Intersection Traffic State Estimation using Speed Transition Matrix and Fuzzy-based Systems

Željko Majstorović[a], Leo Tišljarić[b], Edouard Ivanjko[c] and Tonči Carić[d]
Faculty of Transport and Traffic Sciences, University of Zagreb, Vukelićeva street 4, 10000 Zagreb, Croatia

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Abstract: Urban traffic congestion is a significant problem for almost every city, affecting various aspects of life. Besides increasing travel time, congestion also affects air and life quality causing economic losses. The construction of infrastructure to solve congestion problems is not always feasible, and, at the end, attracts only additional traffic demand. Thus, a better approach for solving the problem of city congestion is by optimal management of the existing infrastructure. Timely detection of traffic congestion on the road level can prevent congestion formation and even improve road network capacity when used for appropriate traffic control actions. Detecting congestion is a complex process that depends on available traffic data. In this paper, for traffic state estimation, including congestion level, at the intersection level, a new method based on Speed Transition Matrix and Fuzzy-Based System is presented. The proposed method utilizes the Connected Vehicle environment. It is tested on a model of an isolated intersection made in SUMO simulation software based on real-world traffic data. The validation results confirm the successful detection of traffic state (congestion level) at intersections.

1 INTRODUCTION

In recent years, the world has faced an increased number of vehicles due to globalization and enlargement of the urban centers. The major challenge for almost every city is traffic congestion, which mainly affects road vehicles and public transportation as they share the same urban infrastructure. On urban roads, traffic congestions occur mainly at intersections where conflicting traffic flows are safely resolved using traffic lights.

In general, urban congestions can be divided into recurrent and non-recurrent. Physical limitations of infrastructure, daily repeating periods of increased traffic demand, and infrastructure management cause recurrent congestions, and non-recurrent congestions are mainly caused by traffic incidents, special events (e.g., sports events, concerts, vehicle breakdown, traffic accident), roadworks, etc (Chow et al., 2014). Recurrent congestions are easier to predict, and appropriate control actions can be planned in advance to alleviate them. For the latter, good traffic state estimation is crucial because the first step towards the solution of congestion is its detection. By successfully detecting the congestion, appropriate actions for congestion soothing like changing the signal or rerouting vehicles can be taken.

Urban roads can operate longer in stop-and-go conditions during rush hours. Especially at intersections where the distinction between congestion and regular traffic light queue is not always obvious. Having accurate and timely information regarding traffic state is crucial for every modern traffic control system (Wang et al., 2018). Such a system has to account for multiple input parameters and describe them with a single value to have reliable data to estimate the traffic state. Researchers often use fuzzy logic because it is close to human reasoning and can also process vague input information. Fuzzy logic is used to describe the uncertainty of things and has wide application in fuzzy reasoning which allows application in different fields (Maldonado et al., 2021). Because of its properties, fuzzy logic has also found its application in traffic engineering (Koukol et al., 2015). It is also common to combine fuzzy logic with different methods for different purposes (Borlea et al., 2021). One example are decision support systems,

[a] https://orcid.org/0000-0002-5242-6607
[b] https://orcid.org/0000-0001-8257-8356
[c] https://orcid.org/0000-0002-2083-0672
[d] https://orcid.org/0000-0001-8564-4304

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variable speed limits, and adaptive traffic signal control. (Pozna and Precup, 2014) created an algorithm for modeling expert systems. The advantage of this algorithm is the systematic and general formulation that allows the modeling of uncertain expert systems.

The appearance of Connected Vehicles (CVs) has opened up new areas of research. CVs have the possibility of operating like mobile sensors that can provide large amounts of data for various traffic analyzes. Unlike historical traffic data that needs to be collected over a period of time to observe patterns in traffic behavior covering only specific measurement points, each CV can provide real-time data. Thus, spatio-temporal traffic data can be collected. While existing infrastructure uses traffic monitoring sensors and cameras to detect congestion and manage traffic infrastructure, networked vehicles can communicate with infrastructure using Vehicle to Infrastructure (V2I) communication and other vehicles using Vehicle to Vehicle (V2V) communication to exchange current traffic parameters. For example, at a city intersection, vehicles can exchange position data, which results in increased traffic safety.

