Power System Operation Modeling, Monitoring and Control using *P*etri Nets

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Abstract: *P*etri nets have been widely used as a tool for the model of Dynamics Discrete Event System (DDES). In this paper, *P*etri nets are used to model, monitor and control Electrical Power Operation (EPO). For that, it will be used a linear transformation to expand the original *P*etri net. The expansion will change the original rules of firing transitions. Its changes impose restrictions on the system's operation. As result, a simplified equation is presented and it is used for monitoring and controlling EPO.

1 INTRODUCTION

In the earlier years, Power Systems Operations (PSO) have been analyzed using logic diagrams to stable functionalities rules for their ways of operation. Nowadays, have been used modern tools for analysis and control that have not been used for this action before. One of these tools is *P*etri nets (PN) (Tekiner-Mogulkoc et al., 2012).

Generally, in the study of a PSO is considered that its operation should occur predominantly in a steady state. In this case, all the load changes, provoked by the operation or not of the circuit breaker or any other occurrence that can generate transients in the power system are not considered. Thus, all variables are manipulated with dependence only on time in the strictly mathematical sense (Weiss and Schulz, 2015). However, when the dynamic of operation of a PSO is analyzed its operation can be considered as a system whose dynamic is event-driven, i.e. it can be manipulated as a Discrete Event System (DES) (Amini et al., 2019).

Originally, a system is considered as a Dynamic Discrete Event System (DDES) if its dynamic is such that it can be considered as having no dependence on time, but it is considered as having dependence on occurrences or events. The elapsed time of the event is usually negligible. There are several tools used in the study DES for different kinds of systems. Among the main tools used in the study of DES, one can point out Finite State Automata, Dioids, Queue Theory, and *P*etri Nets (Papadopoulos et al., 2019; Wu et al., 2019; Komenda et al., 2018; Lin et al., 2016).

The *P*etri nets are as well known in the DES analysis and its popularity comes mainly due to two factors: its compact representation and its graphic representation. Over time, the *P*etri nets have been incorporating several resources to become them, more powerful (Murata, 1989; David and Alla, 1994; Bin et al., 2015; Fendri and Chaabene, 2019).

In this work, *P*etri nets are used for monitoring and synthesis of controllers for PSO. Using matrix algebra and supervisory control theory a *P*etri nets is used to model Power System Operation (Giua, 1992; Krogh and Holloway, 1991; Holloway et al., 1996; Dideban and Alla, 2009).

This work is complemented with the following sections: Theoretical background composed by *P*etri nets concepts and a little review of Power System Operation. In the following section, the use of *P*etri nets in the Power System Operation is presented as the tool for modeling. After that, the original model is expanded to monitoring and control. The section Application is used to validate the theory presented in the preview sections. The Conclusion Section are presented the mains results (Souza et al., 2016).

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2 THEORETICAL BASIS

2.1 *P*etri Nets

A *P*etri net is a particular kind of directed bipartite graph (or digraph) together with an initial state called the initial marking. It contains two types of nodes, places and transitions. In the graphical representation, places are drawn as circles, and transitions as bars or boxes. Arcs links either from a place to a transition or from a transition to a place. Arcs are labeled with their weights (positive integers). Labels for unity weight are usually omitted. A marking (state) assigns to each place(p) is a non-negative integer k; we say that p is marked with k tokens. Pictorially, we place k black dots (tokens) in place p. If k is large, it can simply write the number k inside p to represent k tokens. A marking is denoted by M, an m-vector, where m is the total number of places. The number of marks inside p are denoted by $\sharp M$. (Murata, 1989; David and Alla, 1994; Cassandras and Lafortune, 2009).

Definition 1. *A PN is a five-tuple* $PN = (P,T,I,O,M_0)$*, where*

1. $P = \{p_1, p_2, \ldots, p_m\}$ *is a finite set of places.*
2. $T = \{t_1, t_2, \ldots, t_n\}$ *is a finite set of transitions,* $P \cup T \neq \emptyset, P \cap T = \emptyset$
3. $B^- : PT \to N$ *is an input matrix such that its elements defines the directed arcs from places to transitions, where* $N = \{0, 1, 2, \ldots\}$.
4. $B^+ : PT \to N$ *is an output function that defines the directed arcs from transitions to places.*
5. $M : P \to N$ *is a marking vector representing the numbers of tokens in places with Mo denoting the initial marking.*

In modeling, often places represent conditions and transitions represent events. A transition (event) has a certain number of input and output places representing the preconditions and postconditions, respectively. The presence of a token in places are interpreted as the truth of the condition associated with places.

