Generalized Mutant Subsumption

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Abstract: Mutant Subsumption is an ordering relation between the mutants of a base program, which ranks mutants according to inclusion relationships between their differentiator sets. The differentiator set of a mutant with respect to a base program is the set of inputs for which execution of the base program and the mutant produce different outcomes. In this paper we propose to refine the definition of mutant subsumption by pondering, in turn: what do we consider to be the outcome of a program's execution? under what condition do we consider that two outcomes are comparable? and under what condition do we consider that two comparable outcomes are identical? We find that the way we answer these questions determines what it means to kill a mutant, how subsumption is defined, how mutants are ordered by subsumption, and what set of mutants is minimal.

1 INTRODUCTION

1.1 Program Executions and Outcomes

The mutants of a program are generated by applying localized syntactic alterations to the source code of a program, and are typically used to assess the quality of test suites: a good test suite is one that yields different outcomes (from the base program) for all the mutants that are not semantically equivalent to the program. Mutation testing is a reliable way to assess the effectiveness of test suites, but it is also an expensive proposition. As a consequence, it is sensible to try to reduce the size of mutant sets, without loss of effectiveness. In (Marsit et al., 2021), Marsit et al. propose an algorithm to reduce the size of a set of mutants by partitionning the set of mutants into equivalence classes, modulo semantic equivalence, and retaining one mutant from each equivalence class. While this criterion is non-controversial (two semantically equivalent mutants are as good as just one), it may be sub-optimal: In (Guimaraes et al., 2020; Parsai and Demeyer, 2017; Souza, 2020; Li et al., 2017; Jia and Harman, 2008; Kurtz et al., 2014; Kurtz et al., 2015; Tenorio et al., 2019), a more sophisticated criterion is used: a partial ordering is defined between mutants of a given base program P, whereby a mutant M is said to *subsume* a mutant M' if and only if M produces a different outcome from P for at least one input, and any input for which M produces a different outcome from P, M' produces necessarily a different outcome from P.

The original definition of mutant subsumption (Kurtz et al., 2014) refers to program outcomes being identical or distinct, but does not dwell on what exactly is the outcome of a program and when can we consider that two program outcomes are identical. In this paper we propose to generalize the concept of *subsumption* by seeking to ponder the following questions:

- *What Is the Outcome of a Program?* Programs and mutants do not always terminate normally and produce a well-defined/ agreed upon outcome. Programs may fail to terminate (if they enter an infinite loop, and eventually time out), or they may encounter an exceptional condition, such as an array reference out of bounds, a reference to a nil pointer, a division by zero, an undefined expression (e.g. the logarithm of a negative number), an overflow, an underflow, etc. In fact many mutation operators are prone to create the conditions of divergence even when the original pro-

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gram terminates normally. The question that we must ponder is: when a program fails to converge, do we consider that it has no outcome, or that failure to converge is itself an outcome.

Also, even when a program does terminate normally, it is not always clear what we consider to be its outcome: is it its final state or the output that the program delivers as a projection of the final state? For example, if a program permutes two variables x and y using an auxiliary variable z, what is the outcome of the program? is it the final values of x, y and z, or just the final values of x and y? This can get more complicated/ ambiguous when we consider global variables, parameter passing, communication channels, operations with side effects, etc.

- *Under what condition do we consider that the Outcomes of Two Programs Are Comparable?* When the execution of a program on some input terminates after a finite number of steps without causing any exception, such as an array reference out of bounds, a reference to a nil pointer, a runtime stack overflow, or an illegal operation (such as a square root of a negative number, the log of zero or a negative number, a division by zero, an arithmetic overflow, etc) we say that the execution *converges* (or: *terminates normally*); else we say that the execution *diverges*. When two programs converge, comparing their outcomes poses no problem; but we must decide whether their outcomes are comparbale when one of them or both of them diverge.
- *When Do We Consider That Two Program Outcomes Are Identical or Distinct?* If we consider that the outcome of a program that converges and the outcome of a program that diverges are comparable, then it is sensible to consider that their outcomes are distinct. But if two programs diverge for a given input, do we consider that their outcomes are incomparable, or that they are comparable and identical? What if the divergence is of the same type (e.g. both fail to terminate, or both attempt a division by zero, etc)? What if both fails at the same statement of the source code?

In this paper we argue that the definition of subsumption depends critically on how we answer these three questions. Specifically, we present three possible definitions of subsumption, which correspond to different interpretations of program outcomes and how to compare them. Then we show on a concrete example how these yield different ordering relations on the mutants of a base program, and different minimal mutant sets. It appears that the original definition of mutant subsumption (Kurtz et al., 2014) makes no provision for the possibility of divergence, hence assumes implicitly that programs and mutants converge for all inputs; it also seems to assume that the outcome of a program that converges is well-understood, hence requires no careful consideration.

1.2 Motivation

While the discussion of what is a program outcome and when two program outcomes are considered identical may sound like an academic exercise in hair-splitting, we argue that it is fact an important consideration in mutation testing, because many mutation operators are prone to cause programs to diverge, even when the base program converges:

- if we consider a loop that addresses an array at indices 0 through $N-1$
  ```
  while (i<N) {a[i]=..;...;i++;}
  ```
  and the logical operator `<` is changed to `<=` then the resulting mutant will diverge due to an array index out of bound.
- If we consider a guarded assignment statement such as:
  ```
  if ((x!=0) && (x!=1)) {y=1/x(x-1);}
  ```
  and the `&&` is replaced by a logical OR `||`, then the resulting mutant will diverge for $x = 0$ and for $x = 1$, due to a division by zero.
- If we consider a loop that decrements an integer variable by 2 at each iteration while the variable is positive,
  ```
  x=5; while (x>0) {x=x-2;..;}
  ```
  and we change the comparison operator `>` to `!=`, then (if the initial value is odd) the resulting mutant will diverge due to an infinite loop.

