**Opti-Soft+: A Recommender and Sensitivity Analysis for Optimal Software Feature Selection and Release Planning**

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Abstract: Many approaches have been developed to increase the return on a software investment, but each one has drawbacks. Proposed in this paper is the Opti-Soft+ framework that addresses this problem by producing a software release schedule that maximizes the business value of investments in information systems that automate business processes. The optimal release schedule is the result of solving a mixed integer linear programming problem. Opti-Soft+ is an extension of the Opti-Soft framework proposed earlier with (1) a refined cost model, (2) a technique for sensitivity analysis of the normalized cost per unit of production, and (3) an atomic business process model that is driven by output throughputs in addition to input throughputs.

1 INTRODUCTION

Software development projects that are successful and return to the business a value that justify their investments are not common. According to (The Standish Group, 2018) only 36% of projects are successful. To improve the rate of success, organizations have been using Agile methods. As reported in (Serrador & Pinto, 2015,) Agile does have a statistically significant impact on three dimensions of project success, but adopting Agile is not a guarantee of a return on the investment.

Because the Agile software development lifecycle is based on short iterations, at each iteration the team needs to decide what functionality to include. This process, called Release Planning, provides an opportunity to improve the business value of the software because different functionalities result in different value profiles.

Several release planning approaches have been developed to maximize the business value of software delivery. The highly influential Incremental Funding Methodology (IFM) by (M. Denne & Cleland-Huang, 2004) uses heuristics to select a release schedule that maximizes the business value of software investments. F-EVOLVE*'s approach (Maurice et al., 2006) is to involve stakeholders iteratively to achieve releases that result in the highest degree of satisfaction. A third approach by (Van den Akker et al., 2005) applies integer linear programming to maximize the revenue.

The IFM, F-EVOLVE* and Van den Akker approaches use cash flow as a proxy for business value. They all require the estimation of cash flows for each software feature and that's very challenging due to the difficulty of drawing a direct correlation between a particular business benefit, like a reduction in cost, and a specific piece of software. (Devaraj & Kohli, 2002) have acknowledged this difficulty of isolating the effect of IT on firm performance.

The main pitfall of the existing approaches is imprecision. Also, every dollar of cash flow needs to be allocated to one and only one feature which is not a realistic assumption, because often, realizing a business benefit requires the implementation of multiple software features. Another pitfall is that each cash flow estimate combines the business benefit with the software development cost, which means that all the estimations have to be done externally, which is typically difficult and often inaccurate.

In (Boccanera & Brodsky, 2020) and (Boccanera & Brodsky, 2021), we proposed a new approach, called Opti-Soft, to address the pitfalls of existing methods for a class of software projects that automate a Business Process Network (BPN). Opti-Soft is a decision guidance framework for release planning that maximizes the business value as measured by the Net Present Value (NPV), based on a model of the underlying business process and savings achieved...
due to the combined effect of new software features on improved business process efficiency.

However, the Opti-Soft approach has limitations, as we discovered looking at a number of real software project examples. First, the cost model was based on labor costs only, whereas realistic cost models may be considerably more involved. Second, for stakeholders to have a high confidence on the recommendations on software feature selection and release planning, they often need to know the sensitivity of the recommendations to assumptions on demand on business process throughput, e.g., the number of daily patent applications to be processed by the Patent Office. Third, while the business processes can be hierarchically composed, Opti-Soft only supported a limited atomic (leaf) process in the hierarchy in which the cost is driven by input throughput (e.g., number of patent applications that need to be processed per day). Whereas, atomic processes driven by output throughput were not supported.

Lifting these limitations is exactly the focus of this paper. More specifically, the contributions of this paper are as follows. First, we extend the cost model, of both BPN and software development, beyond labor cost to include a range of variable and fixed costs (i.e., of resources required).

Second, we develop a technique for sensitivity analysis of the normalized cost per unit of production, for a recommended release plan and associated improved BPN, as a function of BPN throughput. The analysis involves fixing some of the decision variables while allowing others to be instantiated by the optimizer. The idea is to determine the delta change in the objective function for a one-unit change of the required BPN throughput.

Third, we develop an atomic service model that is driven by output throughputs in addition to the one driven input throughputs.

Opti-Soft+ is the result of these extensions. It takes advantage of the fact that the implementation of software features leads to more efficient business processes due to a reduction of the time a worker spends, or the elimination of a portion of the process, or the utilization of workers with a lower labor rate. The key idea is that, because the improved business efficiency is a direct consequence of the availability of software features, this relationship can be formally modelled using mixed integer linear programming (MILP) constraint formulation, which allows the use of MILP solvers to find optimal release plan.

The uniqueness of Opti-Soft+ framework is its accurate estimate of business value improvement by formally modeling BPN and associated costs over the investment time horizon, as a function of chosen software features and release plan. Also, Opti-Soft+ removes the limitation of existing approaches that force every investment dollar to be assigned to one and only one software feature. The Opti-Soft+ model allows one software feature to impact multiple processes and allows a process to be impacted by multiple features, on a many-to-many relationship. In Opti-Soft+, the estimation of the business value of software features is not external to the approach, but instead, is at the heart of the cost model. The Opti-Soft+ framework is composed of a methodology, a formal optimization model and a Decision Guidance System (DGS) which implements the formal model and produces an optimal recommendation.

This paper is organized as follows: Section 2 provides an overview of the Opti-Soft+ model, including the cost approach, the BSN modeling and release planning. Section 3 briefly describes the optimization model; Section 4 describes the DGS and the methodology; Section 5 provides an example of the extensions; Section 6 conducts a sensitivity analysis and Section 7 provides concluding remarks. Appendix 1 shows the entirety of the formal model.

2 Opti-Soft+ Model

Overview

In order to maximize the business value associated with new software features, we need to estimate the cost of software development as well as the benefit of the implementation of the software. For software that implements information systems to automate a business process, the benefit of the software is the cost savings in the business process due to automation.

A business process consumes input flows and produces output flows. The cost associated with the business process is a function of the cost drivers such as labor rates and time spent. This means that the benefit (savings) of a software feature that automates a business process can be determined by subtracting the cost of the automated process from the cost of the process before automation.

The above insight, that the implementation of software features allows the adoption of more efficient business process networks (BPN) is key to Opti-Soft+, because each new BPN configuration can be modelled, and its cost measured. In the Opti-Soft+ approach, there is no need to estimate the cost of each individual feature, a feature is just a device that triggers a change in the BPN configuration, while cost is precisely calculated at the level of the BPN.
In Figure 1, we have an initial BPN configuration, called BPN0, that can benefit from automation and has a Net Present Cost NPC(BPN0). A cash investment, NPC(SW1), is made to implement software features SW1 in the first release (r=1). After release 1, the availability of the software features SW1 allows process improvements, so BPN0 transitions to BPN1, resulting in a Net Present Cost NPC(BPN1), which is lower than NPC(BPN0). The procedure continues iteratively, with each investment NPC(SW r) in release r, causing the BPN r-1 to transition to BPN r, resulting in a lower NPC(BPN r).

![Figure 1: BPN Cost Reduction due to the Investment in Software Features.](image)

In order to calculate and optimize the cost savings, we need to model the BPN transitions as well as the enabling software development features.

