

Linear Analysis of National Life Expectancy

Huijia Wang

XJTLU Wisdom Lake Academy of Pharmacy, China

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Abstract: Life expectancy is the most important indicator to measure the population's health status. It is also a comprehensive indicator to measure a country's economic and social development and medical and health service level. It is affected by biological factors, environmental factors, lifestyle, and medical and healthcare systems. This study aims to explore how the national life expectation changes and the differences in regions and potential health care systems. The given dataset and related data are for human development reports. There are 185 countries considered in this study. Besides regional factors, many variables affect life expectancy, such as Total fertility rate (FERTILITY), Health expenditure per capita (HEALTHEXPEND), Public expenditure on education (PUBLICEDUCATION) and so on. In this paper, we developed a simple linear model and a multiple regression model using stepwise regression to explore which potential variables have a more significant effect on life expectancy and test the feasibility and plausibility of the model. We find that region, fertility, health care costs, and public expenditure on education significantly affect national life expectancy. Finally, we developed generalized linear models with different linkage functions for comparison and further analysis.

1 INTRODUCTION

The average life expectancy of the population is an indicator to measure the level of economic development and medical and health services of a society. Two aspects restrict the length of life expectancy. On the one hand, social and economic conditions and health care levels limit people's life spans, so the length of life span varies significantly in different societies and periods. On the other hand, due to personal differences in physique, genetic factors, and living conditions, the length of life of each person also varies greatly. With the improvement of the modern medical system, the health care system has gradually become an essential indicator of life expectancy (Feynman, 1963). Virtually every person, corporation, and government have their perspective on health care; These different perspectives result in a wide variety of systems for managing health care (Dirac, 1953). Comparing different health care systems helps us learn about approvals other than our own, which helps us make better decisions in designing improved systems (Frees, 1993).

An assessment of the poverty, production, and environmental challenges in CARICOM countries

showed that scientists had examined the current human development experience of CARICOM countries, focusing on the interrelated challenges of poverty, production, and environment, which showed that development remained unbalanced. Therefore, the national medical and health services may not be guaranteed accordingly (Lima, 2020). And in some remote and barren countries, the health care system is not perfect. The Brazilian National Cancer Institute (INCA) periodically publishes cancer incidence estimates of the nineteen main cancer sites in Brazil (Perry, 2020). And this report aims to explore how the national life expectancy changes and differs in regions and potential health care systems through R studio.

2 DATA PREPROCESSING

Who is doing health care right? Health care decisions are made at the individual, corporate and government levels. Virtually every person, corporation and government have their own perspective on health care; these different perspectives result in a wide variety of systems for managing health care. Comparing different health

care systems help us learn about approaches other than our own, which in turn help us make better decisions in designing improved systems.

Here, we consider health care systems from n = 185 countries throughout the world from the UNLifeExpectancy.csv for National Life Expectancies from the following website:

<https://instruction.bus.wisc.edu/jffrees/jffreesbooks/Regression%20Modeling/BookWebDec2010/data.html>.

The data is concerned with human development reports. We see <https://hdr.undp.org/en/data> for more details. The details are included in Table 1 below.

Table 1: Data description.

Variable	Number of Obs Missing	Description
REGION	0	Categorical variable for region of the world
COUNTRY	0	The name of the country
LIFEEXP	0	Life expectancy at birth, in years
ILLITERATE	14	Adult illiteracy rate, % aged 15 and older
POP	1	2005 population, in millions
FERTILITY	4	Total fertility rate, births per woman
PRIVATEHEALTH	1	2004 Private expenditure on health, % of GDP
PUBLICEDUCATION	28	Public expenditure on education, % of GDP
HEALTHEXPEND	5	2004 Health expenditure per capita, PPP in USD
BIRTHATTEND	7	Births attended by skilled health personnel (%)
PHYSICIAN	3	Physicians per 100,000 people
SMOKING	88	Prevalence of smoking, (male) % of adults
RESEARCHERS	95	Researchers in R & D, per million people
GDP	7	Gross domestic product, in billions of USD
FEMALEBOSS	87	Legislators, senior officials and managers, % female

As a measure of the quality of care, we use LIFEEXP, the life expectancy at birth. There are 185 countries consider in this study, not all countries

provided information for each variable. Data not available are noted under the column "NA". The descriptive statistics are included in Table 2 below.