The Society of Automotive Engineers (SAE) J3016 standard defines six levels of driving automation (Shuttleworth, 2019). SAE Level 0 implies vehicles without automation to SAE Level 5 which implies full vehicle autonomy. According to the same standard almost fully self-driving (Level 4) and even fully autonomous vehicles (Level 5) are expected within a decade including city driving.

CVs have significant advantages. Modern equipment such as radars, lidars, cameras, and many other sensors makes them excellent mobile data traffic sources. To optimize the driving strategy of the automated vehicles, (Kumm and Schreckenberg, 2019) implemented agent-based simulation in the framework of the three-phase traffic theory using a high number of interacting vehicles. The proposed methodology allowed automated vehicles to merge into the best possible gap between moving vehicles. Moreover, CVs have advantages over existing traffic sensor technology because they are not limited by line of sight like cameras, and are collecting large amounts of data at the microscopic level which is convenient for studying traffic. However, the motivation for this research stems from the question of how to process large amounts of data quickly and efficiently. Such data amounts of data will be generated by future traffic flows containing classic and CVs. The share of the later will rise decreasing the need for classic traffic sensors (inductive loops). Speed Transition Matrices (STMs) are suitable in this context because they simplify data processing and are applied in this paper as well. The motivation for this research stems from the question of how to process large amounts of data quickly and efficiently using the potential of mobile sensors in form of CVs.

Thus, this paper presents a new method for intersection traffic state estimation that is based on the Center of Mass (CoM) of speed data represented in the STM and Fuzzy Inference System. The STMs were computed based on vehicle speed data collected during simulation, and are represented as a speed probability distribution of vehicles traveling between two consecutive road segments. Thus, the scientific contributions of this paper are as follows:

- Traffic data representation on a isolated urban intersection using STMs for state estimation is developed;
- The methodology for the intersection traffic state estimation is proposed based on the attributes extracted from the traffic patterns represented with the STMs;
- The proposed methodology is applied and validated on an isolated intersection in the Simulation of Urban Mobility (SUMO) software.

The rest of the paper is organized as follows. In section 2, existing traffic state estimation methods are described. The proposed methodology applied intersection state estimation and validation methods are presented in section 3. Obtained results of intersection state estimation are analyzed in section 4. Conclusion and future work suggestions are given in last section 5.

2 TRAFFIC STATE ESTIMATION METHODS

Urban traffic state estimation is the subject of interest for many researchers. In the literature, researchers used different data sources and explored various traffic state estimation methods. In general, traffic state estimation methods can be categorized into the following categories: model driven, data driven, and streaming-data driven methods (Seo et al., 2017).

Model driven methods are based on knowledge of physical flows where models represent the physical flow. Models have high explanatory characteristics, which means even if the estimation is inaccurate, it is possible to explain the inaccuracy. However, a poorly calibrated model can affect the performance of the estimation method (Seo et al., 2017).

Data-driven traffic state estimation methods rely on historical data, using statistical methods and machine learning techniques to determine real-time traffic states based on features found in historical data. Depending on historical data has its drawbacks. These methods are prone to failure if an irregular event occurs or traffic trends change longtime. Especially in the case without having these two cases in the historical data (Seo et al., 2017).

In contrast to previously mentioned methods, streaming-data driven methods do not require historical data, which makes them robust to unpredictable events. They rely on streaming data and weak assumptions without capabilities for future prediction. For accurate estimation, these methods require large amounts of streaming data (Seo et al., 2017).