**BPN Modeling**

To intuitively understand BPN modeling, consider the example depicted in Figure 2. It shows a parent process P composed of subprocesses A, B, and C, all of which must be executed. Note that the output from A serves as input to B and the output from B serves as input to A. Subprocess A has three alternatives, AA, AB, and AC, whereas only one of them must be executed. Similarly, B has alternatives BA and BB, and C has alternatives CA and CB. By choosing among the alternatives for each subprocess, a new configuration of P is established.

Note that a valid configuration for P requires one and only one of each of its three subprocesses A, B, and C, which establishes an AND relationship between process P and its subprocesses A, B, and C. The relationship between A and its alternatives (subprocesses) AA, AB, and AC is an OR because either AA or AB or AC can be present in P. B and C also have OR relationships with their subprocesses.

We model the BPN as a Service Network (SN) (Brodskey et al., 2017) which is a “network of service-oriented components that are linked together to produce products”. We use the term Business Service Network (BSN) to refer to the BPN as a SN. The linkage among service components is through inputs and outputs. In Figure 2, P is a composite (parent) service because it is composed of subservices A, B, and C. A, B, and C are also composites while all the other subservices are atomic, that is, indivisible.

**BSN Transition and Release Planning**

The transition from a subprocess alternative to another requires the implementation of specific software functionality called features. For example, subprocess alternative AB requires feature F1. We assume that features are implemented in iterations called releases. At the beginning of each release, the team decides which features to include in the scope. This is called release planning. Note that the implementation of features results in automation of certain aspects of the original business process, allowing it to transition to a more efficient process alternative that results in labor and other savings.

In the example of Figure 2, we assume that AA, BA, and CA are manual processes, and the initial BPN configuration (BPN0) is AA, BA, CA with NPC(BPN0). The top table on the right shows the required features for each process while the table on the bottom shows the BPN configuration after each release. Note that A’s alternative subprocess AB is more cost effective than AA and it requires feature F1. Because F1 is implemented in release 1, after release 1 is completed, BPN0 transitions to BPN1 which is configured with AB, BA, CA, with the cost of NPC(BPN1). Note that F1 ‘activates’ AB and this activation property, which is unique to Opti-Soft+, is used extensively in the formalization of the Mixed Integer Linear Programming problem, described in section 3. At the end of each release, the availability of implemented software features allows the activation of alternative processes that are more cost effective, reducing the overall NPC of the SN. Subprocess AA transitions to AB then to AC, BA transitions to BB and CA transitions to CB. The final, optimal SN configuration is then AC, BB, CB. Note that Fig 2 shows not only the BSN transitions, but also the release plan, that is, the software features implemented in each release.

![Figure 2: Example of BPN Transition as a result of feature delivery.](image)
Cost Model
The NPC over the investment time horizon is the combined NPC of all the BPNs plus the NPC of software development in all releases. Costs are accrued daily and are paid on a set schedule. The NPC is composed of five types of cost:

1. Variable labor costs of the SN
2. Variable non-labor costs of the SN
3. Fixed non-labor costs of the SN
4. Variable labor costs of software features
5. Fixed non-labor costs of software features

Note that in the Opti-Soft+ model, we use the Net Present Value (NPV), which is simply NPC with the negative sign.

Variable Labor Costs of the SN
Each process of the SN is performed by workers with well-defined roles. Each role has a labor rate and each input processed or output produced by the role has a set duration. The cost of a process, or LaborCostPerDay, is the labor rate times the duration to handle all inputs and outputs in a day.

Variable Non-labor Costs of the SN
Variable, non-labor costs are associated with the amount of work produced by an atomic service, that is, driven by the inputs or by outputs and are similar to the calculation of labor costs.

Parameters CostPerInput and CostPerOutput capture the non-labor costs for each input and output. These parameters are used to compute FlowCostPerDay.  

Fixed Non-labor Costs of the SN
Fixed non-labor costs are not driven by inputs or outputs, instead they are driven by the services. An example of a fixed cost associated with a particular service is rent. Parameter ServiceCostPerDay captures the daily cost for each atomic service and is used to calculate the ServiceCostPerDay.

Variable Labor Costs of Software Development
Opti-Soft+ follows an Agile practice called feature-driven, where release planning is done at the feature level, that is, features are removed from the product backlog and assigned to releases. The size of features is estimated in points, which is a unit based on the perceived effort to develop the feature. The release size, that is, the sum of the points of all features in the release, cannot exceed the capacity of the team, which is the average productivity of a developer times the size of the team. Development labor cost, in turn, is computed by multiplying the team’s capacity by the developer cost per effort point. The formal model captures the software cost in SWCostPerDay and then uses a pay schedule to calculate the LaborCashFlow.

Fixed Non-labor Costs of Software Development
Fixed costs associated with features are experienced during software development, where features are produced. They are incurred by resources such as a hardware server, a software tool, etc...

Every feature requires a set of resources. The full set of resources required by a feature f needs to be available prior to the start of the release that implements f. A resource might be paid in the release that implements f or in a prior release. We assume that resource costs are paid on the first day of each release, consequently on the first day of a release, all resources needed by all features in the release must be paid.

To be flexible, we allow multiple features to require the same resource, establishing dependencies among features. Resource dependencies are handled by a Dependency Graph.

The cost of resources is captured in ResCashFlow, whose computation uses the following parameters:
- **ResSet** is the set of all non-labor resources
- **FeatureRes** maps features to resources
- **ResCost** maps a resource to its cost

Computation of the SN Cash Flow
The CostPerDay of each atomic process is the sum of LaborCostPerDay, FlowCostPerDay, and ServiceCostPerDay. CostPerDay is used to calculate the schedule of payments, or CashFlow(d) for each d in the time horizon. The CashFlow(d) for each subprocess of a parent process is aggregated and then rolled up to determine the CashFlow(d) of the entire SN.

Computation of the Software Cash Flow
The CashFlow(d) of the development of software is the sum of LaborCashFlow and ResCashFlow. 

Computation of NPV
The CashFlow(d) for the SN and for the Software Development are combined and discounted to produce the TimeWindowNPV.

3 Opti-Soft+ Optimization Model
Opti-Soft+ produces an optimal release schedule and SN configuration by solving a maximization problem given a set of parameters like the services in the SN, feature sizes, number of releases, time horizon, labor rates, size of the development team, etc... It maximizes the NPV of the total cost of the service network plus the software development cost over the investment horizon, subject to constraints such as the
space of process alternatives. Opti-Soft+’s formal model with its parameters, computations, constraints and maximization formulation is presented in its entirety in Appendix 1.

The formulation of the optimization is of a Mixed-Integer Linear Programming (MILP) problem, because 1) three of the DVs (On(s,r), IBF(r,f), ITF(r,f)) are Boolean, 2) one DV (InputThru(s,i,r)) is real, and 3) the objective function is linear because it is the result of the addition of various cost parameters which themselves are linear. Section A9 of Appendix 1 describes the MIPL formulation, which is summarized below:

Given Parameters
Max \( \text{ReleaseScheduling.TimeWindowNPV} \)
\( \text{s.t. ReleaseScheduling.Constraints} \)

The constraints are the space of SN alternative configurations, the required software features, the capacity of the development team, etc… Each of the six formal components shown in Appendix 1 implements constraints that are then aggregated under ReleaseScheduling.Constraints.