2.1.1 Enabling and Firing Rules

Definition 2. *A PN is said to be an ordinary PN if for any arc in the net its weight is 1. The preset of transition t is the set of all input places to t, i.e.,* $\bullet t = \{p : p \in P \text{ and } B^-(p,t) > 0\}$. *The postset of t is the set of all output places from t, i.e.,* $t^\bullet = \{p : p \in P \text{ and } B^+(p,t) > 0\}$. *Similarly, the preset of p is* $\bullet p = \{t : t \in T : B^+(p,t) > 0\}$ *and postset* $p\bullet = \{t : t \in T : B^-(p,t) > 0\}$.

Definition 3. *A transition* $t \in T$ *in PN is enabled in marking M if for all* $p \in \bullet t$,

$$M(p) \geq B^-(p,t)$$

If a transition is enabled, it can fire. Firing an enabled transition t in marking M yields

$$M'(p) = \begin{cases} M(p) - B^-(p,t), & \text{if} \quad p \in t \\ M(p) + B^+(p,t), & \text{if} \quad p \in t \\ M(p), & \text{if} \quad otherwise \end{cases}$$

2.2 Power Systems Elements: Models

Electric Power Systems(EPS) are a set of equipment that operates in a coordinated manner with the purpose of providing electricity to consumers, within certain standards of quality (reliability, availability), safety and costs, with minimal environmental impact.They share and contribute to the progress and technological advances of humanity. The growth in electricity consumption in the world is considered phenomenal. This makes the EPS increasingly larger and consequently increases the complexity. Typically, electrical power systems can be divided into three large blocks:

Generation - Performs the function of converting some form of energy (hydraulic, thermal, etc.) into electrical energy;

Transmission - Responsible for transporting electric energy from Production Centers to Consumer Centers, or even other electrical systems, interconnecting them;

Distribution - Distributes the electricity received from the transmission system to large, medium and small consumers.

Therefore, the electrical system, as a whole, consists of multiple generation sources that are used to serve the load centers, a process that is done by complex transmission systems. From the above, to keep this complex system operating properly, with quality and safety standards, it is necessary, therefore, monitoring and control.

As modelling tool, *P*etri Nets can be used to model Power System Operation. To do that, it is necessary limit the constructive characteristics of its components. therefore, the degree of complexity of the circuit breaker should consider the following internal components:

1. Axillary contacts
2. Main contacts
3. Protective cover for arc contacts
4. Porcelain wrap

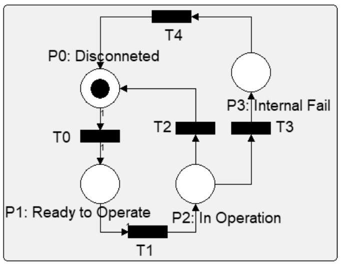

Figure 1: Circuit Breaker Model Using PN.

The states to put in its model will be

In Operation - when the circuit breaker is in the on state

Disconnected - when the circuit breaker is in the off state

Ready to Operate - when the circuit breaker spring is charging

Internal Fail - When the breaker trips because of an internal failure

The circuit breaker operation occur after it is ready to operate. Regarding the shutdown, it can be turned off manually or automatically due to internal failure being recovered ready to restart . So the way to represent the circuit breaker via Ordinary *P*etri is shown below in the Figure 1.

The electric power generator and disconnect switch in this paper will be modeled in two states. In the on-state, the generator and disconnect switch is supplying energy and transferring power respectively. The off-state, the generator shuts down and the disconnect switch suspends transference. In the Figure 2 is shown the *P*etri Net of these components.

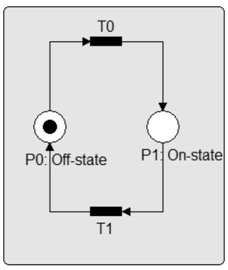

Figure 2: *P*etri Net Model for Generator and Disconnect Switch.

2.3 Expansion of *P*etri Net

In this chapter the use of PN will be introduced in the study of the operation of an EPS. Initially, an PN will be expanded with the addition, in its structure, of one or more places and its consequent transformation from an autonomous to a non-autonomous PN. At the end of this transformation, issues related to monitoring, control and diagnosis will be studied.