In theory, we should also consider cases where the base program itself may fail to converge for some test data; of course one may wonder why we would test a program outside its domain, but it is the domain of the specification, not the domain of the program, that determines what test data to run.

1.3 Agenda

To discuss these questions, we need to introduce a framework for defining and analyzing program functions; this is the subject of section 2. In section 3 we introduce the concept of *differentiator set*, which serves as a basis for redefining subsumption, and in section 4 we present three distinct definitions of subsumption. In section 5 we consider a benchmark program, generate its mutants, then we analyze subsumption relationships between these mutants, using the three definitions introduced in section 4; we show that (not surprisingly) these three definitions give three distinct subsumption graphs, and three distinct minimal mutant sets. In section 6 we summarize our findings and suggest venues for further investigation.

2 MATHEMATICS FOR PROGRAM ANALYSIS

2.1 Sets and Relations

In this paper, we use relations and functions (Brink et al., 1997; Schmidt, 2010) to capture program specifications and program semantics. For the sake of simplicity, and without loss of generality, we consider homogeneous relations on sets represented by program-like declarations. Modeling the program behavior by homogeneous relations encompasses the case where we want to model it by a relation from inputs to outputs: it suffices to add an input stream and an output stream as state variables. We generally denote sets (referred to as *spaces*) by S, elements of S (referred to as *states*) by lower case s, specifications (binary relations on S) by R and programs (functions on S) by P, Q. We denote the domain of a relation R (or a function P) by $dom(R)$ ($dom(P)$). Because we model programs and specifications by homogeneous relations / functions, we usually talk about initial states and final states; we may talk about inputs to refer to the initial value of the input stream and outputs to refer to the final value of the output stream.

2.2 Programs and Specifications

A specification R includes all the initial state / final state pairs that the specifier considers correct; hence the domain of a specification R ($dom(R)$) includes all the initial states for which candidate programs must make provisions. A program P includes all the initial state/ final state pairs (s,s') such that if P starts execution in initial state s it terminates normally (i.e. after a finite number of steps, without raising an exception) in state s'. From this definition, it stems that the domain of program P ($dom(P)$) is the set of initial states s such that execution of P on s terminates after a finite number of steps, and does not raise an exception (such as an overflow, underflow, division by zero, array reference out of bounds, etc).

For the sake of simplicity, we restrict our study to deterministic programs, i.e. programs which map each initial state to at most one final state. It would be interesting to include non-deterministic programs (including concurrent programs), but this would complicate our model, and introduce other mutation-specific difficulties (Vercammen et al., 2021).

2.3 Absolute Correctness

We consider a program P on space S and a specification R on S; without loss of generality, we model the semantics of a program by the (homogeneous) relation that the program defines from its initial states to its final states.

Definition 1. *We say that a program P on space S is* correct *with respect to specification R on S if and only if:*

$$dom(R) = dom(R \cap P).$$

Intuitively, a program P is correct with respect to a specification R if and only if for all s in $dom(R)$, execution of P on s converges and produces a final state that satisfies the condition $(s,P(s)) \in R$. The domain of the intersection of R and P represents the set of initial states for which program P behaves as R mandates; it is called the *competence domain* of P with respect to R. Figure 1 shows a simple example of a (non-deterministic) specification R and two programs P and P' such that P is correct with respect to R and P' is not; the competence domains of P and P' are shown by the ovals.

Definition 2. *We say that a program P on space S is* partially correct *with respect to specification R on S if and only if:*

$$dom(R) \cap dom(P) = dom(R \cap P).$$

Intuitively, a program P is partially correct with respect to a specification R if and only if for all s in $dom(R)$, if execution of P on s converges then it produces a final state that satisfies the condition $(s,P(s)) \in R$. When we want to contrast correctness with partial correctness, we may refer to the former as *total correctness*. Our definitions of total and partial correctness are equivalent, modulo differences of notation, to traditional definitions with respect to pre/post specifications (Hoare, 1969; Gries, 1981; Dijkstra, 1976; Manna, 1974). See Figure 2: Program Q is partially correct with respect to R because for any initial state of $dom(R)$ for which it terminates, program Q delivers a final state that satisfies specification R; by contrast, program Q' is not partially correct with respect to R, even though it terminates normally for all initial states in $dom(R)$, because it does not satisfy specification R; neither Q nor Q' is totally correct with respect to R. We admit without proof that if P is totally correct with respect to R then it is necessarily partially correct with respect to R.

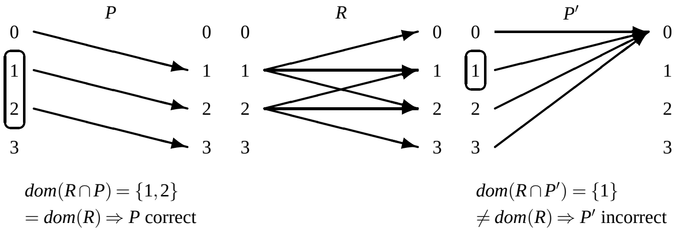


Figure 1: Total Correctness.