Table 2: Descriptive statistics.

	mean	sd	0%	50%	100%	n	NA
LIFEEXP	67.050811	11.081747	40.5	71.0	82.3	185	0
ILLITERATE	17.688304	19.862690	0.2	10.1	76.4	171	14
POP	35.357609	131.698363	0.1	7.8	1313.0	184	1
FERTILITY	3.189503	1.707809	0.9	2.7	7.5	181	4
PRIVATEHEALTH	2.517391	1.329662	0.3	2.4	8.5	184	1
PUBLICEDUCATION	4.694904	2.046379	0.6	4.6	13.4	157	28
HEALTHEXPEND	718.005556	1037.012073	15.0	297.5	6096.0	180	5
BIRTHATTEND	78.252809	26.420077	6.0	92.0	100.0	178	7
PHYSICIAN	146.076923	138.553078	2.0	107.5	591.0	182	3
SMOKING	35.092784	14.399857	6.0	32.0	68.0	97	88
RESEARCHERS	2034.655556	4942.933272	15.0	848.0	45454.0	90	95
GDP	247.551124	1055.692710	0.1	14.2	12416.5	178	7
FEMALEBOSS	29.071429	11.709763	2.0	30.0	58.0	98	87

We conduct the following exploratory analysis concerning the distribution of LIFEEXP and determine if we may do some transformation so as

to symmetrize the distribution. First, we use the histogram to see the distribution of the variable in Figure1. It can be seen from the histogram that this

variable has no obvious symmetrical trend and cannot be considered as normal distribution. Then

we further use the QQ plot to test in Figure2.

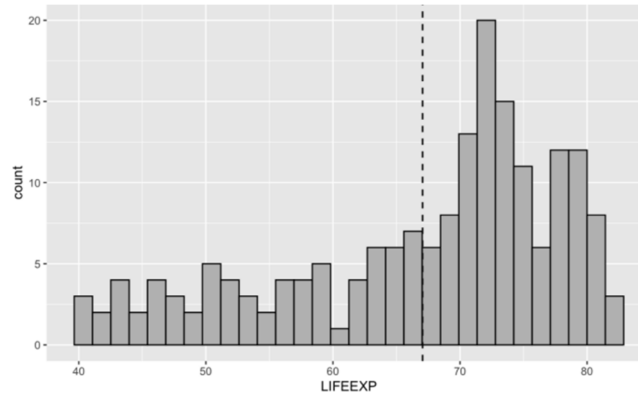


Figure 1: Histogram of LIFEEXP.

When viewing them, the histogram appears to be skewed to the left. The QQ plot indicates a serious departure from normality. If we take a natural log transformation and examine the distribution of this transformed variable when viewing them, the transformation still does little to symmetrize the

distribution. It can also be seen from the correlation matrix that the FERTILITY variable has a strong correlation with LIFEEXP, and the value in the correlation coefficient matrix is the largest, so we can consider establishing a simple linear model of these two variables.

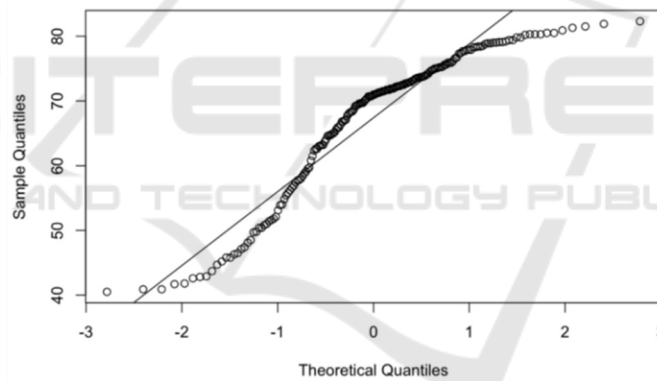


Figure 2: QQplot of LIFEEXP

3 MODEL ANALYSIS

3.1 A simple Linear Model of Life Expectancy

We continue the analysis by examining the relation between $y = LIFEEXP$ and $x = FERTILITY$. Fit a linear regression model of LIFEEXP by FERTILITY. The results are shown as follows.