A Place-Transition type *P*etri net, autonomous, containing m places and n transitions and with initial marking M_0. The *P*etri net can be added with some places (pci) with M_{ci} marking, however, following some premises, which are dictated by the interest in the expansion. In this paper, given the expansion of PN, it is interesting to guarantee two basic premises:

- The marking of the original network must be guaranteed
- The number of marks of the incorporated place should be related to the mark of the original network

The Figure 3 shows a block diagram of the expanded *P*etri Net. From a graphical point of view, this expansion can be interpreted as shown in Figure 3. The *P*etri net gains a new place, therefore, the connection between the new place and the old PN must be through arcs from the new place to PN transitions and from the transitions of the PN to the new place. From the algebraic point of view, the transformation corresponds to an expansion of matrices, that is, a linear transformation over matrices, as seen below

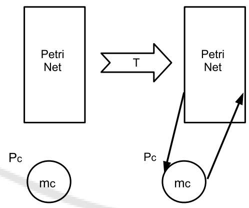

Figure 3: *P*etri Net Expansion Block Diagram.

$$T : \begin{cases} M & \Rightarrow & M_h \\ \mathbb{N}^{(m)} & \Rightarrow & \mathbb{N}^{(m+1)} \end{cases}$$

The transformed matrix M_h will then be of the form:

$$M_h = AM$$

The order of A being as, $[A]^{(m+1)\times m}$. Matrix A can be broken down into two parts, each part meeting one of the premisses:

$$A = \begin{pmatrix} A_1 \\ \dots \\ A_2 \end{pmatrix} \tag{1}$$

So that, it can find the the result in the Equation(2):

$$M_h = \begin{pmatrix} A_1 \\ \dots \\ A_2 \end{pmatrix} M = \begin{pmatrix} A_1 M \\ \dots \\ A_2 M \end{pmatrix} \tag{2}$$

where,

- $A_1.M$ corresponding to marking the old PN. Thus A_1 is the identity matrix of order equal to the number of lines of M, that is, $A_1 = [I]_{m\times m}$.
- $A_2.M$ is the number of outside place marks and the vector A_2 will be treat as the vector line C, whose order will be $[C]_{1\times m}$.
- $C = [c_1, c_2, \ldots, c_{m-1}, c_m]$

$$T(M) = M_h = \begin{pmatrix} I_m \\ \ldots \\ C \end{pmatrix} M = \begin{pmatrix} I_m M \\ \ldots \\ CM \end{pmatrix} \quad (3)$$

Denoting $\sharp M$, number of marks in the in the old PN;

$$\sharp T(M) = \sharp M + \sharp M_c \quad (4)$$

The element M in the Equation(2), can involve only a portion of the places marked in the PN, hence Im is replaced by its main diagonal L, containing $L(i,i)$ belongs to N as desired whether or not the place in the desired operation. Thus, $L(i) = Im(i,i) = 1, 2, \ldots$ means that the place p (i) is included in the desired operation, otherwise L (i) = 0. Taking the vector line L, the Equation(4), of the number of marks of the extended PN becomes, see Equation(5)

$$\sharp T(M) = \sharp LM + \sharp CM = \sharp \begin{pmatrix} L \\ \ldots \\ C \end{pmatrix} M \quad (5)$$

the Equation(5) allows computing the number of marks to the external place from a value of the original PN. Once the transformation is known, the dynamics of the expanded PN is evaluated, that is, how the state of the new PN changes. The Marking of expanded PN will be,

$$\sharp M_h = \sharp T(M) = LM + CM \quad (6)$$

However, considering the dynamics. of the original *P*etri net, this means that the PN remains autonomous, that is, the presence of the external place does not interfere with the change of state of the original PN. From the Original equation,

$$M = M_0 + Bq$$

And using the expansion matrices, A, is find that:

$$M_h = AM_o + ABq = \begin{pmatrix} L \\ \ldots \\ C \end{pmatrix} M_0 + \begin{pmatrix} L \\ \ldots \\ C \end{pmatrix} Bq \quad (7)$$

being,

$$M_h = M_{ho} + ABq = \begin{pmatrix} L \\ \ldots \\ C \end{pmatrix} M_0 + \begin{pmatrix} L \\ \ldots \\ C \end{pmatrix} Bq \quad (8)$$

Until then, the PN remains autonomous. obeying only the rules of formalism, the place p_c just observes, monitors, watches. The next step, then, is to determine the markup value of the inserted place and its incidence matrix, that is, (M_{co}, CB). The equations above allow, therefore, to analyze an expanded PN of a place without losing the information of the PN. This analysis will be extended to the following themes:

- System Monitoring
- Supervisory Control
- Partial Observation by the Controller
- Fault Diagnosis

2.3.1 PSO Monitoring using *P*etri Net

However advanced, from a technological point of view, systems are subject to disturbances in their dynamics. These disturbances can be caused by external or internal sources and can cause problems of different orders. Thus, it is essential to monitor or supervise the systems. Condition for monitoring the system using PN as a model. In this paper, it will use external places to monitor one or more places in a *P*etri net. The following boundary condition will be accepted, see Equation(9)

$$CM - LM = 0 \quad (9)$$

The above equation is equivalent to Equation(10). Where $\sharp Mc$ is Monitor marking and $\sharp M$ is monitored places marking.

$$\sharp Mc = \sharp M \quad (10)$$

That is, the number of marks in the external place $\sharp Mc$ is equal to the number of marks in the monitored places $\sharp M$ of the original *P*etri Net, whatever that marking maybe.