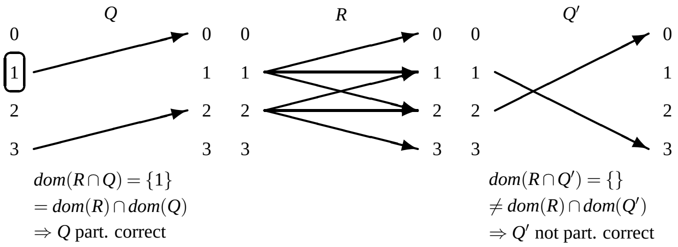


Figure 2: Partial Correctness.

2.4 Relative Correctness

Whereas total correctness and partial correctness define a property between a program and a specification, relative correctness defines a property between two programs and a specification.

Definition 3. *Given a specification R on space S and two programs P and P' on S, we say that P' is* more-correct *than P with respect to R (denoted by: $P' \sqsupseteq_R P$ or $P \sqsubseteq_R P'$) if and only if:*

$$dom(R \cap P) \subseteq dom(R \cap P').$$

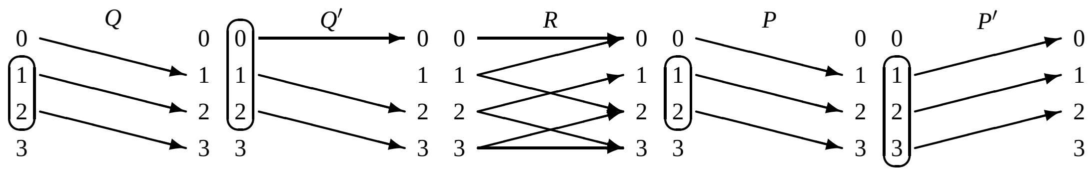
In other words, P' is more-correct than P with respect to R if and only if it has a larger competence domain with respect to R. Whenever we want to contrast correctness (definition 1) with relative correctness, we may refer to the former as *absolute correctness*. See Figure 3; it shows two instances of relative correctness. Q' is more-correct than Q by virtue of imitating the correct behavior of Q; by contrast, P' is more-correct than P by virtue of a different correct behavior; because specification R is non-deterministic, correct behavior is not unique.

3 DIFFERENTIATOR SETS

In section 1, we had argued that while the definitions of mutant subsumption refer to program outcomes and to the condition under which two program outcomes are identical, they are not perfectly clear about what constitutes the outcome of a program, when two program outcomes are comparable, and if they are when can we consider them to be identical. In this section, we address this ambiguity by introducing several definitions of *differentiator sets*, which reflect different interpretations to the questions above.

Given two programs, say P and Q, the *differentiator set* of P and Q is the set of initial states for which execution of P and Q yield different outcomes. For the purposes of this paper, we adopt the three definitions of differentiator sets proposed by Mili in (Mili, 2021):

- *Basic Differentiator Set*. The basic differentiator set of two programs P and Q on space S is defined only if P and Q converge for all s in S; it is the set of states s such that $P(s) \neq Q(s)$. This set is denoted by $\delta_B(P,Q)$ and defined by:
$$\delta_B(P,Q) = \overline{dom(P \cap Q)}.$$
- *Strict Differentiator Set*. The strict differentiator set of two programs P and Q on space S is defined regardless of whether P and Q converge for all initial states. It includes all the states for which executions of P and Q both converge and yield distinct outcomes. This set is denoted by $\delta_S(P,Q)$ and defined by:
$$\delta_S(P,Q) = dom(P) \cap dom(Q) \cap \overline{dom(P \cap Q)}.$$
- *Loose Differentiator Set*. The loose differentiator set of two programs P and Q on space S is defined regardless of whether P and Q converge for all initial states. It includes all the states for which executions of P and Q both converge and yield distinct outcomes, as well as the states for which only one of the programs converges and the other diverges. This set is denoted by $\delta_L(P,Q)$ and defined by:
$$\delta_L(P,Q) = (dom(P) \cup dom(Q)) \cap \overline{dom(P \cap Q)}.$$



Preserving Correct Behavior
$((R \cap Q) \subseteq (R \cap Q'))$

Preserving Correctness
$((R \cap P)L \subseteq (R \cap P')L)$

Figure 3: Relative Correctness: ($Q' \sqsupseteq_R Q$), ($P' \sqsupseteq_R P$).

Figure 4 illustrates the three definitions of differentiator sets (represented in red in each case). To gain an intuitive understanding of these definitions, it suffices to note the following details:

- The domain of program P ($dom(P)$) is the set of initial states on which execution of P converges (i.e. terminates normally after a finite number of steps without raising any exception or attempting any illegal operation). We assume that when a program enters an infinite loop, it gets timed out by the run-time environment, so that non-termination is an observable outcome.
- The domain of $(P \cap Q)$ is the set of inputs for which programs P and Q converge and return the same outcome.
- The complement of the domain of $(P \cap Q)$ is the set of inputs for which program P and Q converge and return distinct outcomes. In other words,

$$\overline{dom(P \cap P)} = \{s : P(s) \neq Q(s)\}.$$

A possible fourth interpretation is to consider that when two programs diverge, they have the same outcome; we illustrate this situation in Figure 5, to highlight its contrast with the situations represented in Figure 4. Notice that it has the same differentiator set as δ_L, but differs from it in the way it interprets simultaneous divergence: whereas δ_L considers that in the case of simultaneous divergence we cannot decide whether the outcomes are identical, the interpretation of Figure 5 considers that simultaneous divergence is a case of identical outcome. Because this interpretation is controversial (and perhaps of limited interest), we do not consider it further in this paper.