Table3. Simple linear model regression results

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	83.7381	1.0439	80.22	<2e-16
FERTILITY	-5.2735	0.2887	-18.27	<2e-16

Through observation, we found that $p\text{-value} < 2.2e-16$, so fertility variables have a significant impact on life expectancy. The obtained linear model formula is as follows.

$$\begin{aligned} LIFEEXP &= \hat{\beta}_0 + \hat{\beta}_1 \times FERTILITY \\ &= -5.2735FERTILITY + 83.7381 \end{aligned} \quad (1)$$

3.2 A Multiple Linear Regression Model of Life Expectancy

After considering establishing a simple univariate linear model, we will consider using the basic multivariate linear regression model. According to the analysis and observation of the missing values of the data, we found that the variables RESEARCHERS, SMOKING, and FEMALEBOSS have many missing values, so these columns of variable values should be deleted when establishing the model.

And we can find that the means and standard deviations of the HEALTHEXPEND variable and the GDP variable in the data set are large, more than

several hundred times those of the other variables. At this point, we should consider a log-transformed form. Taking logarithms of the variables in the linear regression model can reduce the absolute differences between the data and the effect of individual extreme values.

Then we use the filtered variables to establish a multivariate linear model. However, not every variable in the model has significant meaning, so we need to carry out stepwise regression to further screen and analyze. From the results of stepwise regression that after two rounds of regression, the variable FERTILITY, PUBLICEDUCATION, and lnHEALTH are significant in the model. So next we consider fitting the regression model with these three variables. The result is in Table4 as follows.

Table 4: Multiple regression model results.

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	60.7626	4.5865	13.248	< 2e-16
lnHEALTH	3.2777	0.5864	5.589	1.09e-07
PUBLICEDUCATION	-0.5341	0.2549	-2.095	0.0379
FERTILITY	-3.2220	0.4934	-6.531	1.02e-09

As we expected in Table3, each variable is significant in the model. The expression of the established multivariate linear model is as follows.

$$\begin{aligned}
 \text{LIFEEXP} &= \beta_0 + \beta_1 \ln\text{HEALTH} \\
 &+ \beta_2 \text{PUBLICEDUCATION} \\
 &+ \beta_3 \text{FERTILITY}
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 &= 60.7626 + 3.278 \ln\text{HEALTH} \\
 &- 0.534 \text{PUBLICEDUCATION} \\
 &- 3.222 \text{FERTILITY}
 \end{aligned}$$

Also we can do diagnosis plots with this model. The results are in Figure3, Figure4, Figure5, Figure6. It shows that most assumptions on the linear models are consistent with the data information.

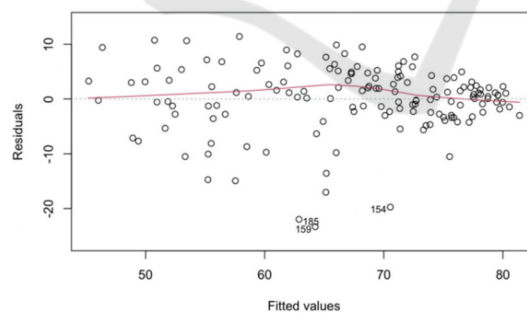


Figure 3: Residuals vs Fitted.

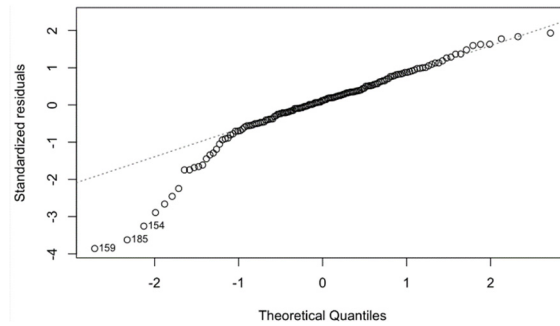


Figure 4: Normal QQplot.