Note that the CashFlow and TimeWindowNPV produced by the formal model are negative numbers consequently maximizing the NPV results in minimizing the cost.

4 **OPTI-SOFT+ METHODOLOGY AND DECISION GUIDANCE SYSTEM**

Opti-Soft+ includes a Decision Guidance System which implements the formal model in Appendix 1 and includes a MILP Solver. It produces:

1. Optimal NPV of the business benefit
2. A release schedule, which is the result of the Solver instantiating DVs IBF(r,f) and ITF(r,f).
3. The optimal service network configuration at the end of each release, which is the result of the Solver instantiating DV On(s,r).

The DGS uses the Parameters in the input file to maximize the NPV, subject to the Constraints. During the maximization, the DGS performs the Computation and chooses the optimal DecisionVariables. The Opti-Soft+ DGS is implemented using Unity (Nachawati, M. O., Brodsky, A., & Luo, J., 2016), (Nachawati, M. O., Brodsky, A., & Luo, J., 2017), a platform for building DGSs from reusable Analytical Models (AMs). Unity exposes an algebra of operators and provides an unified, high-level language called Decision Guidance Analytics Language (DGAL) (Brodsky, Alexander, & Luo, J., 2015).

The Opti-Soft+ framework is composed of the optimization model, the DGS and a methodology. We covered the first two so now we cover the latter. The Opti-Soft+ methodology, which extends the methodology in (Boccanera & Brodsky, 2021), contains the following steps:

1. Generate candidate software features to be implemented
2. Capture the As-Is BPN configuration, and alternative BPN configurations that can be enabled by candidate software features
3. Gather and instantiate input parameters for the optimization model as described in Appendix
4. Compute the baseline NPV for the As-Is BPN
5. Perform Opti-Soft+ DGS optimization to come up with a recommended Release Plan and the associated optimal BPN configuration (To-Be)
6. Calculate the savings, which is the NPV of the To-Be minus the NPV of the As-Is
7. During Release 1:
   a. Operate the BPN according to the optimal BPN configuration.
   b. Implement recommended software features
8. For each release \( r = 2, \ldots, n \):
   a. Update existing software features to include those implemented in the previous release
   b. For updated software features and refined demand/throughput requirements, run operational optimization to find the best BPN configuration. Operate the BPN according to it.
   c. Repeat steps above to update the recommended Release Plan for the remaining releases (starting from \( r + 1 \))
   d. Implement recommended software features

5 **OPTI-SOFT+ PRODUCES EXAMPLE OF EXTENSIONS**

Sections 5 of (Boccanera & Brodsky, 2021) describe an example of a service network composed of 3 parent processes (A, B and C). The optimal release plan and SN configuration, is reproduced in Table 1. The example has 4 releases, each lasting 60 days and a time horizon of 520 days, and a BSN that requires processing 100 user applications per day, that is, for demand=100. In the example, the optimized objective function, or NPV, produced by the DGS is -$6,411,432.73.
Table 1: Optimal release schedule and SN configuration.

<table>
<thead>
<tr>
<th>Software Release #</th>
<th>Features implemented</th>
<th>Optimal SN Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TF1, BF1</td>
<td>AA, BA, CA</td>
</tr>
<tr>
<td>2</td>
<td>BF3</td>
<td>AB, BA, CA</td>
</tr>
<tr>
<td>3</td>
<td>BF2</td>
<td>AB, BA, CB</td>
</tr>
<tr>
<td>4</td>
<td>BF4</td>
<td>AB, BB, CB</td>
</tr>
<tr>
<td>After 4</td>
<td></td>
<td>AC, BB, CB</td>
</tr>
</tbody>
</table>

We now take the example from (Boccanera & Brodsky, 2021) and add the extensions described in section 1. Table 2 shows the extended parameters.

Table 2: Extended Parameters.

<table>
<thead>
<tr>
<th>Formalization</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSch</td>
<td>ResSet</td>
<td>“softwareLicense1”</td>
</tr>
<tr>
<td></td>
<td>ResCost</td>
<td>“softwareLicense1”:20,000</td>
</tr>
<tr>
<td>FeatureRes</td>
<td>TF1:</td>
<td>“”</td>
</tr>
<tr>
<td></td>
<td>BF1:</td>
<td>“”</td>
</tr>
<tr>
<td></td>
<td>BF2:</td>
<td>“”</td>
</tr>
<tr>
<td></td>
<td>BF3:</td>
<td>“”</td>
</tr>
<tr>
<td></td>
<td>BF4:</td>
<td>“softwareLicense1”</td>
</tr>
<tr>
<td>ServiceCost</td>
<td>PerDay</td>
<td>AA:200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AB:200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AC:200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BA:200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BB:200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CA:200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CB:200</td>
</tr>
<tr>
<td>Input Driven</td>
<td>CostPerInput</td>
<td>AA.UserApplication:2</td>
</tr>
<tr>
<td>Atomic Service</td>
<td></td>
<td>AB.UserApplication:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AC.UserApplication:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BA.CompliantApplic:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BB.CompliantApplic:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CA.AdjudicatedApplic:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CB.AdjudicatedApplic:0</td>
</tr>
<tr>
<td></td>
<td>CostPerOutput</td>
<td>AA.CompliantApplic:3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AA.NonComplianceNtc:1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AB.CompliantApplic:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AB.NonComplianceNtc:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AC.CompliantApplic:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AC.NonComplianceNtc:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BA.AdjudicatedApplic:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BB.AdjudicatedApplic:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CA.AdjudApplicLetter:0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CB.AdjudApplicLetter:0</td>
</tr>
</tbody>
</table>

There is a software license that costs $20,000 when feature BF4 is implemented. There are fixed costs per day of $200 for each of the atomic processes (AA, AB, AC, BA, BB, CA, CB). Atomic process AA incurs $2 in cost per “User Application” input, $3 per ‘CompliantApplication” output and $1 per “NonCompliantNotice” output.

Using the parameters in Table 2, plus the parameters in Section 5 of (Boccanera & Brodsky, 2021), the DGS maximizes the objective function and produces an optimal NPV of -$6,748,777.45. The increase from -$6,411,432.73 is expected and is a direct result of the extended costs listed in Table 2.

In order to determine the savings of DGS’ recommendation, we need to compare the NPV of the extended example (-$6,748,777.45), called the To-Be, with the NPV of the As-Is, which is the BPN prior to the development of the software.

To calculate the As-Is recommendation, we change the example parameters as follows: 1) set to zero the parameters used in the Software Development Formal Model and 2) Set the BPN configuration to AA, BA, CA for the entire duration of the time horizon of the investment. Running the DGS with these modified parameters, the resulting NPV for the As-Is is -$9,611,947.49.

The savings is the difference between the To-Be (-$6,748,777.45) and the As-Is (-$9,611,947.49), or $2,863,170.04. Note that this is the maximum savings, i.e., there is no other release plan and BSN configuration that produces a higher savings.