Given that differentiator sets reflect the extent of behavior difference between two programs, we expect that when a differentiator set is empty, the programs have some measure of identity/ similarity. This is discussed in the following Propositions; first, we briefly introduce a lemma from relational algebra (Brink et al., 1997; Schmidt, 2010).

Lemma 1. *If two functions F and G satisfy the conditions $F \subseteq G$ and $dom(G) \subseteq dom(F)$ then $F = G$.*

Proposition 1. *Given two programs P and Q on space S such that $dom(P) = S$ and $dom(Q) = S$. The basic differentiator set of P and Q is empty if and only if $P = Q$.*

Proof. The proof of sufficiency is trivial.

Proof of necessity: If $\delta_B(P,Q) = \emptyset$ then $dom(P \cap Q) = S$. By hypothesis, $dom(P) = S$. By set theory, we have $(P \cap Q) \subseteq P$. By the lemma above, we infer $(P \cap Q) = P$, whence by set theory we infer $P \subseteq Q$. By permuting P and Q in the argument above, we find $Q \subseteq P$. □

For the sake of convenience, we often equate a program with its function; this may give rise to some odd-sounding statements such as the claim that some program is correct with respect to another. When we say that program P is correct with respect to program Q, we really mean that P, as a program written in some programming language, is correct with respect to the *function* of program Q, which we interpret as a specification on space S. With this qualification in mind, we proceed with the next propositions linking differentiator sets with properties of correctness.

Proposition 2. *Given two programs P and Q on space S, the strict differentiator set of P and Q is empty if and only if program P is partially correct with respect to the function of program Q (interpreted as a specification).*

Proof. The proof of sufficiency stems readily from the definition of partial correctness (Definition 2) and the definition of strict differentiator sets.

Proof of Necessity. From $\delta_S(P,Q) = \emptyset$ we infer $dom(P) \cap dom(Q) \subseteq dom(P \cap Q)$. By set theory (and monotonicity of the $dom()$) we infer $dom(P \cap Q) \subseteq dom(P)$ and $dom(P \cap Q) \subseteq dom(Q)$, whence $dom(P \cap Q) \subseteq dom(P) \cap dom(Q)$. From $dom(P \cap Q) = dom(P) \cap dom(Q)$ we infer that P is partially correct with respect to the function of Q. □

Proposition 3. *Given two programs P and Q on space S, the loose differentiator set of P and Q is empty if and only if program P is totally correct with*

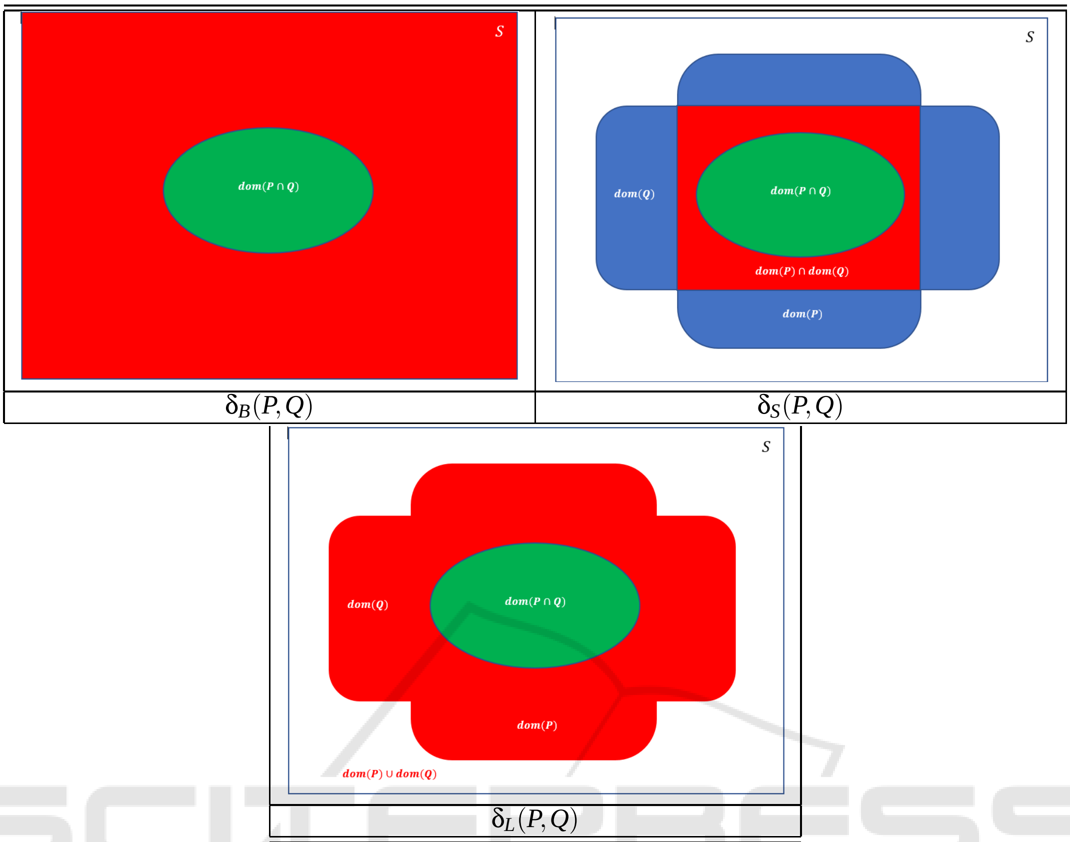


Figure 4: Three Definitions of Differentiator Sets.