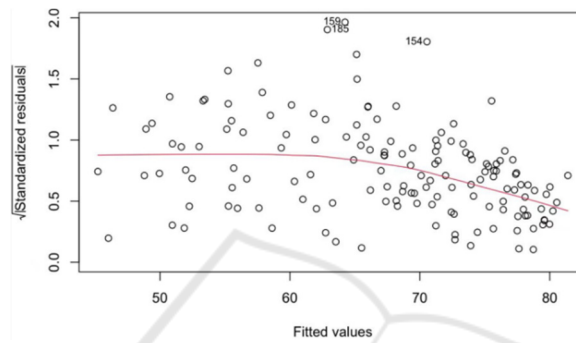


Figure 5: Scale-Location.

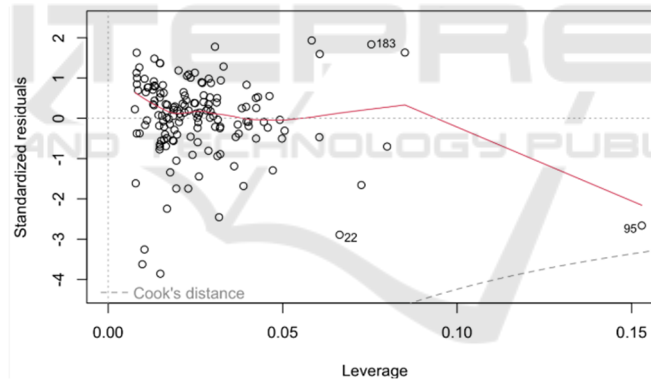


Figure 6: Residuals vs Leverage.

Specifically, we conclude the following:

1.First, the standardized residuals have no apparent trend in its fitted values. Thus, the heteroscedasticity of the response variable is not significantly shown.

2. Second, we see that the normality of the residuals seems acceptable according to the normal QQ plot, besides the 154th,159th and 185th observations. Since the empirical quantile based on normal distribution is fairly close to the theoretical quantile.

3.Finally, we can see that there is no correlation between the residual series and prices. Therefore,

the independence assumption seems to be acceptable.

3.3 Model Improvement and Variable Selection

The study in the previous section shows that national life expectancy is significantly associated with the total fertility rate, public expenditure on education, and 2004 health expenditure per capita. As we want to explore how the national life expectancy changes and differs in regions and

potential health care systems, in this section we will add the region variable to the model.

Then we added the categorical variable REGION to the initial model and again performed stepwise regression to filter the new model. From the results, we found that FERTILITY, REGION, PHYSICIAN, PUBLICEDUCATION and lnHEALTH have a significant impact.

We take the Variance Inflation Factor to check if there is any obvious collinearity. As we know the larger VIF implies high correlation between the independent variable and the remaining variables. It means that different combinations of the covariates

may lead to the same fitted values, which is therefore mainly a problem for interpretation rather than prediction, and cause numerical problems during the fitting process. For this model, variable PHYSICIAN has a VIF of 2.60. and a maximum correlation of 70.1% with the other variables. Therefore, the removal of this variable could be considered for the model.

Then we fit a regression model using three explanatory variables, FERTILITY, PUBLICEDUCATION, and lnHEALTH, as well as the categorical variable REGION. The result is in Table5.

Table 5: Multiple regression model results (add REGION).

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	61.4123	64.3493	14.120	< 2e-16
PUBLICEDUCATION	-0.5492	0.2416	-2.273	0.0245
lnHEALTH	3.8136	0.5702	6.688	4.58e-10
FERTILITY	-3.0003	0.4705	-6.376	2.28e-09
REGION	-0.8802	0.2100	-4.191	4.80e-05

All variables are significant in this model. And the expression of the established multivariate linear model is as follows.

$$\begin{aligned} \text{LIFEEXP} &= \beta_0 + \beta_1 \ln\text{HEALTH} + \beta_2 \text{PUBLICEDUCATION} + \beta_3 \text{FERTILITY} + \beta_4 \text{REGION} \\ &= 61.4123 + 3.8136 \ln\text{HEALTH} - 0.5492 \text{PUB} - 3.000\text{FER} - 0.880\text{REGION} \end{aligned} \quad (3)$$

Clearly, this model explains the data very well. The coefficient of determination is 73% and each variable has a significant impact on life expectancy.