6 Opti-Soft+ Sensitivity Analysis

One aspect that a decision-maker would be interested in, is how sensitive the total NPC is to certain changes in parameters. To answer this, we developed a technique for sensitivity analysis as follows.

The objective function is the NPV of the cash outflow of the service network (SN) plus the cash outflow of developing the software features that allow the SN to transition to more efficient processes. Opti-Soft+ has several parameters that influence the NPV, but the one with the most impact is the demand, which is the required throughput of the SN. In our example, the required demand is 100 applications per day.

The required demand, used as a parameter in the DGS, is an estimation and if there is a high degree of uncertainty in the estimation, a decision maker might not have a lot of confidence in the recommendation. A sensitivity analysis based on the demand parameter is valuable because it helps to understand risk.

In our sensitivity analysis technique, we use the NPC instead of the NPV because it is more intuitive. The goal is to determine the NPC delta, that is, the additional cost for an increase of one unit of demand. Given $d_0$, the original demand through the SN, we vary $d$, the new demand by 1. The delta of the demand is $\delta = d - d_0$. We then calculate UC, the cost per unit of demand $d$ as follows:
We can utilize the above technique to conduct two analyses for a range of \( \delta \): 1) fix the release plan and the BSN configuration, and 2) fix the release plan, allowing the BSN configuration to be optimized. The first analysis will show how the unit cost varies with each \( \delta \), while the second will show the unit cost variation and the stability of the BSN configuration.

**Sensitivity Analysis 1**

The steps to conduct analysis number 1 are as follows: 1) determine a range of \( \delta \), above and below \( d_0 \), to conduct the analysis, 2) run the DGS optimization with \( \text{demand}=d_0 \) to get a recommendation and the value of \( \text{NPV}_0 \), 3) instantiate the ITF\((r,f)\), IBF\((r,f)\) and On\((s,r)\) decision variables with the release planning schedule and SN configuration recommended by the DGS in the previous step, leaving \( \text{InputThru}(s,i,r) \) as a DV, 4) set the demand parameter to \( d_0 + \delta \), where \( \delta_1 \) is the first value in the \( \delta \) range, and run the DGS to get the value for \( \text{NPC}_1 \), 5) repeat steps 2-4 (i.e., now performing operational optimization when software features available are fixed) for all \( \delta_i \) in the range, \( i >1 \), 6) calculate the values of \( \text{UC}(\delta_i) \), and 7) plot a chart with the values of \( \delta_i \) and \( \text{UC}(\delta_i) \).

We now apply our sensitivity analysis technique to the example in Section 4. In step 1, we determine that the estimated demand \( d_0=100 \) has an error of 10%, so we set the range of \( \delta \) to -10 to +10. In step 2 we run the DGS with \( \text{demand}=100 \) and produce the recommendation and \( \text{NPV}_0=6,748,777.45 \), described in Section 4. In step 3 we instantiate the release planning schedule and SN configuration DVs with the recommendation in Section 4. In step 4, we take the first value in the \( \delta \) range, and run the DGS to get the value for \( \text{NPC}_1 \). In step 5, we repeat steps 2-4 (i.e., now performing operational optimization when software features available are fixed) for all \( \delta_i \) in the range, \( i >1 \), 6) calculate the values of \( \text{UC}(\delta_i) \), and 7) plot a chart with the values of \( \delta_i \) and \( \text{UC}(\delta_i) \).

The table and the chart show that as the demand \( d \) increases, the \( \text{UC} \), which is NPC per unit of \( d \), decreases. For a decision maker, this is a desirable behavior because the initial demand \( d_0 \) is just an estimation. If \( d_0 \) was underestimated, then the optimal NPC is even better than the value provided by the original recommendation. If \( d_0 \) was underestimated, it is easy to determine the reduction in NPC. This would help a decision maker to manage the estimation risk of the demand and consequently yield a higher degree of confidence in the DGS recommendation.

**Table 3: Results of the Sensitivity Analysis.**

<table>
<thead>
<tr>
<th>( d )</th>
<th>( \delta )</th>
<th>( \text{NPV}(d_0+\delta) )</th>
<th>( \text{UC}(\delta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>-10</td>
<td>$6,236,485.38</td>
<td>$69,294.28</td>
</tr>
<tr>
<td>91</td>
<td>-9</td>
<td>$6,287,714.60</td>
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**Sensitivity Analysis 2**

To perform analysis number 2, we use the same steps as analysis number 1 with one change. In step 3, we do not instantiate \( \text{On}(s,r) \), that is, we do not fix the BSN configuration, allowing it to be optimized.

We run all the steps, and for every \( \delta \) in the range -10 to +10, the results are the same as in analysis number 1. In addition, the recommended BSN configuration is also the same. This means that for a delta in the range of -10 to +10, the recommendation is stable.
7 CONCLUSION AND FUTURE WORK

In this paper we introduced Opti-Soft+, an extended framework to produce a software release schedule that maximizes the business value of investments in the development of software applications that automate business processes. Opti-Soft+ employs a realistic cost approach, and models the MILP optimization problem formally, which is implemented by a Decision Guidance System. We also conducted a sensitivity analysis that helps a decision maker to understand the range of parameters that the solution would hold.

The contributions of this paper are: 1) extending the cost model, of both BPN and software development, beyond labor cost to include a range of variable and fixed costs (i.e., of resources required), 2) developing a technique for sensitivity analysis of the normalized cost per unit of production, for a recommended release plan and associated improved BPN, as a function of BPN throughput, and 3) developing an atomic service model that is driven by output throughputs in addition to the model driven input throughputs.

The benefits of the above contributions are: 1) making the cost model more realistic and allowing a cost to be incurred multiple features, 2) providing a decision maker with analytical results showing how sensitive the recommendation is to certain changes in parameters, and 3) allowing a natural way to model process that are output driven or that are driven by both input and output, which increases the practicality of the framework.

Potential future work involve comparing Opti-Soft+ with other frameworks such as the popular Incremental Funding Methodology (Cleland-Huang & Denne, 2005) and conducting a case study.

REFERENCES


APPENDIX: FORMAL MODEL WITH EXTENSIONS

A1. Release Scheduling Formalization

ReleaseScheduling (RSch) formalization is a tuple (Parameters, DecisionVariables, Computation, Constraints, InterfaceMetrics)

where:

1. **Parameters**, also denoted Parm, is a tuple (Features, TH, DiscountRate, ReleaseInfo, ResSet, ResCost, FeatureRes, BSN.Parameters, SWD.Parameters)

   *BF* is a set of business features
   *TF* is a set of technical features, such that
   *BF ∩ TF = ∅*
   *DG*, (Dependency Graph), is a partial order over 
   F = BF ∪ TF, (f1, f2) ∈ DG also denoted f1 < f2;
   means that f2 is dependent on f1, that is, feature f1 is a pre-requisite for feature f2.
   *FS: F → ℝ⁺* is a function described as follows:
   (∀ f ∈ F), FS(f) gives the size, in effort point, of each feature f.
   *TH* is the time horizon for analysis in days
   *DiscountRate* is the daily rate to discount cash flows.
   *ReleaseInfo* is a tuple (NR, RD), where:
   *NR* is the number or releases
   *RD: [1..NR] → ℝ⁺* is a function described as follows:
   (∀ r ∈ [1..NR]), RD(r) gives the maximum duration in days for release r.
   *ResSet* is a set of non-labor resources that have a fixed-cost
   *ResCost: ResSet → ℝ⁺* is a function described as follows:
   (∀ e ∈ ResSet), ResCost(e) gives the non-labor fixed cost for resource e.
   *FeatureRes: F → 2<ResSet> is a function described as follows:
   (∀ f ∈ BF ∪ TF, ∀ e ∈ ResSet), FeatureRes(f) gives a set of resources e required by feature f.
   *BSN.Parameters* is defined in section 4.2
   *SWD.Parameters* is defined in section 4.7