Since the dynamics of the extended *P*etri Net is given by

$$M_h = M_{h0} + B_h q, \quad (11)$$

$$M_h - M_{h0} = AB q, \quad (12)$$

$$\begin{pmatrix} L \\ \ldots \\ C \end{pmatrix} B^+ - \begin{pmatrix} L \\ \ldots \\ C \end{pmatrix} B^- = 0 \quad (13)$$

That way is find

$$L(B^+ - B^-) = C(B^+ - B^-) \quad (14)$$

therefore,

$$LB = CB \quad (15)$$

that is, the incidence matrix of the external place p_c is equal to the incidence matrix of the places to be monitored.

Example: It is desired to monitor the energization of a distribution bus (B), powered by an EPS (G), through a disconnect switch (SW), see Figure 4. The

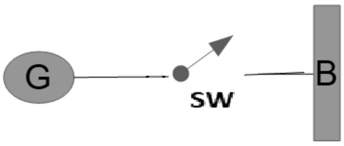

Figure 4: Single-Line Diagram of Electric Power System.

*P*etri net model of each elements that electric power system is shown in the Figure 5.

it is observed that the bus will be energized if the SW switch is closed and the generator is turned on. In this way, a monitor will be synthesized so that it can verify the instants in which the bus is energized. From the *P*etri net shown in Figure 5, extract the incidence matrix of the switch(SW)-generator(G) set and the vector L.

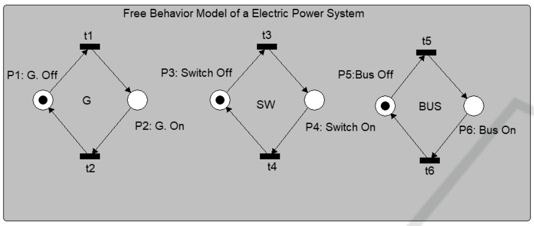

Figure 5: PN Model of The Single-Line Diagram.

The bus will then be energized if the places p_2 and p_4 are marked, that is, $M(p_2)+M(p_4)=2$ is equivalent to the vector L = [0 1 0 1]. the incidence matrix and the calculation of the monitor will be.

$$B = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Using the Equation 15 it is determined that:

$$CB = LB = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$$

CB is then the connections of the external place(the monitor) with the transitions of the *P*etri net as shown in Figure 6. The initial mark of the monitored places is the same as the mark of the monitor, see Equation(10). Therefore, the monitor place p_m will have zero marks initially for the example developed. Whenever the place p_5 has two marks its mean the bus is energized. Bus monitoring is shown in Figure 6

2.4 Modeling Electric Power System Operation using *P*etri Net

The objectives of the control actions depend on the point of operation of the electric power system. Under normal conditions, the purpose of the control is to keep the systems operating as efficiently as possible, with voltage and frequency values close to the nominal (T. et al., 2020). On the other hand, when abnormal conditions are verified, new objectives must be pursued to restore the system to its normal operating conditions, as soon as possible.

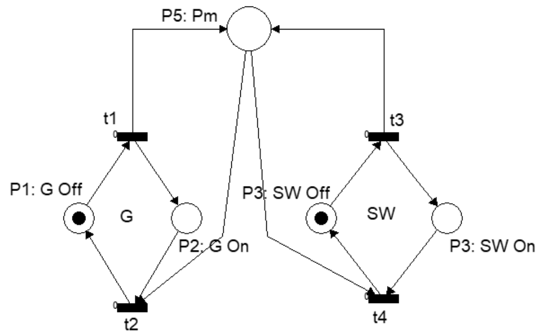

Figure 6: Bus Monitoring.

Control of the operation of an EPS consists of commanding the components of interruption of the system, to keep it under normal conditions, guaranteeing the safety and quality of the energy supplied.

2.4.1 Supervisory Control Theory using *P*etri Nets Applied to EPS Operation

From this point on, the *P*etri Net is no longer autonomous and becomes controllable, that is, the transitions now also depend, for their firing, on the external places connected to transitions. The Equation(16) shows how computing a controller.

$$\sharp M + \sharp M_c = K \tag{16}$$

The marking can be rewrite as the following

$$(L+C)M = K \tag{17}$$

substituting M by *P*etri Net Dynamic Equation, $M = M_o + Bq$ in the Equation(17) and rewiriting it, the final result is shown in the Equation(19). The Equation(16) allows calculate the initial mark value of the controller.

$$\begin{pmatrix} L \\ \dots \\ C \end{pmatrix} B^{+} - \begin{pmatrix} L \\ \dots \\ C \end{pmatrix} B^{-} = 0 \tag{18}$$

$$CB^{+} - CB^{-} = -LB^{+} + LB^{-}$$

In this way, Equation(19) provides the weight and directions of the arcs that connect the controller to the transitions of the original *P*etri net

$$CB = -LB \tag{19}$$

The expanded *P*etri net N_h has a marking $M_h^{(m+1)\times 1}$ such that preserves its marking by a T transformation, $TM \implies M_h$.