For traffic analysis, the Global Positioning System (GPS) is commonly used as a data source. Needed traffic data are mostly collected by GPS-equipped taxi or delivery vehicles. GPS cannot always provide accurate data. Thus, it is mostly used to determine conditions on the road level. (Kan et al., 2019) detected turn-level congestion analyzing features of GPS data. Using clustering, their method identifies congestion events for each turning direction. (D'Andrea and Marcelloni, 2017) also used GPS data to detect traffic states. They detected traffic states based on vehicle speeds extracted from GPS data classified based on speed thresholds. Another example of applying GPS data to estimate queue length and level of service for an intersection is (Tišljarić et al., 2018). All of these methods rely on features such as vehicle speed and position extracted from GPS data, which depend on the number of tracked vehicles and valid GPS data samples. Also, it is important to consider the time delay caused by data processing because it takes some time to collect enough GPS data and process it.

(He et al., 2019) relayed on probe vehicle data consisted of latitude, longitude, speed, moving direction, and the stop-and-go characteristic of traffic passing through the intersection to identify turn-level congestion. Besides GPS data, traffic detectors are also very valuable data sources for research. (Lee et al., 2015) and (Liu et al., 2009) implemented real-time estimation of queue lengths on intersection based on detectors data which resulted with reliable estimation of queue lengths in real-time.

Fusion from multiple data sources can also provide timely and accurate information, which are important for applications such as driver information systems and traffic control systems (Papageorgiou et al., 2003). To estimate queue tail location, (Rostami Shahrbabaki et al., 2018) fused detector data with the location and speed of the CVs in a mixed traffic flow containing classic and CVs.

3 METHODOLOGY

In this section, key steps of the applied research methodology are described. The methodology presented in this paper relies on vehicle speeds which can be extracted from GPS data or vehicle probe data in a CV environment. Graphical visualization of methodology is shown in Fig. 1 which is divided into three main steps: (i) simulation framework, (ii) intersection state estimation, and (iii) validation. Within the simulation framework, real-world data are used as input in SUMO software to create a simulation model, which results with STM and *TimeLoss* parameter. Intersection state estimation incorporates fuzzy logic, which takes STMs as an input to estimate the intersection state. In the validation step, congestion estimation results and *TimeLoss* from the first step are validated using a confusion matrix to check the accuracy of the state estimation. Every step was done by combining Python programming language and SUMO simulation software using real-world traffic data for the creation of the intersection model. More details about every step are given in the continuation.

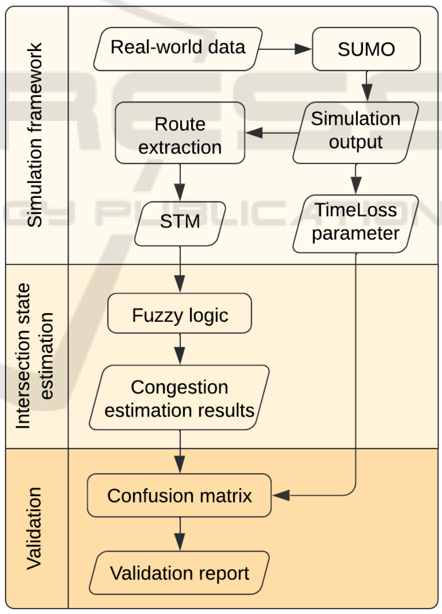


Figure 1: Graphical visualization of proposed methodology for the development of the intersection state estimator.

3.1 Simulation Framework

The simulation model is created in the open-source SUMO microscopic traffic simulator software (Behrisch et al., 2011). It represents the intersection between Heinzelova street and King Zvonimir street in the (capital) City of Zagreb, Croatia (Fig. 3).

The model is made based on real-world data, it is part of the arterial road network of the City of Zagreb, and it is known for congestions during peak hours.

Fig. 2 shows traffic volumes for each direction where it can be observed that the morning and afternoon peak hours are more evident for directions East and West, while that is not the case with directions South and North. Although this intersection is prone to congestion, it has regular traffic flow outside the peak hours without significant congestions.