DecisionVariables, also denoted DV, is a tuple (IBF, ITF, BSN.DecisionVariables, SWD.DecisionVariables) where:

- **IBF: [1..NR] → 2<BF>** is a function described as follows:
  (∀ r ∈ [1..NR]), IBF(r) gives a set of business features planned to be implemented in release r.
- **ITF: [1..NR] → 2<BF>** is a function described as follows:
  (∀ r ∈ [1..NR]), ITF(r) gives a set of technical features planned to be implemented in release r.
- **BSN.DecisionVariables** is defined in section A.2.
- **SWD.DecisionVariables** is defined in section A.8.

Computation

1. Let **SoFarIBF: [1..NR + 1] → 2<BF>** be a function described as follows:
   (∀ r ∈ [1..NR + 1]), SoFarIBF(r) gives the set of all business features implemented up to release r or the period after the last release, computed as follows:
   \[ SoFarIBF(r) = \bigcup_{i=1}^{r-1} IBF(i) \]

2. Let **CombinedCashFlow: [1..TH] → ℝ** be a function described as follows:
   (∀ d ∈ [1..TH]), CombinedCashFlow(d) gives the combined income/expenditure of both the Business Service Network and the Software Development, (∀ d ∈ [1..TH]), computed as follows:
   \[ CombinedCashFlow(d) = BSN.IM.CashFlow(d) + SWD.IM.CashFlow(d) \]

Note that a negative cash flow means that it is a cash outflow.

3. Let **TimeWindowNPV: [1..TH] → ℝ** be a function described as follows:
   (∀ d ∈ [1..TH]),
   TimeWindowNPV(d) gives the Net Present Value (NPV) of the CombinedCashFlow for the time investment window[1..d], computed as follows:
   \[ TimeWindowNPV(d) = \sum_{i=1}^{d} CombinedCashFlow(i) \frac{1}{(1 + DiscountRate)^i} \]

4. Let **F = BF ∪ TF**

5. Let **1F(r) = IBF(r) ∪ ITF(r), (∀ r ∈ [1..NR])**
6. FeatureSetsForReleasesArePairwiseDisjoint constraint is:
\[ \forall i, j \in [1..NR], i \neq j, F(i) \cap F(j) = \emptyset \]

7. DependencyGraphIsSatisfied constraint is:
\[ \forall r \in [1..NR] (\forall f_1, f_2 \in F), \]
\[ (f_1 < f_2 \land f_1, f_2 \in IF(r)) \rightarrow (f_1 \in \bigcup_{i=1}^{r} IF(i)) \]

Constraints
1. FeatureSetsForReleasesArePairwiseDisjoint is defined in computation #6 above.
2. DependencyGraphIsSatisfied is defined in computation #7 above.
3. BSN.Constraints is defined in section A.2.
4. SWD.Constraints is defined in section A.8.

InterfaceMetrics, also denoted IM, is a tuple 
\langle SoFarBF, CombinedCashFlow, TimeWindowNPV, BSN.InterfaceMetrics, SWD.InterfaceMetrics \rangle,
where:
- CombinedCashFlow is defined in computation #2 above.
- TimeWindowNPV is defined in computation #3 above.
- BSN.InterfaceMetrics is defined in section A.2.
- SWD.InterfaceMetrics is defined in section A.8.

A2. Business Service Network Formalization
Due to paper size restriction, this section is published in (Boccanera & Brodsky, 2022), section A2.

A3. Service Formalization
Due to paper size restriction, this section is published in (Boccanera & Brodsky, 2022), section A3.

A4. ANDservice Formalization
Intuitively, an ANDservice is a composite service, that is, an aggregation of sub-services such that all sub-services are activated.

ANDservice formalization is a tuple \( \langle \text{Parameters}, \text{DecisionVariables}, \text{Computation}, \text{Constraints}, \text{InterfaceMetrics} \rangle \),
where:
- Parameters, also denoted Parm, is a tuple \( \langle id, ServiceType(id), I(id), O(id), Subservices(id) \rangle \),
  where:
  - id is the Service id, which must be unique across all services in the ServicesSet.
  - I(id) is a set of inputs
  - O(id) is a set of outputs
  - Subservices(id) is a set of the ids of the sub-services.
  - ServiceType(id) is ANDservice.

DecisionVariables, also denoted DV, is a tuple \( \langle On(id) \rangle \),
where:
- On(id): \([1..NR + 1] \rightarrow \{0,1\}\) is a function that determines whether the Service id is activated or not, for a particular release, i.e., \( (\forall r \in [1..NR + 1]) On(id)(r) \), also denoted by \( On(id,r) \) is as follows:
\[ On(id,r) = \begin{cases} 1 & \text{if service id is activated in release r} \\ 0 & \text{otherwise} \end{cases} \]

Computation
1. AllSubservicesAreActivated constraint:
Let \( n \) be the cardinality of Subservices(id). Then the constraint is:
\[ \sum_{i \in \text{Subservices}(id)} \text{On}(i,r) = n * \text{On}(id,r), \forall r \in [1..NR + 1] \]

2. Let SetAllIO(id) be a set of inputs and outputs, computed as follows:
\[ \text{SetAllIO}(id) = I(id) \bigcup O(id) \bigcup \bigcup_{i \in \text{Subservices}(id)} I(i) \bigcup \bigcup_{i \in \text{Subservices}(id)} O(i) \]

3. Let InternalSupply(id):SetAllIO \times [1..NR + 1] \rightarrow \mathbb{R} be a function described as follows: \( (\forall j \in \text{SetAllIO}(id), \forall r \in [1..NR + 1]) \text{InternalSupply}(id)(j,r) \), also denoted \( \text{InternalSupply}(id, j, r) \), gives the internal supply of flow \( j \) during release \( r \) (and the period after the last release), computed as follows:
\[ \text{InternalSupply}(id, j, r) = \begin{cases} \sum_{s \in \text{Subservices}(id)} \text{OutputThru}(s, j, r) & \text{if } j \in O(s) \\ 0 & \text{otherwise} \end{cases} \]