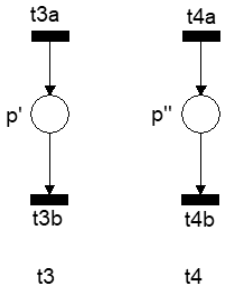

Figure 7: Equivalent PN to transitions.

Example: Now in the Figure 4 is desired to control the opening/closing of SW switch. Opening and closing operation of SW is not allowed when the generator(G) is in On state. The controller must establish restrictions on the free behavior of the *P*etri net models of the G generator and SW switch to not allow those. The PN models free behavior of G and SW are the same as shown in Figure 5. To restrict the states reached from the *P*etri net the following equations can be used:

$$p_2 + t_4 = 1$$

and

$$p_2 + t_3 = 1$$

Adding the two equations will find

$$2p_2 + t_3 + t_4 = 2$$

The restriction found has two transitions (events) and a place (state). This situation eliminates the ability to operationalize terms with different meanings. The way to be able to operationalize this equation is to transform one of the terms into others. Thus, can convert the transitions of the equation by associating them with places or places associated with transitions. Figure 7 shows a way to associate transitions with place. That way, rewriting the restriction find:

$$2p_2 + p' + p'' = 2$$

So using the change and the Equations(19) and (16) is founded:

$$CB = [+1, -1, \pm 1, \pm 1]$$

and

$$m_c = 1$$

The Figure 8 shows the expanded PN with the controller p_c. In this figure, it is observed the controller with a mark releases the SW switch model allowing to change marking. This occurs while the generator PN model is in the off state. When the model of the generator goes into the On state (p_4 with mark) the controller will lose the mark inhibiting the model of the SW switch to change their states.

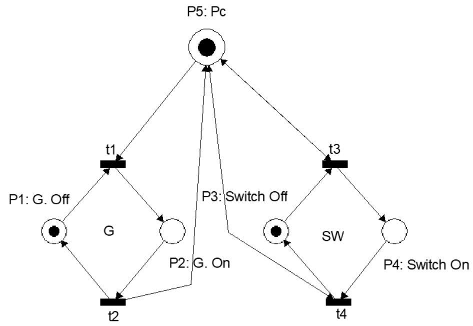

Figure 8: PN: Opening/Closing Controller Operation.

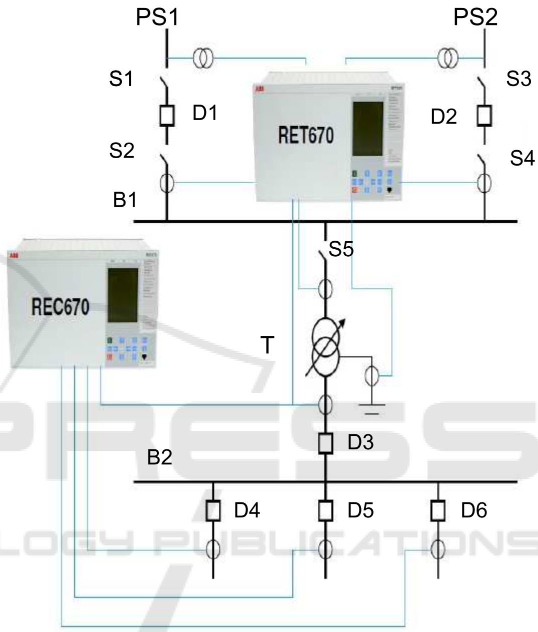

Figure 9: Bus Feeder System and Power Supply (Ltd, 2010).

3 APPLICATION IN SUBSTATIONS

As an application, it is made the modeling, monitoring and control of the EPS represented in Figure 9 by its single line diagram. It is a substation composed of two buses, [B_1 and B_2], six circuit breakers [D_1, $D_2, \cdots, D_6$], five disconnect switches [$S_1, \cdots, S_5$], one transformer and three energy consumers.

