided into two macroperiods, each one with a length of l =480 minutes: these two macroperiods correspond respectively to a morning and evening eight-hour shift. We consider three set of instances of various sizes. Small-size instances involve $J = 3$ products and a two-day planning horizon of $T = 4$ macroperiods. Medium-size instances involve $J = 5$ products and eight days, i.e., $T = 16$ macroperiods. Finally, large-size instances involve $J = 10$ products and a 16-days planning of $T = 32$ macroperiods.

For any item $j = 1,...,J$, a setup of the machine costs $f_j = 200$€ and consumes $e_j^f = 10$kWh. Moreover, producing one unit of this item takes $k_j = 0.05$ minutes and consumes $e_j = 0.1$kWh. The production capacity of the machine within one macroperiod can be computed as $Cap = l/k_j = 480/0.05 = 9600$. The demand for each item $j = 1,...J$ is randomly generated as follows. We consider a utilization rate of the resource of $\rho = 0.8$ and set the mean value of the total demand to be satisfied in each macroperiod $t = 1,,...,T$ to $\rho Cap = 0.8 * 9600$ items. For each item $j = 1,...,J$, the demand $d_{j,t}$ is generated according to a Normal distribution of mean $\frac{\rho Cap}{J}$ and standard deviation $\frac{\rho Cap}{3J}$. In case the randomly generated value is negative, we replace it by 0. We thus have $d_{j,t} \sim \max(int(\mathcal{N}(\frac{\rho Cap}{J}, \frac{\rho Cap}{3J})), 0)$. The initial inventory of item j is randomly generated following the uniform distribution using $\bar{I}_{j,0} \sim randint(0, 2 * d_{j,1})$. The unit inventory holding cost of item j is set to $h_j = 0.05$€ per macroperiod. In the PLSP model, the production-oriented microperiods are identical to the energy-oriented ones. In the GLSP model, each macroperiod t may be flexibly divided into a predefined number $|\mathcal{S}_t|$ of production-oriented microperiods of variable length: we set $|\mathcal{S}_t| = 7$ for each t.

Regarding the energy supply and consumption, each macroperiod t is split evenly into $|\mathcal{R}_t| = 8$ energy-oriented microperiods of l^e =60min length. The energy price is assumed to display intra-day variations but to be otherwise daily periodic. The unit energy price for each hour (indexed from 1 to 16) of a given day in our reference scenario is displayed in Table 1. Moreover, we consider in our experiments two additional scenarios: a low (resp. extreme low) energy price scenario in which the reference unit price in every period is divided by 10 (resp. by 100). The amount of on-site generated electricity depends on the weather and thus varies both within the day and from one day to the next. A microperiod indexed by r corresponds to the hour indexed $r_d = r(mod16)$ of the day. The value of p_r^R for microperiod r is randomly generated using a Normal distribution with an expected value and a standard deviation equal to $Gen[r_d]$: see Table 1. Note that the values of $Gen[r_d]$ are consistent with a power generation by PV panels. We thus have $p_r^R \sim \max(\mathcal{N}(Gen[r(mod16)], Gen[r(mod16)]), 0)$. The energy storage system has a capacity of $c^B =$ 500kWh. Within each microperiod r, the maximum amount of energy that can be charged into or discharged from the battery is set to $m_C = m_D =$ 250kWh. The charging and discharging efficiencies, as well as the grid transformer efficiency, are set to $\eta_C = \eta_D = \eta_G = 0.95$.

4.2 Preliminary Numerical Results

For each considered instance size and energy price level, we randomly generated 10 instances, resulting in a total number of 90 instances. Each instance was solved with the MILP solver CPLEX 20.10 using either the GLSP-based model described in (Wichmann et al., 2019) or the PLSP-based model described in Section 3. The implementation was done in Python. The computational experiments were carried out on a laptop running under Windows 10 with an Intel(R) Core(TM) i5-6300HQ CPU @ 2.30GHz processor and 16 GB RAM. The time limit was set to 20 min-

Table 1: Energy prices and generated electricity in one day.

Time of a day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Energy price	4.8	6.1	6.3	6.0	5.6	4.0	3.7	3.8	4.5	5.1	5.4	5.9	6.4	6.3	5.5	4.5
$Gen[r_d]$	1	4	10	18	25	27	30	30	25	15	5	2	0	0	0	0

Table 2: Optimization results for small-size instances with $J = 3$, $T = 4$.