4. Let InternalDemand(id):SetAllIO \times [1..NR + 1] \rightarrow \mathbb{R} be a function described as follows: \( (\forall j \in \text{SetAllIO}(id), \forall r \in [1..NR + 1]) \text{InternalDemand}(id)(j, r) \), also denoted \( \text{InternalDemand}(id, j, r) \), gives the internal demand of flow \( j \) during release \( r \) (and the period after the last release), computed as follows:
\[ \text{InternalDemand}(id, j, r) = \begin{cases} \sum_{s \in \text{Subservices}(id)} \text{InputThru}(s, j, r) & \text{if } j \in I(s) \\ 0 & \text{otherwise} \end{cases} \]
5. Let \( \text{InputThru}(id) : I(id) \times [1..NR + 1] \rightarrow \mathbb{R}^+ \) be a function described as follows: \((\forall i \in I(id), \forall r \in [1..NR + 1]), \text{InputThru}(id)(i, r)\), also denoted \( \text{InputThru}(id, i, r) \), gives the throughput of \( i \) (or quantity per day) during release \( r \) or the period after the last release, computed as \((\forall i \in I(id), \forall r \in [1..NR + 1])\):
\[
\text{InputThru}(id, i, r) = \text{InternalDemand}(id, i, r) - \text{InternalSupply}(id, i, r)
\]

6. Let \( \text{OutputThru}(id) : O(id) \times [1..NR + 1] \rightarrow \mathbb{R}^+ \) be a function described as follows: \((\forall o \in O(id), \forall r \in [1..NR + 1]), \text{OutputThru}(id)(o, r)\), also denoted \( \text{OutputThru}(id, o, r) \), gives the throughput of \( o \) (or quantity per day) during release \( r \) or the period after the last release, computed as \((\forall o \in O(id), \forall r \in [1..NR + 1])\):
\[
\text{OutputThru}(id, o, r) = \text{InternalSupply}(id, o, r) - \text{InternalDemand}(id, o, r)
\]

7. Let \( \text{TotalDemand}(id) : \text{SetAllIO} \times [1..NR + 1] \rightarrow \mathbb{R} \) be a function described as follows: \((\forall j \in \text{SetAllIO}(id), \forall r \in [1..NR + 1]), \text{TotalDemand}(id)(j, r)\), also denoted \( \text{TotalDemand}(id, j, r) \), gives the total demand of flow \( j \) during release \( r \) (and the period after the last release), computed as:
\[
\text{TotalDemand}(id, j, r) = \text{InputThru}(id, j, r) + \text{InternalDemand}(id, j, r) \text{ if } j \in I(id) \\
- \text{InternalDemand}(id, j, r) \text{ otherwise}
\]

8. Let \( \text{TotalSupply}(id) : \text{SetAllIO} \times [1..NR + 1] \rightarrow \mathbb{R} \) be a function described as follows: \((\forall j \in \text{SetAllIO}(id), \forall r \in [1..NR + 1]), \text{TotalSupply}(id)(j, r)\), also denoted \( \text{TotalSupply}(id, j, r) \), gives the total supply of flow \( j \) during release \( r \) (and the period after the last release), computed as:
\[
\text{TotalSupply}(id, j, r) = \text{OutputThru}(id, j, r) + \text{InternalSupply}(id, j, r) \text{ if } j \in O(id) \\
- \text{InternalSupply}(id, j, r) \text{ otherwise}
\]

9. \( \text{TotalSupplyMatchesTotalDemand} \) constraint is:
\[
\forall j \in \text{SetAllIO}(id), \forall r \in [1..NR + 1], \text{TotalSupply}(id, j, r) = \text{TotalDemand}(id, j, r)
\]

10. Let \( \text{CostPerDay}(id) : [1..NR + 1] \rightarrow \mathbb{R} \) be a function described as follows: \((\forall r \in [1..NR + 1]), \text{CostPerDay}(id)(r)\), also denoted \( \text{CostPerDay}(id, r) \), gives the total dollar cost per day during period \( r \) and the period after the last period, computed as:
\[
\text{CostPerDay}(id, r) = \sum_{i \in \text{Subservices}(id)} \text{Service. IM. CostPerDay}(i, r)
\]

Constraints are as follows:
1. \( \text{AllSubservicesAreActivated} \) (see computation #1)
2. \( \text{TotalSupplyMatchesTotalDemand} \) (see computation #9)

InterfaceMetrics, also denoted IM, is a tuple \( \text{CostPerDay}(id), \text{InputThru}(id), \text{OutputThru}(id) \) where:
- \( \text{CostPerDay}(id) \) is defined in computation #10 above.
- \( \text{InputThru}(id) \) is defined in computation #5 above.
- \( \text{OutputThru}(id) \) is defined in computation #6 above.

A5. ORService Formalization
Due to paper size restriction, this section is published in (Bocanegra & Brodsky, 2022), section A5.

A6. InputDrivenAtomicService Formalization
Intuitively, an InputDrivenAtomicService is an indivisible, atomic, service which’s throughput is driven by the number of inputs that it needs to consume, for example, a process that receives applications and adjudicates them.

InputDrivenAtomicService formalization is a tuple \( \langle \text{Parameters, DecisionVariables, Computation, Constraints, InterfaceMetrics} \rangle \)

Parameters, also denoted Parm, is a tuple \( \langle \text{id}, \text{ServiceType}(id), I(id), O(id), \text{RBF}(id), \text{ServiceRoles}(id), \text{I0thruRatio}(id), \text{RoleTimePerI0}(id), \text{ServiceCostPerDay}, \text{CostPerInput}, \text{CostPerOutput} \rangle \) where:
- \( \text{id} \) is the Service id.
- \( I(id) \) is a set of inputs
- \( O(id) \) is a set of outputs
- \( \text{RBF}(id) \subseteq \text{ReleaseScheduling. Parm.BF} \) is a set of business features required by Service id
- \( \text{ServiceRoles}(id) \subseteq \text{BSN. Parm.LR} \) is a set of roles involved in the business service.
- \( \text{I0thruRatio}(id): I(id) \times O(id) \rightarrow \mathbb{R}^+ \) is a function described as follows: \((\forall i \in I(id)), (\forall o \in O(id)), \text{I0thruRatio}(id)(i, o) \) also denoted as \( \text{I0thruRatio}(id, i, o) \), gives for input \( i \) and output \( o \), the ratio of output throughput based on the input throughput.
- \( \text{RoleTimePerI0}(id), \text{ServiceRoles}(id) \times (I(id) \cup O(id)) \rightarrow \mathbb{R}^+ \) is a function described as follows: \((\forall l \in \text{ServiceRoles}(id), \forall j \in \text{InterfaceMetrics}) \)
Let $I(id) \cup O(id), RoleTimePer10(id)(i,j)$, also denoted as $RoleTimePer10(id,i,j)$, gives the amount of time, in hours, that role $l$ spends per flow $j$.

- **ServiceCostPerDay**: $\text{ServicesSet} \rightarrow \mathbb{R}^+$ is a function described as follows: $(\forall s \in \text{ServicesSet}), ServiceCostPerDay(s)$ gives the non-labor fixed cost of service $s$ for each day.

- **CostPerInput**(id): $I(id) \rightarrow \mathbb{R}^+$ is a function described as follows: $(\forall i \in I(id)),$ CostPerInput(id)$(i)$, also denoted as CostPerInput(id, o), gives the non-labor fixed cost for each input o processed by the service id.

- **CostPerOutput**(id): $O(id) \rightarrow \mathbb{R}^+$ is a function described as follows: $(\forall o \in O(id)),$ CostPerOutput(id)$(o)$, also denoted as CostPerInput(id, o), gives the non-labor fixed cost for each output $o$ processed by the service id.