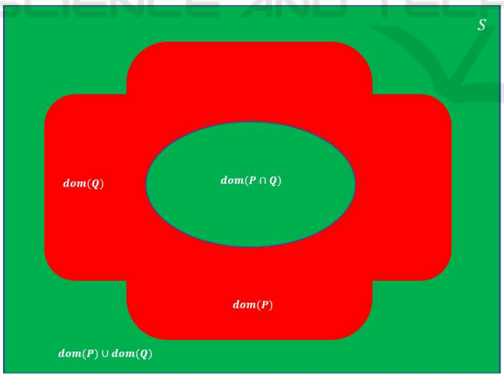


Figure 5: Fourth Interpretation of Identical Outcome.

respect to the function of program Q (interpreted as a specification).

Proof. The proof of sufficiency stems readily from the definition of total correctness (Definition 1) and the definition of loose differentiator sets.

Proof of Necessity. From $\delta_L(P,Q) = \emptyset$ we infer $dom(P) \cup dom(Q) \subseteq dom(P \cap Q)$. From which we infer, a fortiori: $dom(P) \subseteq dom(P \cap Q)$. The reverse incluse is a tautology. From Definition 1 we infer that P is totally correct with respect to the function of Q. $\square$

In Propositions 2 and 3, the roles of P and Q can be permuted: each is (partially/ totally) correct with respect to the function of the other; in the context of mutation testing, we use these propositions asymmetrically,

4 MUTANT SUBSUMPTION

In (Kurtz et al., 2014; Kurtz et al., 2015), Kurtz et al define the concept of *true subsumption* as follows:

Definition 4. *Given a program P on S and two mutants M and M', we say that M* subsumes *M' with respect to P if and only if:*

P1 There exists an initial state s for which P and M produce different outcomes.

P2 For all s in S such that P and M produce different outcomes, so do P and M'.

Since this definition makes no mention of P, M or M' failing to converge, we assume that P, M and

M' are considered to converge for all initial states. The following Proposition formulates subsumption by means of basic differentiator sets.

Proposition 4. *Given a program P on space S and two mutants M and M' of P, M subsumes M' if and only if:*

$$\emptyset \subset \delta_B(P,M) \subseteq \delta_B(P,M').$$

Proof. We consider the first condition

$\emptyset \subset \delta_B(P,M)$

$\Leftrightarrow$ {definition of $\delta_B(P,M)$}

$\exists s : s \in \overline{dom(P \cap Q)}$

$\Leftrightarrow$ {interpreting the complement}

$\exists s : \neg(s \in dom(P \cap Q))$

$\Leftrightarrow$ {interpreting $dom(P \cap Q)$}

$\exists s : \neg(P(s) = Q(s)))$

$\Leftrightarrow$ {definition 4}

Condition P1.

As for the second conditions,

$\delta_B(P,M) \subseteq \delta_B(P,M')$

$\Leftrightarrow$ {definition of $\delta_B(,)$}

$\forall s : s \in \overline{dom(P \cap M)} \Rightarrow s \in \overline{dom(P \cap M')}$

$\Leftrightarrow$ {interpreting the complement}

$\forall s : \neg(s \in dom(P \cap M)) \Rightarrow \neg(s \in dom(P \cap M'))$

$\Leftrightarrow$ {interpreting the domain}

$\forall s : \neg(P(s) = M(s)) \Rightarrow \neg(P(s) = M'(s))$

$\Leftrightarrow$ {definition 4}

Condition P2. $\square$

The following Proposition reformulates subsumption by means of relative correctness.

Proposition 5. *Given a program P on space S and two mutants M and M' of P, M subsumes M' if and only if M is not equivalent to P and it is more-correct than M' with respect to (the function of) P.*

Proof. According to Proposition 1, condition P1 is equivalent to: P and M are not equivalent.

On the other hand, we have shown above that condition P2 is equivalent to:

$\delta_B(P,M) \subseteq \delta_B(P,M')$

$\Leftrightarrow$ {definition of $\delta_B(,)$}

$\forall s : s \in \overline{dom(P \cap M)} \Rightarrow s \in \overline{dom(P \cap M')}$

$\Leftrightarrow$ {Boolean identity}

$\forall s : s \in dom(P \cap M') \Rightarrow s \in dom(P \cap M)$

$\Leftrightarrow$ {set theory}

$dom(P \cap M') \subseteq dom(P \cap M)$

$\Leftrightarrow$ {Definition 3}

$M \sqsupseteq_P M'$. $\square$

Proposition 4 provides an alternative formula to define mutant subsumption in the case where we assume that all programs and mutants terminate for all initial states. This Proposition is formulated in terms of basic differentiator sets, which are defined when the program and its mutants are assumed to converge for all initial states; but in section 3, we have introduced two more definitions of differentiator sets, which do not assume universal convergence of programs and mutants, and take a liberal interpretation of program outcomes and when to consider outcomes as identical or distinct. The following definitions generalize the concept of subsumption to the case when programs and their mutants do not necessarily converge for all initial states.

Definition 5. Strict Subsumption. *Given a program P on space S and two mutants M and M' of P, we say that M* strictly subsumes *M' if and only if:*

$$\emptyset \subset \delta_S(P,M) \subseteq \delta_S(P,M').$$

Definition 6. Loose Subsumption. *Given a program P on space S and two mutants M and M' of P, we say that M* loosely subsumes *M' if and only if:*

$$\emptyset \subset \delta_L(P,M) \subseteq \delta_L(P,M').$$

In the next section,we will see that the distinction between the basic definition of subsumption (Definition 4, (Kurtz et al., 2014; Kurtz et al., 2015)), strict subsumption, and loose subsumption is not a mere academic exercise. These definitions yield vastly different subsumption graphs.