Energy price level	initial		low		extremely low	
Model	PLSP	GLSP	PLSP	GLSP	PLSP	GLSP
# VAR	563	7779	563	7779	563	7779
# BINVAR	259	3875	259	3875	259	3875
# CONS	497.9	9629.9	500.1	9627.9	498.7	9627.7
Z_{best}	15567.60	15567.60	3515.14	3515.10	2732.36	2732.25
Gap_{LP}	4.37%	4.86%	13.74%	14.04%	18.91%	19.01%
Gap_{MIP}	0.01%	0.18%	0.01%	0.31%	0.00%	0.02%
Computation time	1.53	1029.92	1.23	1179.20	1.41	924.67

Table 3: Optimization results for medium-size instances with $J = 5$, $T = 16$.

Energy price level	initial		low		extremely low	
Model	PLSP	GLSP	PLSP	GLSP	PLSP	GLSP
# VAR	3035	175163	3035	175163	3035	175163
# BINVAR	1541	87621	1541	87621	1541	87621
# CONS	2890.4	204144	2913.4	204144	2915	204144
Z_{best}	59912.29		14209.03		9999.33	
Gap_{LP}	3.84%	-	10.63%	-	14.38%	-
Gap_{MIP}	1.41%	No solution	2.47%	No solution	2.62%	No solution
Computation time	1200.68	-	1152.15	-	1097.88	-

Table 4: Optimization results for large-size instances with $J = 10$, $T = 32$.

Energy price level	initial		low		extremely low	
Model	PLSP	GLSP	PLSP	GLSP	PLSP	GLSP
# VAR	10069	1271349	10069	1271349	10069	1271349
# BINVAR	5642	636234	5642	636234	5642	636234
# CONS	12022.5	1387775	12088.4	1387775	14428.2	1387775
Z_{best}	130087.44		41935.94		32371.44	
Gap_{LP}	5.71%	-	13.75%	-	16.29%	-
Gap_{MIP}	4.74%	No solution	10.98%	No solution	12.39%	No solution
Computation time	1201.63	-	1201.71	-	1201.78	-

utes.

The results for small, medium, and large-size instances are displayed in Tables 2, 3, and 4, respectively. Each column provides the average results over the 10 instances corresponding to the given energy price level. We provide, for each set of 10 instances:

- # VAR/ # BINVAR, the number of variables and binary variables involved in the formulation,
- # CONS, the number of constraints in the formulation,
- Z_{best}, the value of the best feasible integer solution found by the solver in 20min,
- Gap_{LP}, the integrality gap, i.e., the relative difference between the lower bound provided by the linear relaxation of the problem and the value of the best integer feasible solution found by the solver,
- Gap_{MIP}, the optimality gap, i.e., the gap between the best lower bound and the best upper bound found by the solver before the time limit is reached (note that this gap is equal to 0% in case

a guaranteed optimal solution could be found),

- the computation time (in seconds).

We first note that the computational effort needed to solve the problem with a mathematical programming solver is drastically reduced when using the PLSP-based model instead of the GLSP-based model. Thus, for the small instances, the proposed PLSP-based model can be solved to optimality within less than 2s, while the GLSP-based model cannot converge to the optimal solution after 20 minutes of computation: see Table 2. Moreover, for the set of medium and large instances, no feasible solution can be found by the solver in 20min when using the GLSP-based model. In contrast, the PLSP-based model is able to provide feasible solutions of acceptable quality. The integrality gaps seem to be similar for both formulations (see in particular Table 2). Thus, the observed improvement in the computational efficiency may be mainly explained by the significant decrease in the size of the MILP formulation obtained when using the PLSP-based model.

In terms of solution quality, the GLSP-based formulation is more flexible regarding the production plan than the PLSP-based formulation. Namely, the GLSP-based model allows the production of more than 2 different types of items in a given energy-oriented microperiod, which is forbidden in the PLSP-based model. Thus, theoretically, the GLSP-based model may provide less expensive production and energy supply plans. However, in practice, as can be seen in the results reported in Table 2, the value of the optimal solution found by the PLSP-based model is equal or very close to the one found by the GLSP-based model. Thus, the GLSP-based model is able to provide a lower cost production plan than the PLSP-based model in only four instances out of the 30 corresponding instances. Moreover, the maximum observed relative cost difference over these four instances is only 0.05%. This indicates that in most cases, the optimal solution obtained by the PLSP model is of the same cost as the one obtained by the GLSP model.

5 CONCLUSION AND PERSPECTIVES

This paper investigated an integrated lot-sizing and energy supply planning problem in a single-machine multi-product setting. A new MILP model based on an extension of the PLSP was proposed to handle this problem. Our numerical results showed that the proposed model outperforms a previously published GLSP-based model with respect to the computation times. However, the availability of the renewable energy resources heavily depends on weather conditions, so that the corresponding power generation cannot be predicted precisely. Therefore, future work should focus on explicitly taking into account these uncertainties in the problem modeling.

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