- **ServiceType**(id) is InputDrivenAtomicService

**DecisionVariables**, also denoted $DV$, is a tuple $(On(id), InputThru(id))$

- **On**(id): $[1..NR+1] \rightarrow \{0,1\}$ is a function that determines whether the Service id is activated or not, for a particular release, i.e., $(\forall r \in [1..NR+1]), On(id)(r)$, also denoted by On(id, r) is as follows:

  $On(id, r) = \begin{cases} 
  1 & \text{if service id is activated in release r} \\
  0 & \text{otherwise}
  \end{cases}$

- **InputThru**(id): $I(id) \times [1..NR+1] \rightarrow \mathbb{R}^+$ is a function described as follows: $(\forall i \in I(id), \forall r \in [1..NR+1]), InputThru(id)(i,r)$, also denoted InputThru(id, i, r), gives the throughput of $i$ (or quantity per day) during release $r$ or the period after the last release.

**Computation**

1. **FeatureDependencyIsSatisfied** constraint:

   $On(id, r) = 1 \rightarrow RBF(id) \subseteq RSh. IM. SoFarIBF(r)$

   $(\forall r \in [1..NR+1])$

2. **DeactivatedServicesIsSatisfied** constraint:

   $\forall i \in I(id), \forall r \in [1..NR+1],$

   $On(id, r) = 0 \rightarrow InputThru(id, i, r) = 0$

3. Let **OutputThru**(id): $O(id) \times [1..NR+1] \rightarrow \mathbb{R}^+$ be a function described as follows: $(\forall o \in O(id), \forall r \in [1..NR+1]),$

   $OutputThru(id)(o,r)$, also denoted OutputThru(id, o, r), gives the throughput of $o$ (or quantity per day) during release $r$ or the period after the last release, computed as

   $\forall o \in O(id), \forall r \in [1..NR+1],$

   $OutputThru(id, o, r) = \sum_{i \in I(id)} (10 thru Ratio(id, i, o) \times InputThru(id, i, r))$

4. Let **TimePerDay**(id): $[1..NR+1] \times ServiceRoles(id) \rightarrow \mathbb{R}^+$ be a function described as follows: $(\forall l \in ServiceRoles(id), r \in [1..NR+1]), TimePerDay(id)(l, r)$, also denoted TimePerDay(id, l, r), gives the total duration per day for role $l$ and release $r$ (and the period after the last release), computed as:

   $TimePerDay(id, l, r) = \sum_{j \in I(id)} (RoleTimePer10(id, l, j) \times InputThru(id, j, r)) + \sum_{j \in O(id)} (RoleTimePer10(id, l, j) \times OutputThru(id, j, r))$

5. Let **LaborCostPerDay**(id): $[1..NR+1] \rightarrow \mathbb{R}$ be a function described as follows: $(\forall r \in [1..NR+1]), LaborCostPerDay(id)(r)$, also denoted LaborCostPerDay(id, r), gives the total labor cost per day during release $r$, computed as follows:

   $LaborCostPerDay(id, r) = \sum_{i \in ServiceRoles} (BSN. Parm. Rate(I) \times TimePerDay(id, l, r))$

6. Let **FlowCostPerDay**(id): $[1..NR+1] \rightarrow \mathbb{R}$ be a function described as follows: $(\forall r \in [1..NR+1]), FlowCostPerDay(id)(r)$, also denoted FlowCostPerDay(id, r), gives the total non-labor cost per day for all input and output flows processed during release $r$, computed as follows:

   $FlowCostPerDay(id, r) = \sum_{j \in I(id)} (CostPerInput(id, f) \times InputThru(id, j, r)) + \sum_{j \in O(id)} (CostPerOutput(id, j) \times OutputThru(id, j, r))$

7. Let **CostPerDay**(id): $[1..NR+1] \rightarrow \mathbb{R}$ be a function described as follows: $(\forall r \in [1..NR+1]), CostPerDay(id)(r)$, also denoted CostPerDay(id, r), gives the total cost per day during release $r$, computed as follows:
\[ \text{CostPerDay}(id, r) = \text{LaborCostPerDay}(id, r) + \text{FlowCostPerDay}(id, r) + \text{ServiceCostPerDay}(id) \times \text{On}(id) \]

**Constraints** are as follows:
1. **FeatureDependencyIsSatisfied** (see computation \#1)
2. **DeactivatedServicesIsSatisfied** (see computation \#2)

**InterfaceMetrics**, also denoted 1M, is a tuple \((\text{CostPerDay}(id), \text{InputThru}(id), \text{OutputThru}(id))\) where:
- \(\text{CostPerDay}(id)\) is defined in computation \#7.
- \(\text{InputThru}(id)\) is defined in DecisionVariables.
- \(\text{OutputThru}(id)\) is defined in computation \#3.

**A.7 OutputDrivenAtomicService Formalization**

Intuitively, an OutputDrivenAtomicService is an indivisible, atomic service which’s throughput is driven by the number of outputs that it needs to produce, for example, a service that produces a report.

**OutputDrivenAtomicService** formalization is a tuple \((\text{Parameters}, \text{DecisionVariables}, \text{Computation}, \text{Constraints}, \text{InterfaceMetrics})\)

**Parameters**, also denoted \(\text{Parm}\), is a tuple \((id, \text{ServiceType}(id), 1(id), O(id), RBF(id), \text{RoleTimePerOI}(id))\):
- \(id\) is the Service id.
- \(I(id)\) is a set of inputs
- \(O(id)\) is a set of outputs
- \(RBF(id)\) \(\subseteq\) \(\text{ReleaseScheduling}.\text{Parm}.BF\) is a set of business features required by Service id
- \(\text{ServiceRoles}(id)\) \(\subseteq\) \(\text{BSN}.\text{Parm}.LR\) is a set of roles involved in the business service.
- \(\text{O thru Ratio}(id): O(id) \times I(id) \rightarrow \mathbb{R}^+\) is a function described as follows: \((\forall o \in O(id), (\forall i \in I(id)), \text{O thru Ratio}(id)(i, o))\) also denoted as \(\text{O thru Ratio}(id, i, o)\), gives for output \(o\) and input \(i\), the ratio of input throughput based on the output throughput.
- \(\text{RoleTimePerOI}(id): \text{ServiceRoles}(id) \times (O(id) \cup I(id)) \rightarrow \mathbb{R}^+\) is a function described as follows: \((\forall I \in \text{ServiceRoles}(id), \forall j \in O(id) \cup I(id)), \text{RoleTimePerOI}(id)(i, j)\), also denoted as \(\text{RoleTimePerOI}(id, i, j)\), gives the amount of time, in hours, that role \(l\) spends per flow \(j\).

**ServiceCostPerDay**: \(\text{ServicesSet} \rightarrow \mathbb{R}^+\) is a function described as follows: \((\forall s \in \text{ServicesSet}), \text{FixedCostPerDay}(s)\) gives the non-labor fixed cost of service \(s\) for each day.

**CostPerInput**: \(I(id) \rightarrow \mathbb{R}^+\) is a function described as follows: \((\forall i \in I(id))\), \(\text{CostPerInput}(id)\) gives the non-labor fixed cost for each input \(i\) processed by the service id.