First of all, considering the substation de-energized [i.e. the disconnect switches and circuit-breaker assemblies are all disconnected or open]. With this, the following procedure[1] can be proposed

[1]Any procedure proposed must be in agreement with local standards and aligned with the concessionary that holds the formal authorization to distribute electricity in the re-

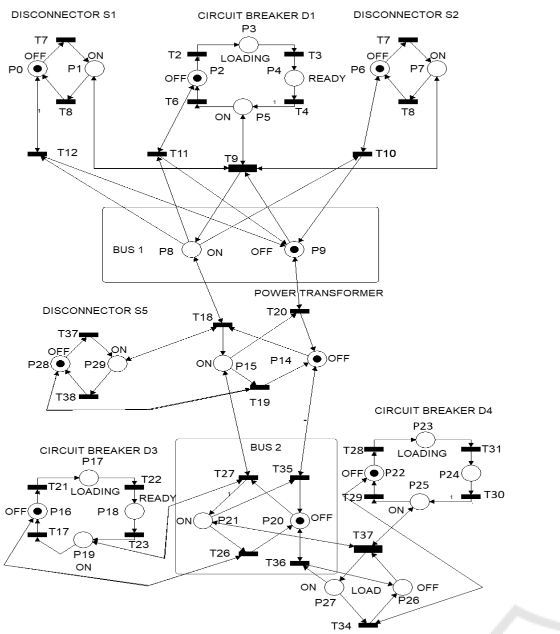

Figure 10: Substation Free Behaviour PN Modeling.

to energize the buses B_1 and B_2:

1. To power on the Bus B_1 via PS_1, must be given priority to close the disconnecting switches S_1 and S_2 and after these the circuit breaker D_1 must be closed;
2. To power on the Bus B_1 via PS_2, must given priority to close the disconnecting switches S_3 and S_4 and after these the circuit breaker D_2 must be closed;
3. To power on the Transformer T after the Bus B_1 is on, it must be given priority to close the disconnect switch S_5. So after that, the circuit breaker D_3 can be closed and consequently the bus B_2 will be on;
4. To energize some electric loads after bus B_2 is on, one circuit breaker must switch on D_4 to D_6 without setting an order of priority;
5. The procedure for shutdown the substation must obey an inverse prioritization order those done for startup it [i.e. first of all, it must disconnect the loads by switching off D_4 to D_6, then isolating the bus B_2 through D_3, following with the de-energizing of the transformer through S_5 and so on];

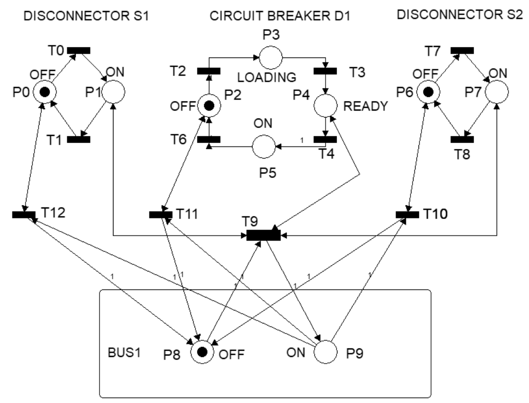

Figure 11: PS_1 Free Behaviour PN Modeling.

3.1 Free Behavior Modeling for Substation

Some specifications are adopted for modeling of the substation as following. The disconnect switches have two possible states: (i) the connected state; and (ii) the disconnected state. The circuit breakers have 4 states: (i) the Off state; (ii) Carrying the Spring State; (iii) ready to go Into Operation State; and the On State. For this modeling, abnormal conditions of the equipment are not considered [for instance, the broken state]. The buses also are represented by two states: (i) de-energized; and (ii) energized.

The presence of the token depicts the current state of the equipment. For instance, considering a disconnector switch where P_1 is the state that symbolizes when it is disconnected and P_2 the state representing when it is connected, then if a token is in P_2 it represents that the disconnector switch is closed, otherwise [token in P_1] it is open.

In Figure 10, a summarized version of the free behavior for the substation presented in Figure 9 is presented. The substation *P*etri net model consists of an input PS_1 composed of two disconnect switches S_1 and S_2 and a circuit breaker D_1, followed by the representation of the input bus B_1. After this, there is the model of the disconnect switch (S_5), responsible for energizing the primary winding of the transformer. To the right side of S_5 model is the representation of the possible states of the power transformer. In the bottom of the *P*etri net, all of the other models are presented: the model to the circuit breaker D_3, the output bus B_2 and the energizing circuit breaker of a load D_4.

gion where that the substation is installed.