5 ILLUSTRATION

We consider the Java benchmark program of *jTerminal* (available online at http://www.grahamedgecombe.com /projects /jterminal), an open-source software product routinely used in mutation testing experiments (Parsai and Demeyer, 2017). We apply the mutant generation tool *LittleDarwin* in conjunction with a test generation and deployment class that includes 35 test cases (Parsai and Demeyer, 2017); we augmented the benchmark test suite with two additional tests, intended specifically to *trip* the base program *jTerminal*, by causing to diverge. We let T designate the test augmented test suite codified in this test class; all our analysis of mutant equivalence, mutant redundancy, mutant survival, etc is based on the outcomes of programs and mutants on this test suite (and carefully selected subsets thereof). Execution of LittleDarwin on jTerminal yields 94 mutants, numbered m1 to m94; the test of these mutants against the original using the selected test suite kills 48 mutants; for the sake of documentation, we list them below:

`m1, m2, m7, m8, m9, m10, m11, m12, m13,`

m14, m15, m16, m17, m18, m19, m21, m22, m23, m24, m25, m26, m27, m28, m44, m45, m46, m48, m49, m50, m51, m52, m53, m54, m55, m56, m57, m58, m59, m60, m61, m62, m63, m83, m88, m89, m90, m92, m93.
The remaining 46 mutants are semantically equivalent to the pre-restriction of jTerminal to T. The first order of business is to partition these 48 mutants into equivalence classes modulo semantic equivalence; we find that these 48 mutants are partitioned into 31 equivalence classes, and we select a member from each class; we let μ be the set of selected mutants: $\mu =$
m1, m2, m7, m11, m13, m15, m19, m21, m22, m23, m24, m25, m27, m28, m44, m45, m46, m48, m49, m50, m51, m52, m53, m55, m56, m57, m60, m63, m92, m93.

We resolve to draw the subsumption graphs of these mutants according to the three definitions:

- Basic/ True Subsumption:

$$\emptyset \subset \delta_B(jTerminal, M) \subseteq \delta_B(jTerminal, M').$$

- Strict Subsumption:

$$\emptyset \subset \delta_S(jTerminal, M) \subseteq \delta_S(jTerminal, M').$$

- Loose Subsumption:

$$\emptyset \subset \delta_L(jTerminal, M) \subseteq \delta_L(jTerminal, M').$$

To this effect, we must compute the differentiator sets $\delta_B(jTerminal, M)$, $\delta_S(jTerminal, M)$, $\delta_L(jTerminal, M)$ for all 31 mutants selected above, with respect to *jTerminal*. For the sake of illustration, we show in Table 1 the output file of the base program *jTerminal*, as well as that of mutant M22; the number at the start of each line identifies the input. Using this table, we can derive the differentiator set of M22 with respect to *jTerminal*; this is shown in Table 2, under all. three interpretations (basic, strict, loose) of differentiator sets; these sets can be inferred from the definitions, by analyzing the output files of the base program and the mutant. The first observation that we can make about these output files is that divergence, far from being an exceptional circumstance, is in fact a very frequent occurrence; notwithstanding the two instances of divergence that we have triggered on purpose at lines 9 and 10, mutant M22 fails to converge for several other inputs, which are part of the original benchmark test suite.

Note that this experiment is artificial in the sense that whereas the strict and loose definitions of differentiator sets can be applied to the same combination of program and test suite, the basic definition can only be applied when we know, or assume, that the base program and all the mutants converge for all the elements of the test suite. In the case of *jTerminal* and its mutants, this assumption does not hold, as virtually all of them fail to converge on at least some elements of T. We obviate this difficulty by considering that divergence is itself an execution outcome, but this is merely a convenient assumption for the sake of the experiment.

By computing the basic, strict and loose differentiator sets of all the mutants with respect to *jTerminal* and comparing them for inclusion, we derive the subsumption relations between the mutants, which we can represent by graphs; these graphs are given in, respectively, Figures 6, 7 and 8. Nodes in these graphs represent mutants and arrows represent subsumption relations: whenever there is an arrow from mutant M to mutant M', it means that M subsumes M' (hence M' can be eliminated from the mutant set without affecting its effectiveness). When two mutants subsumes each other (for example $M27$ and $M28$ in 7), this means that though these mutants are distinct from each other (they compute functions functions), they have the same differentiator set with respect to *jTerminal*.

From these graphs, we derive minimal mutant sets by selecting the maximal nodes in the subsumption ordering. Once we have the minimal mutant sets, we derive minimal test suites that kill all the mutants in these sets. We verify, in each case, that the test suites that kills all the mutants of the minimal mutant sets actually kill all the 48 non-equivalent mutants derived in our experiment; this comes as no surprise, since this precisely the rationale for deleting subsumed mutants.

For strict subsumption, for example, we find the following minimal mutant set:
m22, m23, m27, m28, m44, m45, m48, m50, m51, m54, m56, m61, m83, m92, m93.
Using this mutant set, we derive minimal test suites that kill all these mutants; we find 6 minimal test suites, of size 7:
Suite 1: {t7,t16,t18,t20,t21,t22,t25}
Suite 2: {t7,t16,t18,t20,t21,t22,t26}
Suite 3: {t16,t18,t20,t21,t22,t23,t25}
Suite 4: {t16,t18,t20,t21,t22,t25,t27}
Suite 5: {t16,t18,t20,t21,t22,t23,t26}
Suite 6: {t16,t18,t20,t21,t22,t26,t27}
By virtue of subsumption, these test suites kill all 31 mutants selected above; by virtue of equivalence, they necessarily kill all 48 killable mutants of *jTerminal*.