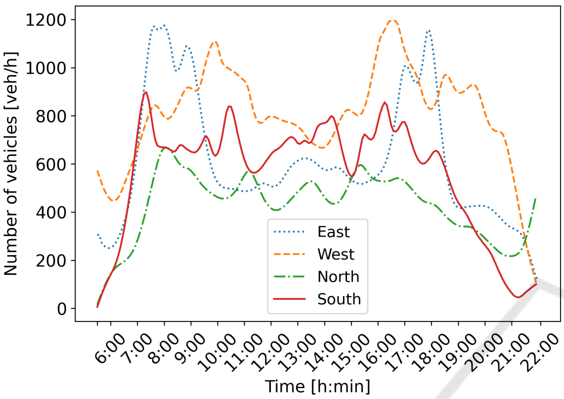


Figure 2: Traffic volume for all directions of the simulated intersection.

Data for traffic demand generation are used from (Vujić, 2013) and project SORDITO (Erdelić and Ravlić, 2016) augmented with manual measurements. Generated traffic demand in the form of traffic flow rate (veh/h) and turn probabilities is used as input for the SUMO simulator. It covers 16.5 h of a typical working day, from $5:30$ AM to $10:00$ PM, including accurate daily traffic signal programs consisting of four different ones based on the Fixed Traffic Signal Control (FTSC) regime. This simulation does not include pedestrians, but traffic light signal programs include minimum green light safety intervals for pedestrians (Miletić et al., 2020).

The configuration of the chosen isolated intersection is shown in Fig. 3. It is a traffic light signalized cross intersection with left and right turns for each traffic direction. To get more precise measurements the simulation model is divided into 50 m segments (edges) according to (Tišljarić et al., 2022) and data sampling is synchronized with traffic signal control. The speed limit is set to 50 km/h.

During the simulation, two parameters (*speed* and *TimeLoss*) are extracted for traffic state estimation. Vehicles were monitored during the simulation, and their speeds were collected from the simulation to create STM. Also, the *TimeLoss* parameter is recorded from the simulation. The *TimeLoss* parameter shows the total number of seconds vehicles have lost due to driving slower than the desired speed, and it is used for the validation of the method (Behrisch et al., 2011). Total *TimeLoss* for a single edge is the sum of all values for each vehicle crossed over the edge in a given period of time.

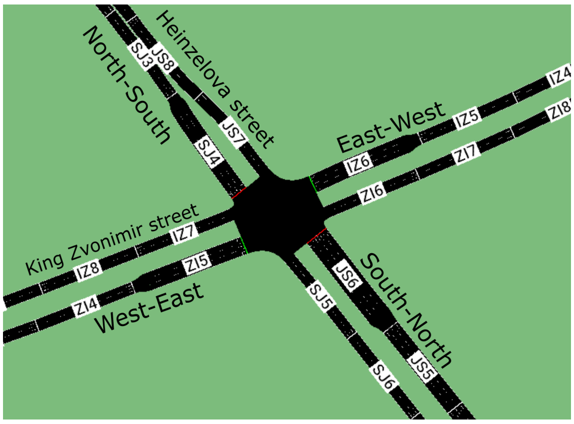


Figure 3: Configuration of the isolated intersection used in this research.

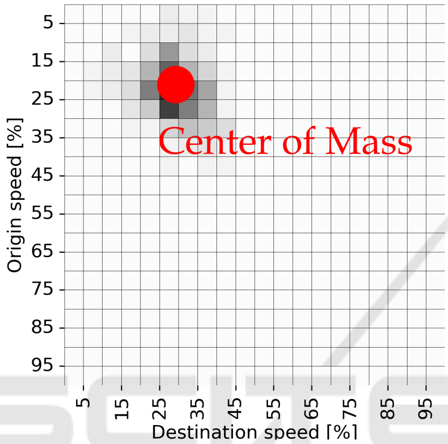
3.2 Intersection State Estimation

Intersection traffic state is represented by applying a fuzzy-based method that uses attributes extracted from the STM as input parameters. Road traffic is modeled using STMs, which are a cell transition model as proposed in (Tišljarić et al., 2020). The STM is a matrix that represent the probability of the speed change when vehicles are traveling between two consecutive road segments in the observed time period Δt. Every transition is defined with two consecutive road segments, e_i representing the origin segment, and e_{i+1}, representing destination segment. Harmonic mean vehicle's speed is computed on the transition point between e_i and e_{i+1} and placed in the corresponding STM cell in the (j,k) position, where j represents the speed computed for origin segment, and k represents the speed computed on the destination segment. Fig. 4 shows two representative examples of the extracted STMs. Thus, the final form of the STM is:

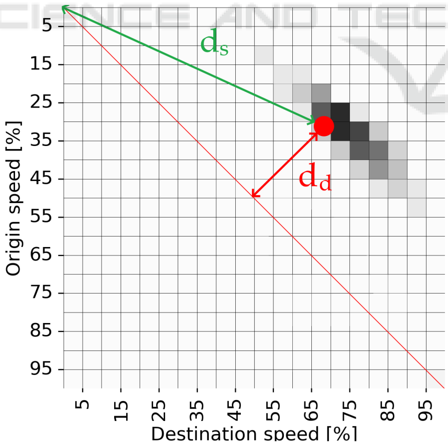
$$X(\Delta t) = \begin{pmatrix} p_{(11)} & p_{(12)} & \cdots & p_{(1k)} \\ p_{(21)} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ p_{(j1)} & \cdots & \cdots & p_{(jk)} \end{pmatrix}, \quad (1)$$

where $p_{(jk)}$ represents the probability of the speed change from j to k on the observed transition at interval Δt.

After processing the road traffic data creating the STMs, the fuzzy-based traffic state estimation method, adopted from (Tišljarić et al., 2022) is applied. The method is based on the traffic pattern position extraction, which is represented within the STM. The traffic pattern position is represented by CoM shown in Fig. 4a. The CoM's position is the most important information when working with the traffic data modeled using STMs because it shows the observed traffic parameter type, which implies the traffic state. In Fig. 4a, congestion as one of the traffic states can be detected because transitions indicate small speeds on origin and destination segments.



(a) Center of mass estimation for congested traffic flow



(b) Extraction of attributes from STM

Figure 4: Examples of STMs representing road traffic states.

There are two attributes extracted from the STM as input parameters for the fuzzy-based traffic state estimation (shown in Fig. 4b): distance from the STM's source d_s, and distance from the STM's diagonal d_d, both computed as the Euclidean distances. Thus, d_s is used as a parameter for the congestion estimation, and d_d parameter for the anomaly potential (Tišljarić et al., 2021). The d_s and d_d parameter computation process is described in (Tišljarić et al., 2022). The set of $IF-THEN$ fuzzy rules used for bottleneck probability estimation is shown in Table 1.

Table 1: Set of fuzzy rules used for bottleneck probability estimation (Tišljarić et al., 2022).

	d_D		d_S		p_b
IF	d_D is "small"	AND	d_S is "small"	THEN	p_b is "large"
IF	d_D is "small"	AND	d_S is "medium"	THEN	p_b is "medium"
IF	d_D is "small"	AND	d_S is "large"	THEN	p_b is "small"
IF	d_D is "medium"	AND	d_S is "small"	THEN	p_b is "medium"
IF	d_D is "medium"	AND	d_S is "medium"	THEN	p_b is "medium"
IF	d_D is "medium"	AND	d_S is "large"	THEN	p_b is "small"
IF	d_D is "large"	AND	d_S is "small"	THEN	p_b is "large"
IF	d_D is "large"	AND	d_S is "medium"	THEN	p_b is "medium"
IF	d_D is "large"	AND	d_S is "large"	THEN	p_b is "large"

The membership functions of the input and output variables of the fuzzy-based system are shown in Fig. 5. The bottleneck probability is the output of the system with input variables modeled with d_d and d_s. Input and output fuzzy based-system variables are represented as linguistic variables with values "small", "medium", and "large", with the corresponding membership functions z-type, Gaussian, and s-type functions. The output variable p_b then represents the bottleneck probability of the observed transition at the intersection approach scale to the value from the interval $[0,1]$ (Tišljarić et al., 2022).