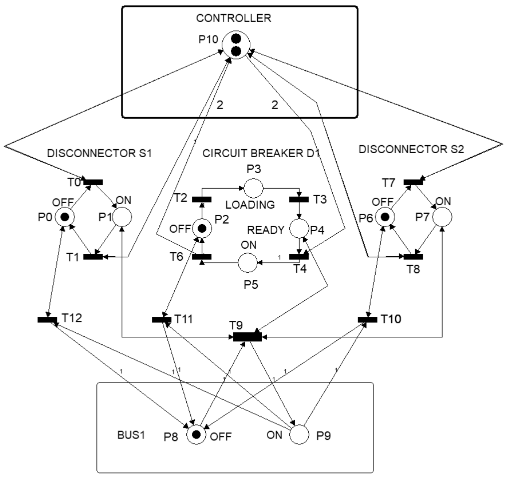

Figure 12: PS_1 System Control PN Modeling.

3.2 Controller Design for Sequence Operation

In Figure 11 is shown the *P*etri net model of the free behavior for the input PS_1 energizing the bus B_1. This representation does not contemplate the restrictions for the operation of the disconnect switches and circuit breakers that affect the safe energization or shutdown of the bus B_1. These constraints are intimately related to the constructive aspects of these operational control elements. For example, the manipulation of the disconnect switches must not be carried out with the circuit-breaker switched on. Thus, the correct way to energize the bus B_1 is closing the disconnect switches S_1 and S_2 and after that close the circuit breaker D_1.

For the energizing and de-energizing operation of the PS_1, it is necessary to compute a controller that can restrict some actions of the free behavior model. For this, it is necessary to specify the controller actions to guarantee the correct sequence of events firing. A control system is presented in Figure 12. The control specifications are represented by a set of linear equations where their variables represent the marking of the controlled places. The constant on the other side of the equation will mean that the controlled places will form a place invariant with the controllers [see Equation(16)].

To specify the controllers, it is necessary to find the incidence matrix B_c of these places and also their initial marking M_{c0}. The incidence matrix B_c is obtained from both the control specifications and the incidence matrix of the original petri net.

From Figure 12, the constraints on this feeder can be found by making the following considerations with respect to feeder free behavior model PS_1:

1. If the current state of the circuit-breaker is closed [i.e., P_5 with marks], it forces that the current states of the disconnect switches are also on states [P_1 and P_7];
2. Generating equation $P_5 + P_1 + P_7 = 3$;
3. If the current state of one of the connect switches is opened [i.e., P_0 or P_6 with marks] the breaker can not to close [$P_5 = 0$];
4. Generating equation $P_5 + P_0 + P_6 = 1$;

   Adding the two equations above one can get

$$2P_5 + P_0 + P_1 + P_6 + P_7 = 4$$

and consequently

$$L = [1, 1, 0, 0, 0, 2, 1, 1, 0, 0].$$

With L and the incidence matrix, the weights of the arc that interconnects the controller with the transitions of the original *P*etri net are obtained from

$$CB = -LB = [\pm 1, \pm 1, 0, 0, -2, 2, \pm 1, \pm 1, 0, 0].$$

The determination of the initial marking of the controller is obtained through Equation(16). With the value of the initial marking of the controlled places [i.e., P_0, P_1, P_5, P_6 and P_7], the vector L and the constant K, $K = 4$. Found: $M_{c0} = 2$. With this information the *P*etri net of Figure 12 is drawn.

The terms represented by $\pm$ correspond to the nonzero position in the vector L that resulted in zero in the calculation of Equation(19). In the formation of the controlled *P*etri net, these positions will be represented by autoloop. see Figure 12. The autoloop present between the transitions of the connect switches models and the controller allows these transitions to fire without changing the controller marking. This condition releases the models of the connect switches to move from the off state (P_0 and P_7 with tokens) to on state (P_1 and P_8 with tokens) freely. When the model representing the circuit breaker goes to the on state (P_5 with mark), the T_4 transition removes two marks from controller (P_{10}). The controller place, P_{10}, without marking inhibits firing of the transitions belonging to the switch models (T_0, T_1, T_7, and T_8). This condition (P_{10} without tokens) keeps the model of the disconnect switches in the state that were before the T_4 transition firing.

In agreement with the constraints imposed to determine the controller, the following sequence of events are possible: $T_0T_7T_2T_3T_4$ representing the on states of the switches, breaker and bus(P_1, P_5, P_7 and P_9 with marks) and another sequences are $T_5T_1T_8$ or $T_5T_8T_1$ representing the states off the disconnect switches, circuit breaker and bus (M_0). In relation to the bus, it already loses the on status as soon as

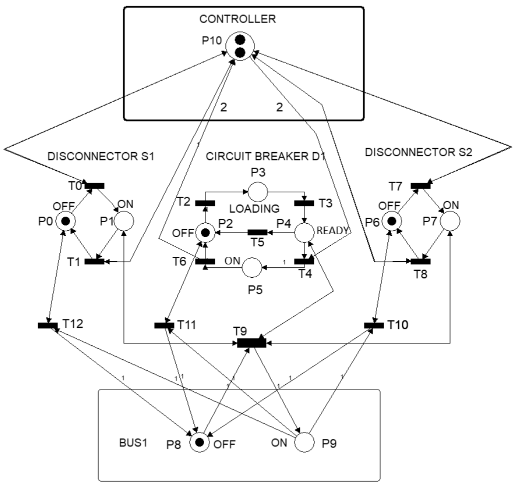

Figure 13: PN Model for PS1 with Uncontrolled and Observable Transition.