Using the basic interpretation of subsumption, we find 96 minimal test suites, all of them of size 12; for the loose interpretation of subsumption, we find 48 minimal test suites, all of them of size 11. Due to

Table 1: Outputs of jTerminal and Mutants.

t3, null	t3, null
t4, c	t4, java.lang.NullPointerException
t5, c	t5, java.lang.NullPointerException
t6, a	t6, java.lang.NullPointerException
t7, null	t7, com.grahamedgecombe.jterminal.TerminalCell@5f8ed237
t8, B	t8, java.lang.NullPointerException
t1, h	t1, h
t23, com.grahamedgecombe.jterminal.TerminalCell@47089e5f	t23, null
t24, X	t24, java.lang.NullPointerException
t25, java.awt.Color[r=255,g=0,b=0]	t25, java.lang.NullPointerException
t26, java.awt.Color[r=255,g=255,b=0]	t26, java.lang.NullPointerException
t27, com.grahamedgecombe.jterminal.TerminalCell@4141d797	t27, null
t28, X	t28, java.lang.NullPointerException
t29, java.awt.Color[r=0,g=0,b=0]	t29, java.lang.NullPointerException
t30, java.awt.Color[r=192,g=192,b=192]	t30, java.lang.NullPointerException
t33, H	t33, java.lang.NullPointerException
t34, i	t34, java.lang.NullPointerException
t35, null	t35, java.lang.NullPointerException
t36, 1	t36, 1
t37, 3	t37, 3
t2, h	t2, h
t9, java.lang.IndexOutOfBoundsException	t9, java.lang.ArrayIndexOutOfBoundsException
t10, java.lang.IndexOutOfBoundsException	t10, java.lang.ArrayIndexOutOfBoundsException
t31, 3	t31, 3
t32, 17	t32, 17
t11, A	t11, java.lang.ArrayIndexOutOfBoundsException
t12, null	t12, java.lang.ArrayIndexOutOfBoundsException
t13, 5	t13, 5
t14, 2	t14, 2
t15, 7	t15, 7
t16, 3	t16, 3
t17, 0	t17, 0
t18, 3	t18, 3
t19, 0	t19, 0
t20, 2	t20, 2
t21, 3	t21, 3
t22, 7	t22, 7
Output of Base Program, jTerminal	Output File, mutant M22

Table 2: Differentiator Sets of Mutant M22.

Mutant	δ_B	δ_S	δ_L
M22	{t4, t5, t6, t7, t8, t23, t24, t25, t26, t27, t28, t29, t30, t33, t34, t35, t11, t12}	{t23, t27}	{t4, t5, t6, t7, t8, t23, t24, t25, t26, t27, t28, t29, t30, t33, t34, t35, t11, t12}

space limitations, we do not include these test suites. Suffice it to say that their number and their size are vastly different from those found under the strict interpretation.

6 CONCLUSION

In this paper, we seek to generalize the concept of mutant subsumption by analyzing what can be considered as the outcome of a program, when two program outcomes can be compared, and under what condition two comparable program outcomes can be considered as identical or distinct. To this effect, we introduce the concept of differentiator set, and find that we can define three versions of differentiator sets, depending on how we answer the above-cited questions. Specifically, we consider the following definitions:

- *Basic Differentiator Set*. The basic differentiator set of two program P and Q (denoted by $\delta_B(P,Q)$) is defined only if P and Q converge for all initial states, and it includes all initial states for which P and Q yield distinct final states.

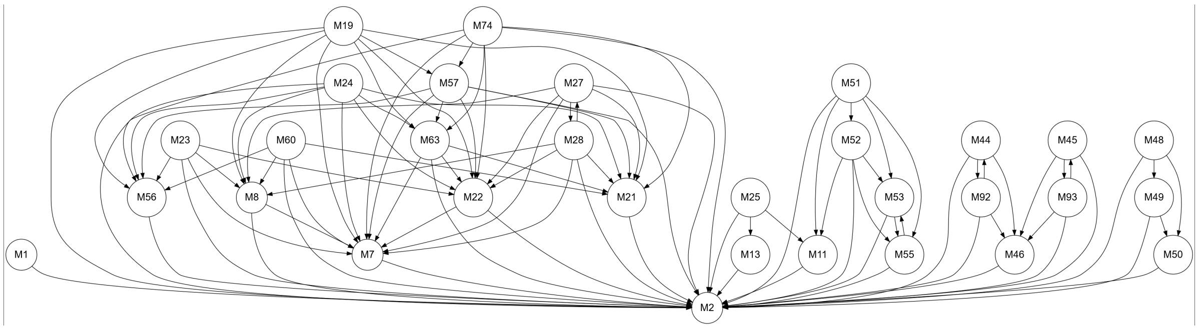


Figure 6: Basic Subsumption Graph, jTerminal Mutants.