We also can use Cook's distance to further draw the influence diagram to observe some outliers in Figure 7.

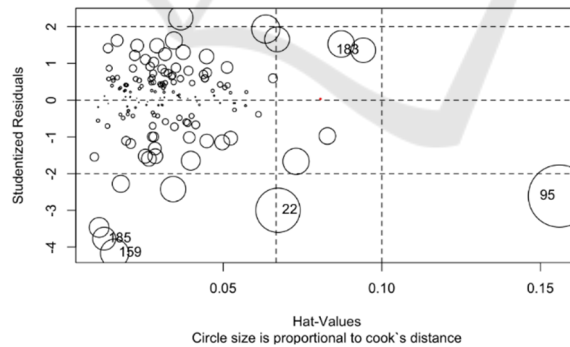


Figure 7: Influence Plot.

3.4 Modeling and Screening of Generalized Linear Models

The generalized linear model is an extension of the linear model and establishes the relationship between the mathematical expectation of the response variable and a linear combination of predictor variables through a linkage function. It is

characterized by the fact that the natural measure of the data is not forcibly altered and the data can have a non-linear and non-constant variance structure. It is a development of the linear model in the study of non-normal distributions of response values and the concise and direct linear transformation of non-linear models.

In this problem, we can also consider the modelling of generalised linear models. We use different link functions and families to build the corresponding models and thus compare them to obtain the best model. Because the joint function involves a logarithmic link function, the

HEALTHEXPEND variable is not treated logarithmically in this question.

Do the glm LIFEEXP and PUBLICEDUCATION+HEALTHEXPEND+FERTILITY+REGION. The result is in Table6 below.

Table 6: Results of generalized linear model.

EDM	$g(\mu)$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	AIC
Gaussian	Identity	-0.451	0.004	-4.246	-1.369	950.2
Gamma	Identity	-0.608	0.005	-4.043	-1.531	982.9
Inverse Gaussian	Identity	-0.699	0.005	-3.928	-1.625	1004
Gaussian	Log	-5.092 e-03	5.255 e-05	-7.115 e-02	-2.012 e-02	954.4
Gamma	Log	-7.395 e-03	6.002 e-05	-6.988 e-02	-2.269 e-02	985.6
Inverse Gaussian	Log	-8.779 e-03	6.465 e-05	-6.884 e-02	-2.416 e-02	1006

Then we can find the gaussian response with identity function seems most appropriate, since both regression parameters are significant for modelling LIFEEXP, and the AIC is the smallest among all the models.

Interestingly, we know that the Gaussian distribution is approximated as a normal distribution. That said, if we use the above variables to build a linear model, a multivariate linear model might work better than a generalised linear model, as not every variable is suitable for modelling using a logarithmic link function. Whether there is a more appropriate generalised linear model deserves further research and investigation.

different link functions were developed for comparison and further analysis.

However, this article still has some shortcomings, such as the treatment of the selection of variables by deleting columns with many missing values. It is worth further debating how to supplement the missing values. As well as in the generalized linear model, there is no better choice of linking function, and the form of the link function still needs further determination. Finally, I believe that the established multivariate linear model R^2 can still be further improved, and in the future, we may conduct further research.

4 CONCLUSION AND DISCUSSION

In this paper, we first conducted a descriptive analysis of the data, observing the missing value characteristics of some variables. The correlation matrix of the data was then derived, and a simple linear model was developed and analyzed for the most highly correlated variables. We then built a multiple regression model using stepwise regression to explore which potential variables had a more significant effect on life expectancy and test the model's feasibility and plausibility. After analyzing this model, we added the region variable, a categorical variable with significantly different means across regions. After building a new model using stepwise regression, we found that region, fertility rate, healthcare costs, and public education expenditure significantly affected national life expectancy. Finally, generalized linear models with

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