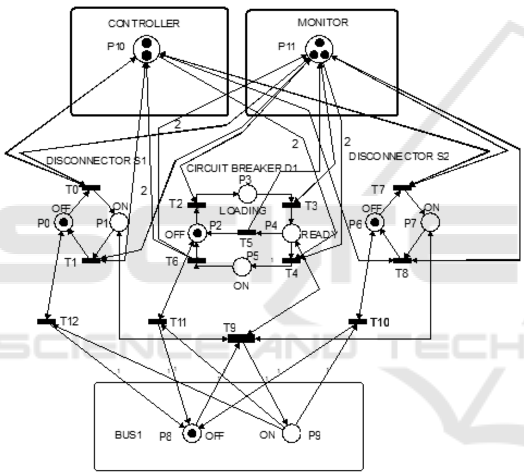

Figure 14: Structure of a monitor for PS1 with Failure Transition.

T_5 fires since the model contemplates the way the switches and circuit breakers are physically interconnected(series association).

3.3 Development of the Monitor

In figure 10 is shown the *P*etri net model for the input power supply PS 1 of the sub-station presented in Figure 6. A new transition T_5 has been inserted to represent a failure event. This event connects both the place that represents the Ready Circuit Breaker status and the place that represents off State Circuit Breaker Status. Thus, T_5 emulates possible problems that may appear on the circuit breaker, such as loss of elastic characteristics of the spring assembly, jammed or worn contacts, etc. Consider T_5 be uncontrollable and observable means that the controller can not intervene in its firing but the monitor alarms if it comes to fire.

So, given the Incidence matrix B of the *P*etri net of Figure 13 and knowing that when specifying a monitor for the main places those represents S_1, S_2 and D_1 the vector L is will be.

$$L = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

And applying the condition for monitoring in the Equation(15). It is determined that the weight of the arches between the monitor place and the Perti net will be:

$$B_m = \left(\pm1 \pm1 \pm1 \pm1 -2 2 \pm1 \pm1 \pm1 \pm1 0 0 \pm1\right)$$

Transition T_5 is occupying the last column of the incidence matrix B. It is a non-controllable transition and therefore the monitor will not be able to intervene in your fire. This condition eliminates in the computation of vector B_m the arc that will link the place monitor to the transition T_5. That way, the monitor will know if the transition T_5 has fired but it does not interfere in your fire. The last field of the B_m vector loses the value -1 and retains the +1 value. See B_m below.

$$B_m = \left(\pm1 \pm1 \pm1 \pm1 -2 2 \pm1 \pm1 \pm1 \pm1 0 0 1\right)$$

The initial marking for the monitor will be obtained through Equation(10). Thus, the new *P*etri net is shown in Figure 14. By analyzing the *P*etri net of Figure 14 are extracted the following informations:

1. The firing of all controllable transitions remains invariant between the monitored and monitor places.
2. The fire of the uncontrolled transition causes the breaking of the invariant place, allowing the identification of the failure.
3. The number of monitor marks expresses the number of times the failure occurred.
4. The occurrence of the fault causes loss of control by the controller P_{10}.
5. The Monitor does not interfere in the actions of the controller.

4 CONCLUSIONS

In this work, the modeling framework of an EPS using *P*etri nets Place-Transition was adopted to analyze issues relevant to monitoring and control in the operation of an EPS. In the development of the model, a linear transformation was used, which allows formally solving monitoring and control problems. Using Linear Algebra techniques, an equation was developed that for boundary conditions, formalizes such techniques. For monitoring analysis, the conservation of the number of marks in the extended *P*etri net was ensured, which leads the expanded *P*etri net to maintain the balance between the input and output incidence matrix of the original *P*etri net, thus ensuring the ownership of place invariants. As for the control of the operation, it was established that the marking of the extended *P*etri net is kept constant, which leads to an incidence matrix of the expanded part as opposed to the incidence matrix of the places to be controlled. The work is completed with an application study that was carried out using an electrical power system substation. The selection of the places to be monitored is made from a line vector, L vector, whose non-null elements point to their positions. The product of this vector with the incidence matrix of the original *P*etri net generates the incidence matrix of the expansion of the *P*etri net. This result determines the weight, the direction of the arc and which events of the original *P*etri net will link the Monitor or Controller place.

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