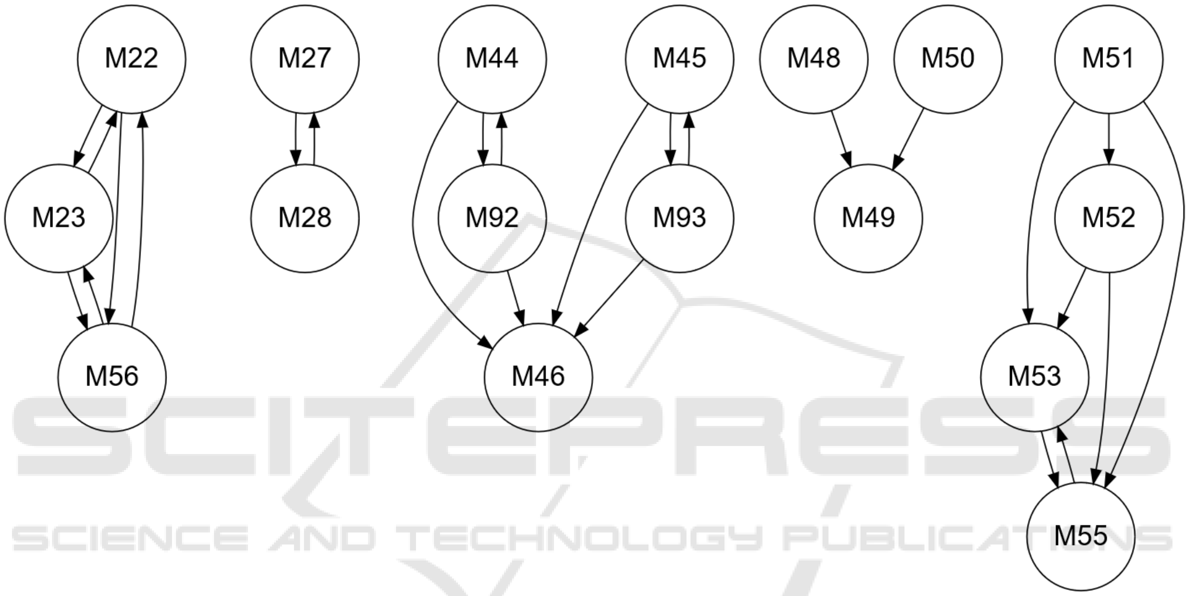


Figure 7: Strict Subsumption Graph, jTerminal Mutants.

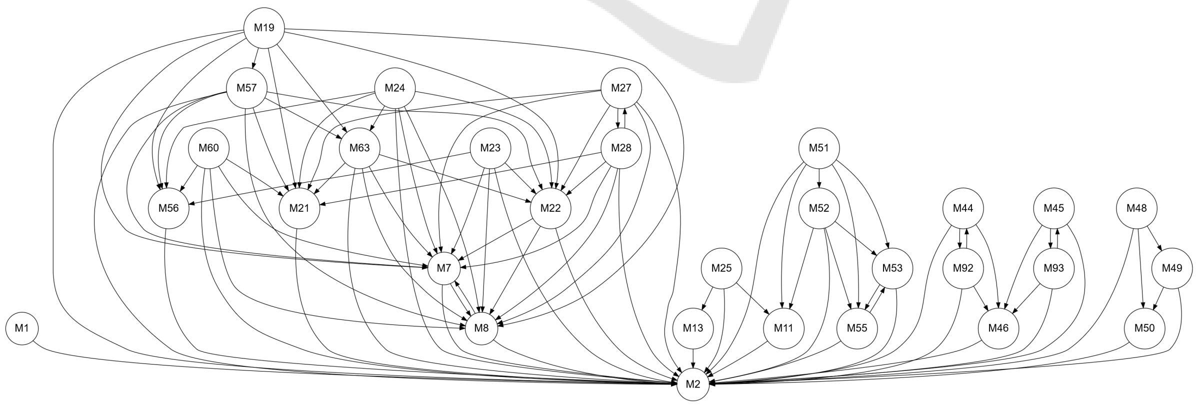


Figure 8: Loose Subsumption Graph, jTerminal Mutants.

- *Strict Differentiator Set*. The strict differentiator set of two programs P and Q (denoted by $\delta_S(P,Q)$) can be defined regardless of whether P and Q converge for all initial states; it contains all initial states for which P and Q converge and produce distinct final states.
- *Loose Differentiator Set*. The loose differentiator set of two programs P and Q (denoted by $\delta_L(P,Q)$) can be defined regardless of whether P and Q converge for all initial states; it contains all

initial states for which P and Q converge and produce distinct final states, as well as all initial states for which one of them diverges and the other converges, regardless of the final state produced by the program that converges.

Using these three definitions of differentiator sets, we get three distinct versions of mutant subsumption: Mutant M subsumes mutant M' with respect to base program P if and only if:

$$\emptyset \subset \delta_B(P,M) \subseteq \delta_B(P,M')$$

or

$$\emptyset \subset \delta_S(P,M) \subseteq \delta_S(P,M')$$

or

$$\emptyset \subset \delta_L(P,M) \subseteq \delta_L(P,M')$$

depending on our interpretation of program outcomes, the condition under which we consider that two outcomes are comparable, and the condition under which two comparable outcomes are identical.

These three definitions of differentiator sets yield three distinct definitions of what it means for a test to kill a mutant; they also yield three distinct subsumption graphs, and three distinct minimal mutant sets. Indeed, a test t kills a mutant M with respect to base program P if and only if

$$t \in \delta(P,M),$$

where $\delta(P,M)$ can be $\delta_B(P,M)$, $\delta_S(P,M)$, $\delta_L(P,M)$, depending on the interpretation we adopt. Also, a test suite T kills a mutant M with respect to base program P if and only if

$$T \cap \delta(P,M) \neq \emptyset,$$

where $\delta(P,M)$ can be $\delta_B(P,M)$, $\delta_S(P,M)$, $\delta_L(P,M)$, depending on the interpretation we adopt.

We show by means of a simple example that different interpretations yield different subsumption graphs. Among other things, this example highlights the need to take into consideration the possibility of divergence, since many of its mutants diverge for many of the tests included in its benchmark test suite.

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