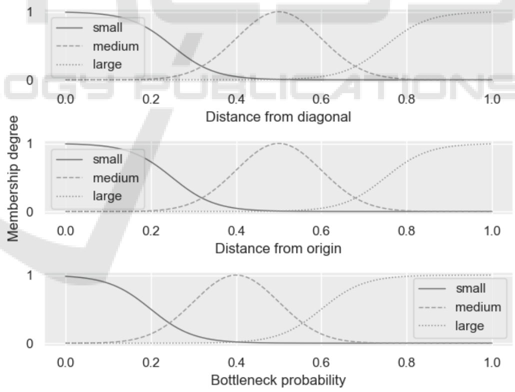
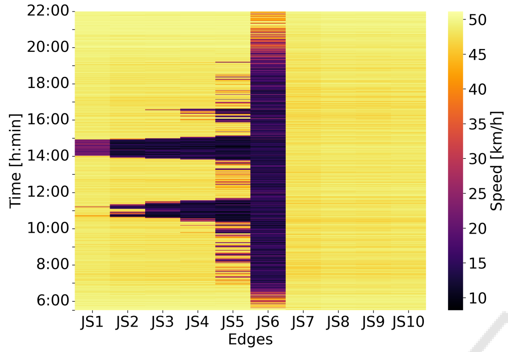


Figure 5: Fuzzy-based system setup for the bottleneck probability estimation.

4 RESULTS

4.1 Intersection State Estimation Results

A representative example of intersection state estimation results are shown in Fig. 6. The Fig. 6a shows speeds of vehicles captured with SUMO simulator software for each edge along a single direction (in this case, direction East-West). It can be observed that the speeds in front of a traffic light (*edge JS6* presented in Fig. 6a) are lower during the whole day, which is expected since the vehicles have to slow down or stop and wait for the green light. Thus, during the rush hour, slower speeds propagate towards the left side of the image at specific parts of the day, indicating the traffic queue propagation due to increased traffic demand and non-optimal traffic signal control.



(a) Harmonic mean vehicles speed extracted from the simulation

(b) Bottleneck probability

Figure 6: Example of intersection state estimation results.

Fig. 6b is the result of fuzzy inference system where peak hours can be observed. Additionally, it can be observed that during the day, the bottleneck probability is increased on edge *JS7*. Increased bottleneck probability on edge *JS7* is because an STM reflects speed change. When the vehicles start moving on the green traffic light signal, the CoM of that STM will be positioned in the upper right corner. In that case, d_s and d_d are high which results with increased bottleneck probability.

Fig. 7 represents the example of *TimeLoss* results, which were collected for each edge during the whole simulation. It can be observed that the *TimeLoss* parameter is increased during peak hours and propagates towards the left side of the figure in the same way in Figs. 6a and 6b.

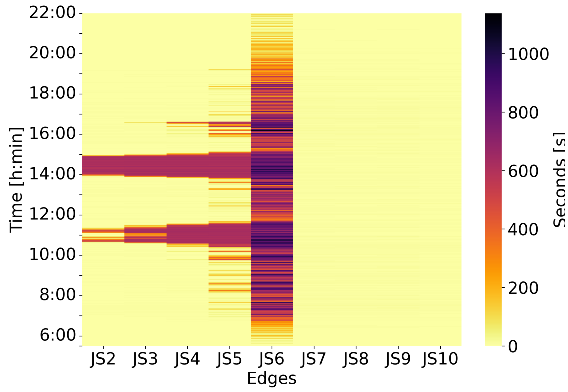


Figure 7: Example of results of *TimeLoss* parameter.

4.2 Validation

The validation process was conducted using confusion matrices, and results were presented using a confusion matrix and a classification report. The classification report contains the total accuracy of the model, precision, recall, and F1-score for each class. In this case, class is particular traffic state, and two classes are detected. Congestion class if there is a traffic light induced queue forming, and free flow class without a traffic light queue. Values in the confusion matrix are represented as a percentage of each class's total number of data instances.