**CostPerOutput**: \(O(id) \rightarrow \mathbb{R}^+\) is a function described as follows: \((\forall o \in O(id))\), \(\text{CostPerOutput}(id)\) gives the non-labor fixed cost for each output \(o\) processed by the service id.

**ServiceType(id)** is \(\text{InputDrivenAtomicService}\)

**DecisionVariables**, also denoted \(\text{DV}\), is a tuple \((On(id), \text{OutputThru}(id))\) where:
- \(\text{On}(id)\): \([1..NR+1] \rightarrow \{0,1\}\) is a function that determines whether the Service id is activated or not, for a particular release, i.e., \((\forall r \in [1..NR+1]), \text{On}(id)(r)\), also denoted by \(\text{On}(id,r)\) is as follows:
  \[ \text{On}(id,r) = \begin{cases} 1 & \text{if service id is activated in release r} \\ 0 & \text{otherwise} \end{cases} \]
- \(\text{OutputThru}(id): O(id) \times [1..NR+1] \rightarrow \mathbb{R}^+\) is a function described as follows: \((\forall o \in O(id), (\forall r \in [1..NR+1]), \text{OutputThru}(id)(o,r)\), also denoted \(\text{OutputThru}(id, o, r)\), gives the throughput of \(o\) (or quantity per day) during release \(r\) or the period after the last release.

**Computation**

1. **FeatureDependencyIsSatisfied** constraint:
   \[ \text{On}(id, r) = 1 \rightarrow \text{RBF}(id) \subseteq \text{Rsch}.\text{I}.\text{SoFarBF}(r) \]
   \(\forall r \in [1..NR+1]\)

2. **DeactivatedServicesIsSatisfied** constraint:
   \[ \forall o \in O(id), \forall r \in [1..NR+1], \text{On}(id,r) = 0 \rightarrow \text{OutputThru}(id, o, r) = 0 \]

3. Let \(\text{InputThru}(id): I(id) \times [1..NR+1] \rightarrow \mathbb{R}^+\) be a function described as follows: \((\forall i \in I(id), (\forall r \in [1..NR+1]), \text{InputThru}(id)(i,r)\), also denoted \(\text{InputThru}(id, i, r)\), gives the throughput of \(i\) (or quantity per day) during release \(r\) or the period after the last release, computed as \(\forall i \in I(id), \forall r \in [1..NR+1], \text{InputThru}(id)(i,r) = \sum_{o \in O(id)} (\text{O thru Ratio}(id, o, i) \times \text{OutputThru}(id, o, r))\)
4. Let \( T_{i}(\text{id}) \): \([1..NR + 1] \times \text{ServiceRoles}(\text{id}) \rightarrow \mathbb{R} \) be a function described as follows: \( \forall l \in \text{ServiceRoles}(\text{id}), r \in [\text{1..NR + 1}] \), \( T_{i}(\text{id})(l, r) \), also denoted \( T_{i}(\text{id}, l, r) \), gives the total duration per day for role \( l \) and release \( r \) (and the period after the last release), computed as:

\[
T_{i}(\text{id}, l, r) = \sum_{j \in \text{ServiceRoles}(\text{id})} (\text{RoleTimePerOI}(\text{id}, l, j) \times \text{InputThru}(\text{id}, j, r)) + \sum_{j \in \text{ServiceRoles}(\text{id})} (\text{RoleTimePerOI}(\text{id}, l, j) \times \text{OutputThru}(\text{id}, j, r))
\]

5. Let \( L_{i}(\text{id}) \): \([1..NR + 1] \rightarrow \mathbb{R} \) be a function described as follows: \( \forall r \in [\text{1..NR + 1}] \), \( L_{i}(\text{id})(r) \), also denoted \( L_{i}(\text{id}, r) \), gives the total labor cost per day during release \( r \), computed as follows:

\[
L_{i}(\text{id}, r) = \sum_{l \in \text{ServiceRoles}(\text{id})} (\text{BSN}.\text{Parm}.\text{Rate}(l) \times T_{i}(\text{id}, l, r))
\]

6. Let \( F_{i}(\text{id}) \): \([1..NR + 1] \rightarrow \mathbb{R} \) be a function described as follows: \( \forall r \in [\text{1..NR + 1}] \), \( F_{i}(\text{id})(r) \), also denoted \( F_{i}(\text{id}, r) \), gives the total non-labor cost per day for all input and output flows processed during release \( r \), computed as follows:

\[
F_{i}(\text{id}, r) = \sum_{j \in \text{ServiceRoles}(\text{id})} (\text{CostPerInput}(\text{id}) \times \text{InputThru}(\text{id}, j, r)) + \sum_{j \in \text{ServiceRoles}(\text{id})} (\text{CostPerOutput}(\text{id}) \times \text{OutputThru}(\text{id}, j, r))
\]

7. Let \( C_{i}(\text{id}) \): \([1..NR + 1] \rightarrow \mathbb{R} \) be a function described as follows: \( \forall r \in [\text{1..NR + 1}] \), \( C_{i}(\text{id})(r) \), also denoted \( C_{i}(\text{id}, r) \), gives the total cost per day during release \( r \), computed as follows:

\[
C_{i}(\text{id}, r) = L_{i}(\text{id}, r) + F_{i}(\text{id}, r) + \text{ServiceCostPerDay}(\text{id}) \times \text{On}(\text{id})
\]

**Constraints** are as follows:

1. \( \text{FeatureDependencyIsSatisfied} \) (see computation #1)
2. \( \text{DeactivatedServicesIsSatisfied} \) (see computation #2)
3. \( \text{InterfaceMetrics} \), also denoted \( \text{IM} \), is a tuple \( (\text{CostPerDay}(\text{id}), \text{InputThru}(\text{id}), \text{OutputThru}(\text{id})) \) where:
   - \( \text{CostPerDay}(\text{id}) \) is defined in computation #7.
   - \( \text{InputThru}(\text{id}) \) is defined in DecisionVariables.
   - \( \text{OutputThru}(\text{id}) \) is defined in computation #3.

**A8. Software Development Formalization**

Due to paper size restriction, this section is published in (Boccanera & Brodsky, 2022), section A8.

**A9. Optimization Formalization**

The formalizations in the previous sections are building blocks; we now use them to formulate the optimization of the NPV of the final BPN configuration. Given the top-level formal optimization model

\[
\text{RSch}\{\text{Parameters, DecisionVariables, Computation, Constraints, IM}\}
\]

the optimal NPV BPN, for a time horizon of \( t \) days, is:

\[
\text{NPV}_{\text{BPN}} = \text{Max} \text{ RSch.IM.TimeWindowNPV}(t) \text{ s.t. } \text{RSch.Constraints}
\]

Each of the six formal components implements constraints that are then aggregated under \( \text{RSch.Constraints} \).

The solution produces:

4. Optimal NPV of the business benefit
5. A release schedule, which is the result of the Solver instantiating \( \text{IBF}(r,f) \) and \( \text{ITF}(r,f) \).
6. The service network configuration at the end of each release, which is captured by the instantiated variables \( \text{On}(s,r) \).