For validation purpose, the *TimeLoss* measure obtained from the simulation tool is used as the ground truth data, and the predicted value is the bottleneck probability. The values in the matrix represent accuracy as the number of data instances that are correctly classified. In binary classification, precision is calculated as the number of true positives (TP) divided by the total number of TP and false positives (FP) according to the equation (Brownlee, 2016):

$$Precision = \frac{TP}{TP + FP}. \quad (2)$$

The recall value is calculated as the number of TP divided by the total number of TP and false negatives (FN) according to the following equation (Brownlee, 2016):

$$Recall = \frac{TP}{TP + FN}. \quad (3)$$

F1-score is calculated as the harmonic mean of precision and recall according to the following equation (Brownlee, 2016):

$$F1 - score = \frac{2 * Precision * Recall}{Precision + Recall}. \quad (4)$$

The accuracy parameter is used to measure the accuracy of the model. It is computed as the number of data instances that were predicted correctly divided by the number of all predictions made on the test set. The accuracy parameter is calculated as (Brownlee, 2016):

$$Accuracy = \frac{TP+TN}{TP+TN+FP+FN}, \quad (5)$$

where TN is the number of true negative predictions. Table 2 represents validation results for East-West, West-East, South-North, and North-South directions. All four traffic directions have high precision and recall rate parameters for the Free flow class, which results in a high value of F1-score. The Congestion class precision values range from 89% for direction South-North to 98% for direction North-South. Although South-North's direction has the lowest precision of 89%, the recall value is 91%, resulting in a 90% F1-score, similar to the other two directions, East-West and West-East.

Table 2: Validation results for all directions.

Direction		Precision	Recall	F1-score
East-West	Free flow	0.99	0.99	0.99
	Congestion	0.95	0.90	0.92
	Accuracy	0.98		
West-East	Free flow	0.98	0.99	0.99
	Congestion	0.93	0.87	0.90
	Accuracy	0.98		
South-North	Free flow	0.98	0.98	0.98
	Congestion	0.89	0.91	0.90
	Accuracy	0.97		
North-South	Free flow	0.99	1.00	0.99
	Congestion	0.98	0.96	0.97
	Accuracy	0.99		

Table 3 represents confusion matrices for the earlier mentioned four directions. The free flow state is correctly classified at least 98% times. The classification is worst performing for the West-East direction, with 87% correct classification for congestion class which means in 12% cases, it miss-classifies congested state as free-flow state, which is overall a pretty decent result.

Table 3: Confusion matrices for all directions.

Direction	Predicted / Known	Free flow	Congestion
East-West	Free flow	0.99	0.01
	Congestion	0.09	0.90
West-East	Free flow	0.99	0.01
	Congestion	0.12	0.87
South-North	Free flow	0.98	0.02
	Congestion	0.08	0.92
North-South	Free flow	0.99	0.01
	Congestion	0.04	0.96

5 CONCLUSIONS

In this paper, the possibility of using STM in combination with fuzzy logic to estimate the intersection state was presented. The research is conducted in the SUMO simulator and the intersection model is made based on real-world data. The results from the microscopic simulator are validated with additional *TimeLoss* parameter and confusion matrix. The validation of the results indicated that this method is successful in intersection state estimation with a total accuracy score of 98%. From the results, we can precisely detect congestion's temporal and spatial characteristics which match real-world situations.

However, this method has its drawbacks. STM is sensitive to vehicle speed changes. Whether the vehicle is slowing down or speeding up, it tends to show increased bottleneck probability which can be misleading. Example of that behavior is shown in results section. Increased bottleneck probabilities can be observed on the edge $JS7$ in Fig. 6b, and that edge is after the traffic light where the vehicles are accelerating. The impact of such behavior on intersection management systems should be considered in future applications. Another drawback is that it cannot be directly determined how many vehicles are waiting ahead of the traffic lights. Although it is possible to detect bottleneck on a specific edge, it cannot be determined how many vehicles are on that road segment. Also, the exact length of the waiting queue cannot be determined. Thus, the future work will be focused on STM improvement and applying the presented method to multiple intersections, where the intersection traffic state estimation will be used as an input parameter for the intersection control system.

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